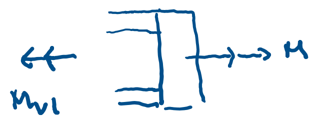
a)



Jämvikt:

$\rightarrow\!\!\rightarrow: \; M - M_{v1} = 0$

$\Rightarrow M_{v1} = M$

största spänning (3.10)

$$\max(|\tau|) = \frac{M_{v1} \cdot D_1/2}{K_{v1}} = (3.11) = \frac{M_{v1}}{W_{v1}} = \frac{M_{v1}}{\frac{\pi}{2}\left((D_1/2)^3 - \underbrace{\frac{(d_1/2)^4}{D_1/2}}_{(d_1/2)^3 \frac{d_1}{D_1}}\right)}$$

med $d_1 = 4D_1/5$, $\max(|\tau|) = \tau_s$, $M_{v1} = M \Rightarrow$

$$\tau_s = \frac{M \cdot 2^4}{\pi \cdot D_1^3 \cdot (1 - (4/5)^4)} \Rightarrow D_1 = \left(\frac{M \cdot 2^4}{\pi \cdot (1-(4/5)^4) \cdot \tau_s}\right)^{1/3}$$

med $\tau_s = 250$ MPa, $M = 10^6$ Nmm $\Rightarrow \underline{\underline{D_1 \approx 32{,}6 \text{ mm}}}$

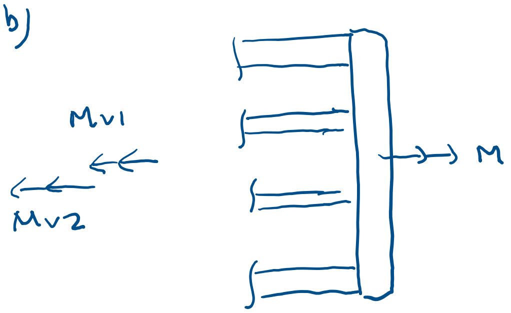
Vridningen blir $\varphi = \dfrac{M_{v1} \cdot \ell}{G\, K_{v1}}$

där $\ell = 1000$ mm, $G = \dfrac{E}{2(1+\nu)} = \dfrac{200 \cdot 10^3}{2 \cdot 1{,}3} \approx 77 \cdot 10^3$ MPa

$$K_{v1} = \frac{\pi}{2}\left((D_1/2)^4 - (\underbrace{d_1/2}_{D_1/2 \cdot 4/5})^4\right) \approx 6.51 \cdot 10^4 \text{ mm}^4$$

$$\Rightarrow \varphi = \frac{10^6 \cdot 10^3}{77 \cdot 10^3 \cdot 6{,}51 \cdot 10^4} \approx 0{,}2 \text{ rad} \approx 11{,}4^\circ$$

b)



$\rightarrow\!\!\rightarrow: \; M - M_{v1} - M_{v2} = 0 \quad (*)$

Vridning $\quad \varphi_1 = \dfrac{M_{v1} \cdot \ell}{G\, K_{v1}} \qquad \varphi_2 = \dfrac{M_{v2} \cdot \ell}{G\, K_{v2}}$

Deformationssamband $\quad \varphi_1 = \varphi_2 \Rightarrow \dfrac{M_{v1}}{K_{v1}} = \dfrac{M_{v2}}{K_{v2}} \quad (**)$

insatt i $(*)$ $\Rightarrow$

$$M - M_{v1} - M_{v1}\frac{K_{v2}}{K_{v1}} = 0 \Rightarrow M_{v1} = \frac{M \cdot K_{v1}}{K_{v1} + K_{v2}}$$

$$(**) \Rightarrow \quad M_{v2} = \frac{M \cdot K_{v2}}{K_{v1} + K_{v2}}$$

$K_{v1} = \{\text{enligt a)}\} \approx 6.51 \cdot 10^4 \text{ mm}^4$

$$K_{v2} = \pi/2 \; \Big(\underbrace{(D_2/2)}_{D_1/4}{}^4 - \underbrace{(d_2/2)}_{d_1/4 \,=\, D_1/4 \cdot 4/5}{}^4 \Big) \approx 4.07 \cdot 10^3 \text{ mm}^4 = K_{v1}/16$$

största skjuvspänningar $\quad \tau_{max,1} = \dfrac{M_{v1} \cdot D_1/2}{K_{v1}} = \dfrac{M \cdot D_1/2}{K_{v1} + K_{v2}} \approx 235 \text{ MPa}$

$$\tau_{max,2} = \frac{M_{v2} \cdot D_2/2}{K_{v2}} = \frac{M \cdot D_2/2}{K_{v1} + K_{v2}} \approx 118 \text{ MPa}$$

Vridningen $\quad \varphi = \varphi_1 = \dfrac{M_{v1} \cdot \ell}{G\, K_{v1}} = \dfrac{M_{v2} \cdot \ell}{G\, K_{v2}} \approx \dfrac{M \cdot \ell}{G\,(K_{v1} + K_{v2})} \approx 0{,}188 \text{ rad}$

$\approx 10{,}8° \;//$

a) Enl (6.5)

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ \nu & 1-\nu & \nu & 0 & 0 & 0\\ \nu & \nu & 1-\nu & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

med $E = 200\cdot 10^3$ MPa, $\nu = 0{,}3$

$\varepsilon_x = 0{,}002$, $\varepsilon_y = -0{,}001$, $\varepsilon_z = 0{,}001$

$\gamma_{xy} = 0{,}005$, $\gamma_{yz} = \gamma_{xz} = 0$

fås $\sigma_x \approx 538$ MPa, $\sigma_y \approx 76{,}9$ MPa, $\sigma_z \approx 385$ MPa

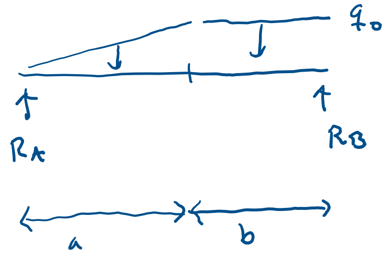
$\tau_{xy} \approx 385$ MPa, $\tau_{xz} = \tau_{yz} = 0$ //

b) Med $S = \begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{bmatrix}$ och Matlabkod 4.7.1 resp 4.7.2

fås $\sigma_e^{vM} \approx 781$ MPa

$\sigma_e^{T} \approx 897$ MPa //

Stödreaktioner



$$\uparrow: \; R_A + R_B - q_0 \cdot a/2 - q_0 \cdot b = 0$$

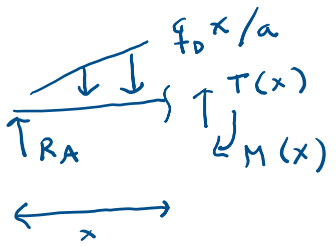
$$A\circlearrowright: \; -R_B \cdot (a+b) + q_0 \cdot a/2 \cdot 2a/3 + q_0 \cdot b \cdot \left(a + \frac{b}{2}\right) = 0$$

$$\Rightarrow \begin{cases} R_B = \dfrac{q_0 \left[a^2/3 + b\,(a + b/2) \right]}{a+b} \\[2ex] R_A = q_0 \left[a/2 + b \right] - R_B \end{cases}$$

$$\Rightarrow R_A \approx 722 \text{ N}, \quad R_B \approx 1{,}28 \text{ kN}$$

Snitta

$0 \leq x \leq a$



$$\uparrow: \; R_A - (q_0 x/a)\,\frac{1}{2} \cdot x + T(x) = 0$$

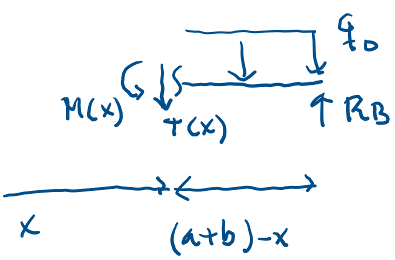
$$\Rightarrow T(x) = q_0 x^2/(2a) - R_A$$

$$x\circlearrowright: \; R_A \cdot x - (q_0 x/a)\,\frac{1}{2}\,x \cdot \frac{1}{3}\,x + M(x) = 0$$

$$\Rightarrow M(x) = q_0 x^3/(6a) - R_A \cdot x$$

$$// \; T(x) = M'(x) = 0 \text{ då } x = \sqrt{\frac{R_A \cdot 2a}{q_0}}$$

$a \leq x \leq (a+b)$



$$\uparrow: \; R_B - q_0\,(a+b-x) - T(x) = 0$$

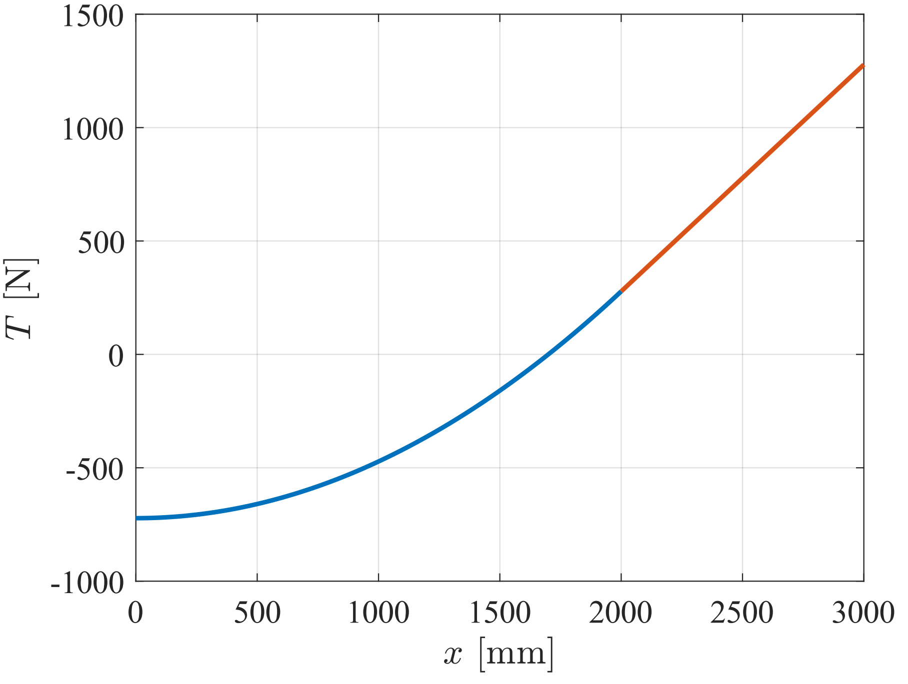
$$\Rightarrow T(x) = R_B - q_0\,(a+b-x)$$

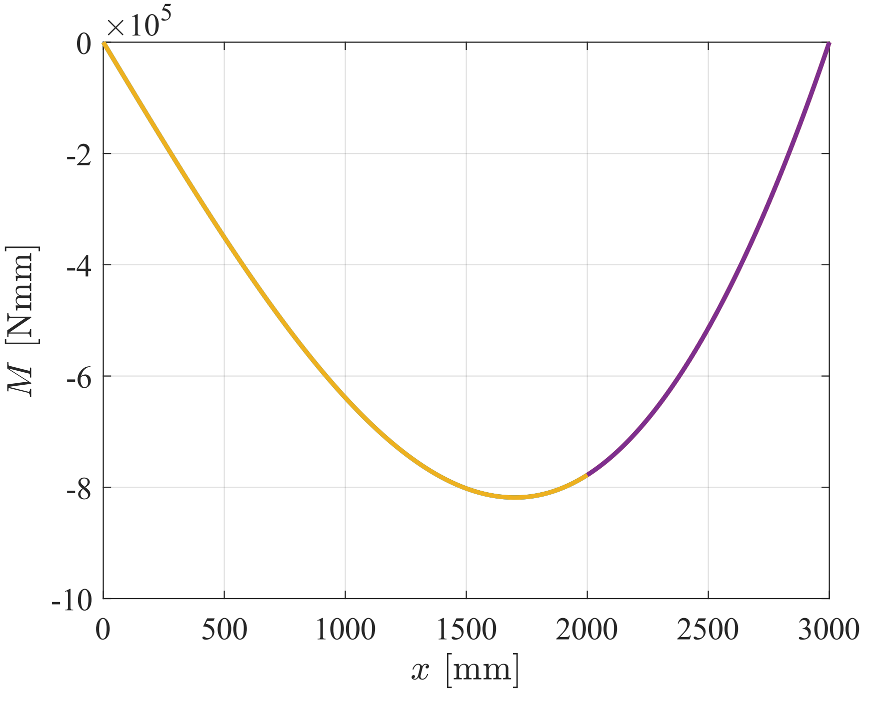
$$x\circlearrowright: \; -R_B\,(a+b-x) + q_0\,(a+b-x)\,\frac{a+b-x}{2} - M(x) = 0$$

$$\Rightarrow M(x) = R_B\,(x-a-b) + q_0\,(a+b-x)^2/2$$

```
x=linspace(0,a,100);
T1=q0*x.^2/(2*a)-RA; M1=q0*x.^3/(6*a)-RA*x;
figure(1)
plot(x,T1,'linewidth',2)
set(gca,'FontSize',14,'fontname','Times New Roman')
xlabel('$x$ [mm]','FontSize',16,'interpreter','latex')
ylabel('$T$ [N]','FontSize',16,'interpreter','latex')
hold on
x=linspace(a,a+b,100);
T2=RB-q0*(a+b-x); M2=RB*(x-a-b)+q0*(a+b-x).^2/2;
plot(x,T2,'linewidth',2)

x=linspace(0,a,100);
figure(2)
plot(x,M1,'linewidth',2)
set(gca,'FontSize',14,'fontname','Times New Roman')
xlabel('$x$ [mm]','FontSize',16,'interpreter','latex')
ylabel('$M$ [Nmm]','FontSize',16,'interpreter','latex')
hold on
x=linspace(a,a+b,100);
plot(x,M2,'linewidth',2)
g
```





Största (till belopp) böjmoment

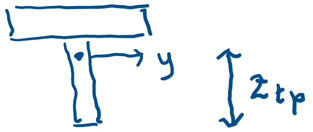
$x = \sqrt{\frac{R_A \cdot 2a}{q_0}}$

≈ 1700 mm

$\Rightarrow$

$|M|_{max} \approx$

$8{,}18 \cdot 10^5$ Nmm



Ytcentrum

$$z_{tp} = \frac{b_1 h_1 \cdot h_1/2 + b_2 h_2 \cdot (h_1 + h_2/2)}{b_1 h_1 + b_2 h_2} \simeq$$

$$\simeq 38{,}75 \text{ mm}$$

Yttröghetsmomentet (Steiners sats)

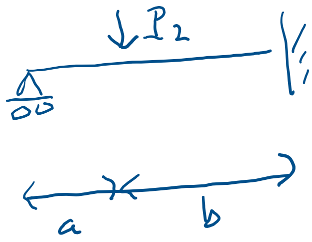
$$I_y = \frac{b_1 h_1^3}{12} + b_1 h_1 (z_{tp} - h_1/2)^2 +$$

$$+ \frac{b_2 h_2^3}{12} + b_2 h_2 (h_1 + h_2/2 - z_{tp})^2 \simeq 1{,}471 \cdot 10^5 \text{ mm}^4$$

Största normalspänningen

$$|\sigma|_{max} = \frac{|M|_{max} \cdot z_{tp}}{I_y} \simeq 216 \text{ MPa} //$$

Enl FS 6.5 fås

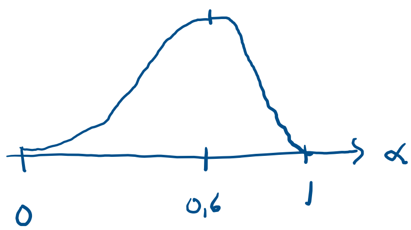


$$p_2 = \frac{P_2\, a^2 b^3}{12\, EI\, \ell^2}\left[4 - \frac{b}{\ell}\right]$$

eftersom spegelvänd $b = \alpha\ell$, $a = (1-\alpha)\ell$ $\Rightarrow$ utböjning vid lasten

$$p = \frac{P(1-\alpha)^2\alpha^3 \cdot \ell^3}{12 \cdot EI}\left[4-\alpha\right] = \frac{P\ell^3}{12\,EI}\underbrace{(1-\alpha)^2\alpha^3(4-\alpha)}_{f(\alpha)}$$

plotta $f(\alpha)$ i Matlab



maxvärde
fås enligt
Matlabgraf vid $\alpha \simeq 0{,}59$

Alternativ:

kolla ändpunkter

$f(0) = 0$, $f(1) = 0$

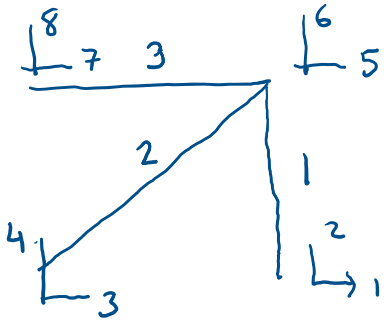
$$f'(\alpha) = -2(1-\alpha)\alpha^3(4-\alpha) + (1-\alpha)^2 3\alpha^2(4-\alpha) - (1-\alpha)^2\alpha^3 =$$

$$= (1-\alpha)\alpha^2\left[-2\alpha(4-\alpha) + (1-\alpha)3(4-\alpha) - (1-\alpha)\alpha\right] = 0$$

$$\Rightarrow -8\alpha + 2\alpha^2 + 3\cdot(4 - 5\alpha + \alpha^2) - \alpha + \alpha^2 = 0 \Rightarrow$$

$$\Rightarrow 6\alpha^2 - 24\alpha + 12 = 0 \Rightarrow \alpha^2 - 4\alpha + 2 = 0 \Rightarrow$$

$\alpha = 2 \pm \sqrt{4-2} = 2 \pm \sqrt{2}$ men $0 \le \alpha \le 1 \Rightarrow \alpha \simeq 2 - \sqrt{2} \simeq 0{,}59$



element och frihetsgrader

Se Matlabkod nedan

Utan element 3 fås $u_5 \approx 0{,}67$ mm

$u_6 \approx -0{,}087$ mm

Med element 3 fås $u_5 \approx 0{,}14$ mm

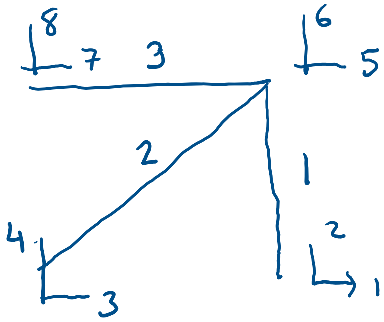
$u_6 \approx 0{,}05$ mm

Inre normalkrafter

utan element 3 : $\begin{cases} N_1 \approx -3{,}66 \text{ kN} \\ N_2 \approx 12{,}2 \text{ kN} \end{cases}$

Euler 2 $\quad P_{kr} = \dfrac{\pi^2 EI}{\ell^2}$

$\Rightarrow$ säkerhetsfaktor $\approx 1{,}8$ //



element och frihetsgrader

Se Matlabkod nedan

Utan element 3 fås $u_5 \approx 0{,}67$ mm

$u_6 \approx -0{,}087$ mm

Med element 3 fås $u_5 \approx 0{,}14$ mm

$u_6 \approx 0{,}05$ mm

Inre normalkrafter

utan element 3 : $\begin{cases} N_1 \approx -3{,}66 \text{ kN} \\ N_2 \approx 12{,}2 \text{ kN} \end{cases}$

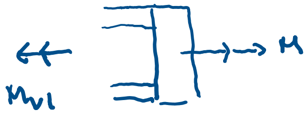
Euler 2 $\quad P_{kr} = \dfrac{\pi^2 EI}{L^2}$

$\Rightarrow$ säkerhetsfaktor $\approx 1{,}8$ //

```
clear all
syms a1 a2 a3 a4 a5 a6 P1 P2 P3 P4 P5 P6
%givna storheter
P=10e3;
Em=210e3; A=100;
EA=Em*A; L=500;
%givet
a1=0; a2=0; a3=0; a4=0;
P5=P*cos(pi/6); P6=P*sin(pi/6);
%definiera avektor pvektor
avektor = [a1; a2; a3; a4; a5; a6];
Pvektor = [P1; P2; P3; P4; P5; P6];
%elementstyvheter
%%%Element 1:
L1=L; EA1=EA; alpha1=pi/2; c=cos(alpha1);s=sin(alpha1);
Ke1=EA1/L1*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris1=sym(zeros(6,6));
Kmatris1([1 2 5 6],[1 2 5 6]) = Ke1;
%%%Element 2:
L2=L*sqrt(2); EA2=EA; alpha2=pi/4; c=cos(alpha2);s=sin(alpha2);
Ke2=EA2/L2*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris2=sym(zeros(6,6));
Kmatris2([3 4 5 6],[3 4 5 6]) = Ke2;
%addera
Ktot=Kmatris1+Kmatris2;
%Lös de obekanta
Sol = solve(double(Ktot)*avektor==Pvektor,[a5,a6,P1,P2,P3,P4])
%Skriv ut resultat
double(Sol.a5)
double(Sol.a6)
double(Sol.P1)
double(Sol.P2)
double(Sol.P3)
double(Sol.P4)
%förlängningarna av stängerna enl s.208
ubar1=[cos(alpha1) sin(alpha1) 0 0; 0 0 cos(alpha1) sin(alpha1)]*[a1; a2; Sol.a5; Sol.a6]
delta1=ubar1(2)-ubar1(1);
eps1=delta1/L1;
N1=double(Em*eps1*A)
ubar2=[cos(alpha2) sin(alpha2) 0 0; 0 0 cos(alpha2) sin(alpha2)]*[a3; a4; Sol.a5; Sol.a6]
delta2=ubar2(2)-ubar2(1);
eps2=delta2/L2;
N2=double(Em*eps2*A)
%beräkning av d: pi d^2/4=A och sedan av yttröghetsmoment (enl FS)
d=sqrt( A*4/pi); Iy=pi*(d/2)^4/4;
%Knäckning enligt Euler 2 av stång 1:
Pkr=pi^2*Em*Iy/L1^2
%säkerhetsfaktor mot knäckning
Pkr/(-N1)
```

```
clear all
syms a1 a2 a3 a4 a5 a6 a7 a8 P1 P2 P3 P4 P5 P6 P7 P8
%givna storheter
P=10e3;
Em=210e3;
EA=Em*100; L=500;
a1=0; a2=0; a3=0; a4=0; a7=0; a8=0;
P5=P*cos(pi/6); P6=P*sin(pi/6);
%definiera avektor pvektor
avektor = [a1; a2; a3; a4; a5; a6; a7; a8];
Pvektor = [P1; P2; P3; P4; P5; P6; P7; P8];
%elementstyvheter
%%%Element 1:
L1=L; EA1=EA; alpha1=pi/2; c=cos(alpha1);s=sin(alpha1);
Ke1=EA1/L1*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris1=sym(zeros(8,8));
Kmatris1([1 2 5 6],[1 2 5 6]) = Ke1;
%%%Element 2:
L2=L*sqrt(2); EA2=EA; alpha2=pi/4; c=cos(alpha2);s=sin(alpha2);
Ke2=EA2/L2*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris2=sym(zeros(8,8));
Kmatris2([3 4 5 6],[3 4 5 6]) = Ke2;
%%%Element 3:
L3=L; EA3=EA; alpha3=0; c=cos(alpha3);s=sin(alpha3);
Ke3=EA3/L3*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris3=sym(zeros(8,8));
Kmatris3([7 8 5 6],[7 8 5 6]) = Ke3;
%addera
Ktot=Kmatris1+Kmatris2+Kmatris3;
%Lös de obekanta
Sol = solve(double(Ktot)*avektor==Pvektor,[a5,a6,P1,P2,P3, P4, P7,P8])
%Skriv ut resultat
double(Sol.a5)
double(Sol.a6)
double(Sol.P1)
double(Sol.P2)
double(Sol.P3)
double(Sol.P4)
double(Sol.P7)
double(Sol.P8)
```

a)



Jämvikt:

$\rightarrow\!\!\rightarrow: \; M - M_{v1} = 0$

$\Rightarrow M_{v1} = M$

största spänning (3.10)

$$\max(|\tau|) = \frac{M_{v1}\cdot D_1/2}{K_{v1}} = (3.11) = \frac{M_{v1}}{W_{v1}} = \frac{M_{v1}}{\frac{\pi}{2}\left((D_1/2)^3 - \underbrace{\frac{(d_1/2)^4}{D_1/2}}_{(d_1/2)^3 \frac{d_1}{D_1}}\right)}$$

med $d_1 = 4D_1/5$, $\max(|\tau|) = \tau_s$, $M_{v1} = M \Rightarrow$

$$\tau_s = \frac{M\cdot 2^4}{\pi\cdot D_1^3\cdot\left(1-(4/5)^4\right)} \Rightarrow D_1 = \left(\frac{M\cdot 2^4}{\pi\cdot\left(1-(4/5)^4\right)\cdot\tau_s}\right)^{1/3}$$

med $\tau_s = 250$ MPa, $M = 10^6$ Nmm $\Rightarrow$ $\underline{\underline{D_1 \approx 32{,}6 \text{ mm}}}$

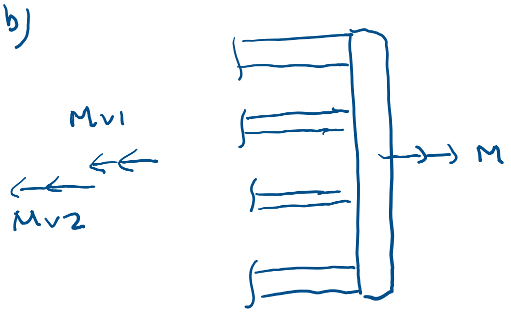
Vridningen blir $\varphi = \dfrac{M_{v1}\cdot \ell}{G\, K_{v1}}$

där $\ell = 1000$ mm, $G = \dfrac{E}{2(1+\nu)} = \dfrac{200\cdot 10^3}{2\cdot 1{,}3} \approx 77\cdot 10^3$ MPa

$$K_{v1} = \frac{\pi}{2}\left((D_1/2)^4 - (\underbrace{d_1/2}_{D_1/2\cdot 4/5})^4\right) \approx 6.51\cdot 10^4 \text{ mm}^4$$

$$\Rightarrow \varphi = \frac{10^6\cdot 10^3}{77\cdot 10^3\cdot 6{,}51\cdot 10^4} \approx 0{,}2 \text{ rad} \approx 11{,}4^\circ$$

b)



$\rightarrow\!\!\rightarrow: M - M_{v1} - M_{v2} = 0 \quad (*)$

Vridning $\varphi_1 = \dfrac{M_{v1} \cdot \ell}{G\, K_{v1}} \qquad \varphi_2 = \dfrac{M_{v2} \cdot \ell}{G\, K_{v2}}$

Deformationssamband $\varphi_1 = \varphi_2 \Rightarrow \dfrac{M_{v1}}{K_{v1}} = \dfrac{M_{v2}}{K_{v2}} \quad (**)$

insatt i $(*)$ $\Rightarrow$

$$M - M_{v1} - M_{v1}\frac{K_{v2}}{K_{v1}} = 0 \Rightarrow M_{v1} = \frac{M \cdot K_{v1}}{K_{v1} + K_{v2}}$$

$$(**) \Rightarrow \quad M_{v2} = \frac{M \cdot K_{v2}}{K_{v1} + K_{v2}}$$

$K_{v1} = \{\text{enligt a)}\} \approx 6.51 \cdot 10^4 \text{ mm}^4$

$K_{v2} = \pi/2 \left((\underbrace{D_2/2}_{D_1/4})^4 - (\underbrace{d_2/2}_{d_1/4 = D_1/4 \cdot 4/5})^4 \right) \approx 4.07 \cdot 10^3 \text{ mm}^4 = K_{v1}/16$

största skjuvspänningar $\tau_{max,1} = \dfrac{M_{v1} \cdot D_1/2}{K_{v1}} = \dfrac{M \cdot D_1/2}{K_{v1} + K_{v2}} \approx 235 \text{ MPa}$

$$\tau_{max,2} = \frac{M_{v2} \cdot D_2/2}{K_{v2}} = \frac{M \cdot D_2/2}{K_{v1} + K_{v2}} \approx 118 \text{ MPa}$$

Vridningen $\varphi = \varphi_1 = \dfrac{M_{v1} \cdot \ell}{G\, K_{v1}} = \dfrac{M_{v2} \cdot \ell}{G\, K_{v2}} \approx \dfrac{M \cdot \ell}{G\,(K_{v1} + K_{v2})} \approx 0{,}188 \text{ rad}$

$\approx 10{,}8^\circ$ //

a) Enl (6.5)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

med $E = 200 \cdot 10^3$ MPa, $\nu = 0{,}3$

$\varepsilon_x = 0{,}002$, $\varepsilon_y = -0{,}001$, $\varepsilon_z = 0{,}001$

$\gamma_{xy} = 0{,}005$, $\gamma_{yz} = \gamma_{xz} = 0$

fås $\sigma_x \approx 538$ MPa, $\sigma_y \approx 76{,}9$ MPa, $\sigma_z \approx 385$ MPa

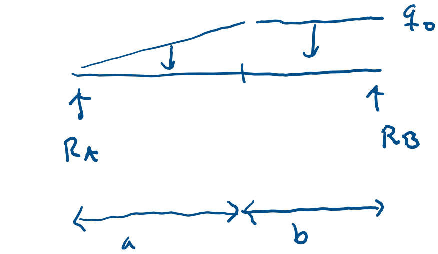
$\tau_{xy} \approx 385$ MPa, $\tau_{xz} = \tau_{yz} = 0$ //

b) Med $S = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$ och Matlabkod 4.7.1 resp 4.7.2

fås $\sigma_e^{vM} \approx 781$ MPa

$\sigma_e^{T} \approx 897$ MPa //

Stödreaktioner



$\uparrow: \; R_A + R_B - q_0 \cdot a/2 - q_0 \cdot b = 0$

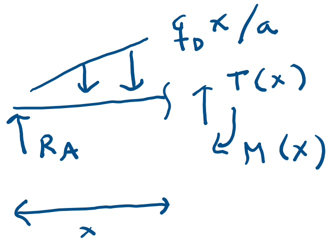
$A\curvearrowright: \; -R_B \cdot (a+b) + q_0 \cdot a/2 \cdot 2a/3 + q_0 \cdot b \cdot \left(a + \frac{b}{2}\right) = 0$

$$\Rightarrow \begin{cases} R_B = \dfrac{q_0\left[a^2/3 + b\,(a+b/2)\right]}{a+b} \\ R_A = q_0\left[a/2 + b\right] - R_B \end{cases}$$

$\Rightarrow R_A \approx 722 \text{ N}, \quad R_B \approx 1{,}28 \text{ kN}$

Snitta

$0 \le x \le a$



$\uparrow: \; R_A - (q_0 x/a)\frac{1}{2} \cdot x + T(x) = 0$

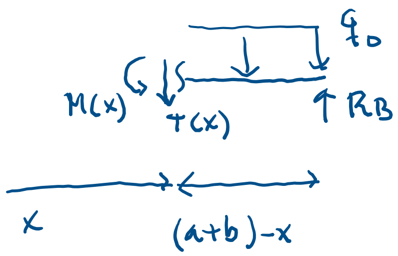
$\Rightarrow T(x) = q_0 x^2/(2a) - R_A$

$x\curvearrowright: \; R_A \cdot x - (q_0 x/a)\frac{1}{2} x \cdot \frac{1}{3} x + M(x) = 0$

$\Rightarrow M(x) = q_0 x^3/(6a) - R_A \cdot x$

// $T(x) = M'(x) = 0$ då $x = \sqrt{\dfrac{R_A \cdot 2a}{q_0}}$

$a \le x \le (a+b)$



$\uparrow: \; R_B - q_0(a+b-x) - T(x) = 0$

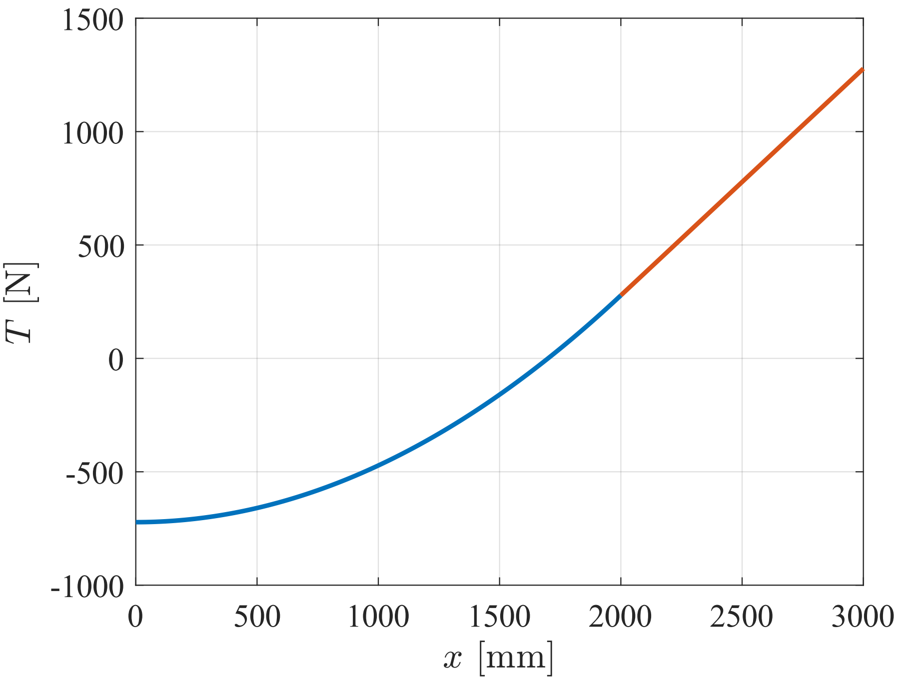
$\Rightarrow T(x) = R_B - q_0(a+b-x)$

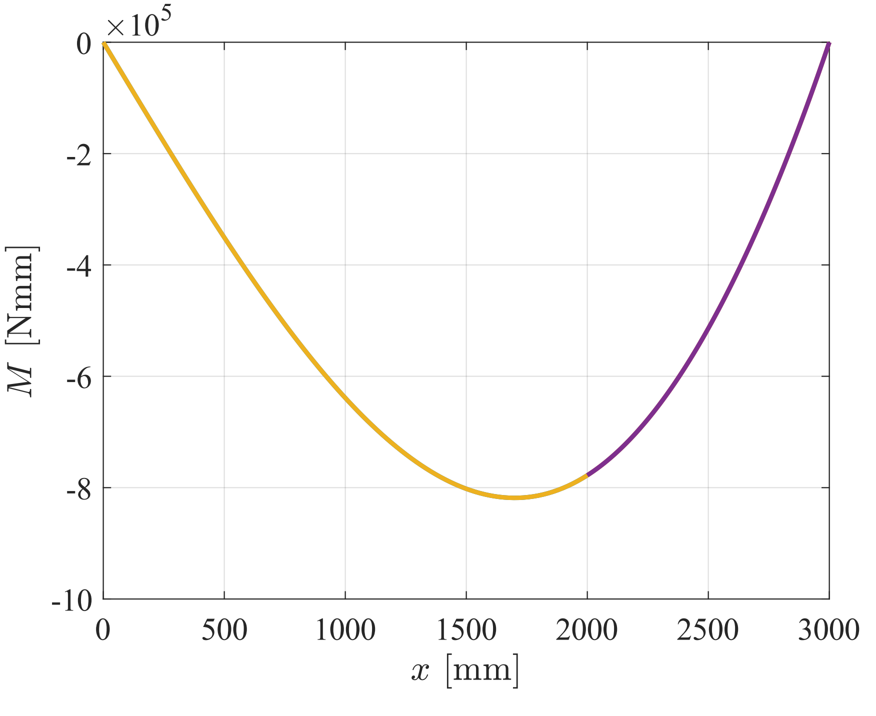
$x\curvearrowright: \; -R_B(a+b-x) + q_0(a+b-x)\dfrac{a+b-x}{2} - M(x) = 0$

$\Rightarrow M(x) = R_B(x-a-b) + q_0(a+b-x)^2/2$

```
x=linspace(0,a,100);
T1=q0*x.^2/(2*a)-RA; M1=q0*x.^3/(6*a)-RA*x;
figure(1)
plot(x,T1,'linewidth',2)
set(gca,'FontSize',14,'fontname','Times New Roman')
xlabel('$x$ [mm]','FontSize',16,'interpreter','latex')
ylabel('$T$ [N]','FontSize',16,'interpreter','latex')
hold on
x=linspace(a,a+b,100);
T2=RB-q0*(a+b-x); M2=RB*(x-a-b)+q0*(a+b-x).^2/2;
plot(x,T2,'linewidth',2)

x=linspace(0,a,100);
figure(2)
plot(x,M1,'linewidth',2)
set(gca,'FontSize',14,'fontname','Times New Roman')
xlabel('$x$ [mm]','FontSize',16,'interpreter','latex')
ylabel('$M$ [Nmm]','FontSize',16,'interpreter','latex')
hold on
x=linspace(a,a+b,100);
plot(x,M2,'linewidth',2)
g
```





Största (till belopp) böjmoment

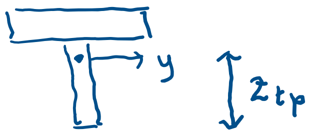
$x = \sqrt{\frac{R_A \cdot 2a}{q_0}}$

≈ 1700 mm

$\Rightarrow$

$|M|_{max} \approx$

$8{,}18 \cdot 10^5$ Nmm



Ytcentrum

$$z_{tp} = \frac{b_1 h_1 \cdot h_1/2 + b_2 h_2 \cdot (h_1 + h_2/2)}{b_1 h_1 + b_2 h_2} \approx$$

$$\approx 38,75 \text{ mm}$$

Yttröghetsmomentet (Steiners sats)

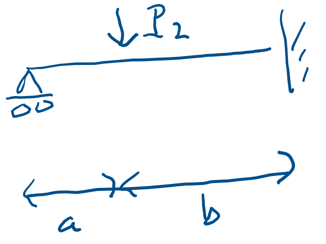
$$I_y = \frac{b_1 h_1^3}{12} + b_1 h_1 (z_{tp} - h_1/2)^2 +$$

$$+ \frac{b_2 h_2^3}{12} + b_2 h_2 (h_1 + h_2/2 - z_{tp})^2 \approx 1,471 \cdot 10^5 \text{ mm}^4$$

Största normalspänningen

$$|\sigma|_{max} = \frac{|M|_{max} \cdot z_{tp}}{I_y} \approx 216 \text{ MPa} //$$

Enl FS 6.5 fås

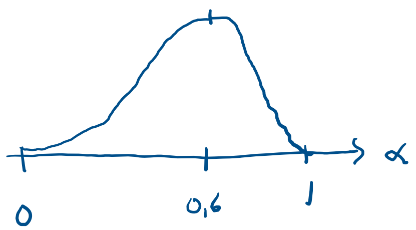


$$p_2 = \frac{P_2\, a^2 b^3}{12\, EI\, \ell^2}\left[4 - \frac{b}{\ell}\right]$$

eftersom spegelvänd $b = \alpha \ell$, $a = (1-\alpha)\ell \Rightarrow$ utböjning vid lasten

$$p = \frac{P\,(1-\alpha)^2 \alpha^3 \cdot \ell^3}{12 \cdot EI}\left[4 - \alpha\right] = \frac{P\ell^3}{12\, EI}\underbrace{(1-\alpha)^2 \alpha^3 (4-\alpha)}_{f(\alpha)}$$

plotta $f(\alpha)$ i Matlab



maxvärde
fås enligt
Matlabgraf vid $\alpha \simeq 0{,}59$

Alternativ:

kolla ändpunkter

$f(0) = 0$, $f(1) = 0$

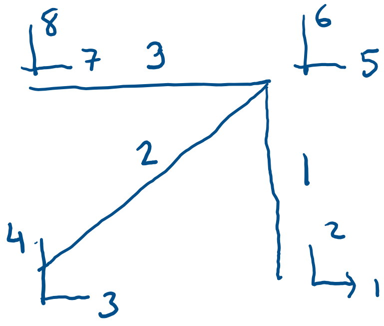
$$f'(\alpha) = -2(1-\alpha)\alpha^3(4-\alpha) + (1-\alpha)^2 3\alpha^2 (4-\alpha) - (1-\alpha)^2\alpha^3 =$$

$$= (1-\alpha)\alpha^2\left[-2\alpha(4-\alpha) + (1-\alpha)3(4-\alpha) - (1-\alpha)\alpha\right] = 0$$

$$\Rightarrow -8\alpha + 2\alpha^2 + 3\cdot(4 - 5\alpha + \alpha^2) - \alpha + \alpha^2 = 0 \Rightarrow$$

$$\Rightarrow 6\alpha^2 - 24\alpha + 12 = 0 \Rightarrow \alpha^2 - 4\alpha + 2 = 0 \Rightarrow$$

$\alpha = 2 \pm \sqrt{4-2} = 2 \pm \sqrt{2}$ men $0 \le \alpha \le 1 \Rightarrow \alpha \simeq 2 - \sqrt{2} \simeq 0{,}59$



element och frihetsgrader

Se Matlabkod nedan

Utan element 3 fås $u_5 \approx 0{,}67$ mm

$u_6 \approx -0{,}087$ mm

Med element 3 fås $u_5 \approx 0{,}14$ mm

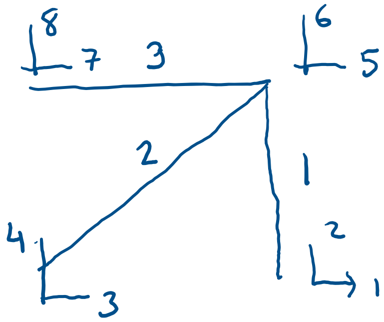
$u_6 \approx 0{,}05$ mm

Inre normalkrafter

utan element 3 : $\begin{cases} N_1 \approx -3{,}66 \text{ kN} \\ N_2 \approx 12{,}2 \text{ kN} \end{cases}$

Euler 2 $\quad P_{kr} = \dfrac{\pi^2 EI}{\ell^2}$

$\Rightarrow$ säkerhetsfaktor $\approx 1{,}8$ //



element och frihetsgrader

Se Matlabkod nedan

Utan element 3 fås $u_5 \approx 0{,}67$ mm

$u_6 \approx -0{,}087$ mm

Med element 3 fås $u_5 \approx 0{,}14$ mm

$u_6 \approx 0{,}05$ mm

Inre normalkrafter

utan element 3 : $\begin{cases} N_1 \approx -3{,}66 \text{ kN} \\ N_2 \approx 12{,}2 \text{ kN} \end{cases}$

Euler 2 $\quad P_{kr} = \dfrac{\pi^2 EI}{\ell^2}$

$\Rightarrow$ säkerhetsfaktor $\approx 1{,}8$ //

```
clear all
syms a1 a2 a3 a4 a5 a6 P1 P2 P3 P4 P5 P6
%givna storheter
P=10e3;
Em=210e3; A=100;
EA=Em*A; L=500;
%givet
a1=0; a2=0; a3=0; a4=0;
P5=P*cos(pi/6); P6=P*sin(pi/6);
%definiera avektor pvektor
avektor = [a1; a2; a3; a4; a5; a6];
Pvektor = [P1; P2; P3; P4; P5; P6];
%elementstyvheter
%%%Element 1:
L1=L; EA1=EA; alpha1=pi/2; c=cos(alpha1);s=sin(alpha1);
Ke1=EA1/L1*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris1=sym(zeros(6,6));
Kmatris1([1 2 5 6],[1 2 5 6]) = Ke1;
%%%Element 2:
L2=L*sqrt(2); EA2=EA; alpha2=pi/4; c=cos(alpha2);s=sin(alpha2);
Ke2=EA2/L2*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris2=sym(zeros(6,6));
Kmatris2([3 4 5 6],[3 4 5 6]) = Ke2;
%addera
Ktot=Kmatris1+Kmatris2;
%Lös de obekanta
Sol = solve(double(Ktot)*avektor==Pvektor,[a5,a6,P1,P2,P3,P4])
%Skriv ut resultat
double(Sol.a5)
double(Sol.a6)
double(Sol.P1)
double(Sol.P2)
double(Sol.P3)
double(Sol.P4)
%förlängningarna av stängerna enl s.208
ubar1=[cos(alpha1) sin(alpha1) 0 0; 0 0 cos(alpha1) sin(alpha1)]*[a1; a2; Sol.a5; Sol.a6]
delta1=ubar1(2)-ubar1(1);
eps1=delta1/L1;
N1=double(Em*eps1*A)
ubar2=[cos(alpha2) sin(alpha2) 0 0; 0 0 cos(alpha2) sin(alpha2)]*[a3; a4; Sol.a5; Sol.a6]
delta2=ubar2(2)-ubar2(1);
eps2=delta2/L2;
N2=double(Em*eps2*A)
%beräkning av d: pi d^2/4=A och sedan av yttröghetsmoment (enl FS)
d=sqrt( A*4/pi); Iy=pi*(d/2)^4/4;
%Knäckning enligt Euler 2 av stång 1:
Pkr=pi^2*Em*Iy/L1^2
%säkerhetsfaktor mot knäckning
Pkr/(-N1)
```

```
clear all
syms a1 a2 a3 a4 a5 a6 a7 a8 P1 P2 P3 P4 P5 P6 P7 P8
%givna storheter
P=10e3;
Em=210e3;
EA=Em*100; L=500;
a1=0; a2=0; a3=0; a4=0; a7=0; a8=0;
P5=P*cos(pi/6); P6=P*sin(pi/6);
%definiera avektor pvektor
avektor = [a1; a2; a3; a4; a5; a6; a7; a8];
Pvektor = [P1; P2; P3; P4; P5; P6; P7; P8];
%elementstyvheter
%%%Element 1:
L1=L; EA1=EA; alpha1=pi/2; c=cos(alpha1);s=sin(alpha1);
Ke1=EA1/L1*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris1=sym(zeros(8,8));
Kmatris1([1 2 5 6],[1 2 5 6]) = Ke1;
%%%Element 2:
L2=L*sqrt(2); EA2=EA; alpha2=pi/4; c=cos(alpha2);s=sin(alpha2);
Ke2=EA2/L2*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris2=sym(zeros(8,8));
Kmatris2([3 4 5 6],[3 4 5 6]) = Ke2;
%%%Element 3:
L3=L; EA3=EA; alpha3=0; c=cos(alpha3);s=sin(alpha3);
Ke3=EA3/L3*[ c^2 c*s -c^2 -c*s;
c*s s^2 -c*s -s^2;
-c^2 -c*s c^2 c*s;
-c*s -s^2 c*s s^2];
Kmatris3=sym(zeros(8,8));
Kmatris3([7 8 5 6],[7 8 5 6]) = Ke3;
%addera
Ktot=Kmatris1+Kmatris2+Kmatris3;
%Lös de obekanta
Sol = solve(double(Ktot)*avektor==Pvektor,[a5,a6,P1,P2,P3, P4, P7,P8])
%Skriv ut resultat
double(Sol.a5)
double(Sol.a6)
double(Sol.P1)
double(Sol.P2)
double(Sol.P3)
double(Sol.P4)
double(Sol.P7)
double(Sol.P8)
```