Fluid Dynamik

TME055, 21 December 2007

Typgodkänd miniräknare får användas.

1 Flersvarsfrågor

(*3 poäng för rätt svar, -1 poäng för varje fel svar*)

1. Behovet av att använda sig av *stresser* i *kontinuummekaniken* är en konsekvens av

   a konceptet existerar redan i mekaniken hos de idividuella molekylerna och bevaras enkelt i kontinuummekanik

   b Newtons lag kan inte beskriva rörelsen hos de individuella molekylerna korrekt

   c efter medelvärdering över de individuella molekylerna, existerar utbytet av momentum (rörelsemängd) inte enbart av kroppskrafter. Det finns andra mekanismer där utbyte av momentum mellan fluidpartiklarna sker

   d behovet av att följa rörelsen hos varje individuell molekyl

2. Vilket av följade påståenden är *sant*

   a i stationärt flöde, sammanfaller aldrig partikelbanorna och strömlinjerna

   b strömlinjerna slutar alltid i fluiden

   c strömlinjerna kan sluta på fluiddomänens rand

   d i instationärt flöde, sammanfaller alltid partikelbanorna och strömlinjerna

3. $\frac{D}{Dt}$ operatorn som används för att beskriva fluid-partikel rörelsen representerar

   a en Lagransk representation av flödet

   b faktumet att en fluidpartikel är mindre än en individuell molekyl

   c faktumet att de makroskopiska förändringarna inte är relevanta för fluidpartikelrörelsen

   d faktumet att fluidpartikelrörelsen inte beror på medelflödet

4. Vilket av flöljande påståenden är *sant* för krafterna som verkar på ett fluidelement

a kroppskrafter är interna krafter och verkar inte genom volymen

b kontaktkrafter är interna krafter som verkar genom fluidelementets yta

c kroppskrafter är externa krafter och verkar inte genom volymen

d kroppskrafter är externa krafter som verkar genom volymen

5. Vilket av följande påståenden är *inte sant*

   a Cauchys rörelseekvation relaterar fluidpartikels accelerationen med netto kraften vid en viss punkt

   b det är ingen skillnad mellan det termodynamiska och det mekaniska trycket för alla fluider (både inkompressibla och kompressibla)

   c per definition, är stresstensorn stymmetrisk

   d formuleringen av stresstensorn beror inte på orienteringen av ytan

6. Vilket av följande påståenden är *sant* för den mekansika energiekavtionen

   a tryckets bidrag är alltid positivt

   b effekten av den viskösa dissipationen kan antingen vara positiv eller negativ

   c tecknet på effekten av den viskösa dissipationen representerar minskningen av mekanisk energi och ökningen av intern energi

   d effekten av kroppskrafter kan försummas i ekvationen

7. När kallas fluidrörelsen barotropisk?

   a densiteten är konstant i hela fluiden

   b densiteten är endast en funktion av trycket och vise versa

   c densiteten är inte konstant över allt, endast längs med en partikelbana

   d precessen måste vara isotermisk och inte isentropisk eller polytropisk

2 öppna frågor

(a) (*20 poäng*) Starta från Cauchys rörelseekvation

$$\rho\frac{Du_i}{Dt} = \rho \mathrm{g}_i + \frac{\partial \tau_{ij}}{\partial x_j}$$

a Derivera den mekaniska energiekvationen och diskutera de erhållna termerna. *Ledtråd:* Cauchys ekvation ska multipliceras med u_i.

b Använd ekvationen erhållen från a och lägg till kontinutetsekvationen ($\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0$) multiplicerad med $\frac{1}{2}\rho u_i^2$. Förklara de erhållna termerna.

c Diskutera ytkrafternas totala arbete på fluidelementet. Vilken del av denna finns inte med i den mekaniska energibalansen? Vad representerar denna del?

d Genom att använda den generella formen av den Newtoniska stresstensorn

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij} - \frac{2}{3}\mu\left(\nabla \cdot \vec{u}\right)\delta_{ij}$$

härled termen som representerar deformationsarbetet. *Ledtråd till härledningen:* den sammandragna (contracted) produkten av en symmetrisk och en antisymmetrisk tensor är noll. Använd skjuvspänningstensorn (strain rate tensor).

e Diskutera teknen på de erhållna termerna. Vad representerar de?

(b) (*20 poäng*) Starta från Euler ekvationen

$$\rho\frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \rho \mathrm{g}_i$$

a Härled Bernuoulli ekvationen. *Ledtråd:* Använd faktumet att kroppskrafterna kan uttryckas som en gradient av en potential funktion (gz). Vidare, uttryck den advektiva termen i termer av vorticiteten och tänk på att vektorprodukten av två vektorer ($\vec{u}$ och $\vec{v}$) yttryckt i tensornotation är $\epsilon_{ijk}e_i u_j v_k$, där ϵ_{ijk} är den alternerande tensorn och e_i är enhets vektorn.

b Förklara de erhållna termerna.

c Vilken typ av fluid har vi antagit här? Vilka konsekvenser leder detta antagnde till?

d Diskutera i detalj Bernulli ekvationens olika specialfall (stationärt flöde, rotationsfritt flöde). Vad är skillnaden mellan specialfallen?

(c) (*20 poäng*) Vilken kropp som helst, av godtycklig form nedsänkt i en strömmande fluid kommer att erhåll krafter och moment från fluiden. Låt oss tänka på ett specifikt fall där fluiden strömmar längs med en koordinat axel. Börja från Navier-Stokes inkompressibla ekvation i x-riktingen,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

a Skriv om denna ekvation på dimensionslös form för problemet ritat i figur 1. *Ledtråd:* Kroppen är helt och hållet nedsänkt i fluiden.

b Identifiera de dimensionslösa grupp(erna) som uppkommer i den dimensionslösa formen av Navier-Stokes ekvation, förlara deras mening.

c Skorstenen på en gammal tegelfabrik ligger på avsnivå ($\rho_{luft} = 1.23\text{kg/m}^3$, $\mu_{luft} = 1.85 \times 10^{-5}\text{kg/m}\cdot\text{s}$) är 2m i diameter och 40m hög. En grupp ingenjörer behöver din hjäp för att utvärdera om skorstenen är tillräckligt stark för att uthärda en storm med kraftiga vindar. Baserat på ett Reynolds nummer på 10^6 och en dragkoefficient (C_D) på 0.3, uppskatta dragkraften (F_D) som ges av $F_D = \frac{1}{2}C_D\rho U^2 A$.

d Vad är det uppskattade vindinduserade böjandemomentet M vid botten av skorstenen? *Ledtråd:* Antag att flödet är uniformt.

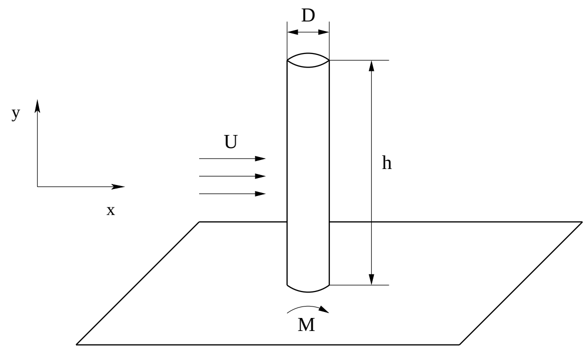

Figure 1: Flöde runt en skorsten.

(d) (*19 poäng*) Luft vid 60°C och 1 atm ($\rho_{luft} = 1.06\,\text{kg/m}^3$, $\nu_{luft} = 1.86\times10^{-5}\,\text{m}^2/\text{s}$) strömmar med 19 m/s över en platt platta enligt figur 2. Ett Pitot rör, placerat 3 mm från plattan, utvecklar en manometerhöjd $\Delta h = 10$ mm av Glycerine, $\rho_G = 1260\,\text{kg/m}^3$.

a Beräkna hastigheten hos fluiden vid nedströmspositionen x.
Ledtråd: Använd Bernoullis ekvation,

$$\frac{1}{2}q^2 + \int \frac{dp}{\rho} + gz = \text{constant}$$

b Uppskatta nedströmspositionen x för Pitot röret. Antag laminär flöde, så att du kan använda Blausis lösningen given i figur 3.

c Beräkna Re_x. Har vi verkligen laminärt flöde vid denna punkt?

d Skjuvspännigen vid väggen av plattan kan beräknas som,

$$\tau_{wall} = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

Vad är skjuvspänningen vid väggen? *Ledtråd:* använd figur 3 för att beräkna derivatan.

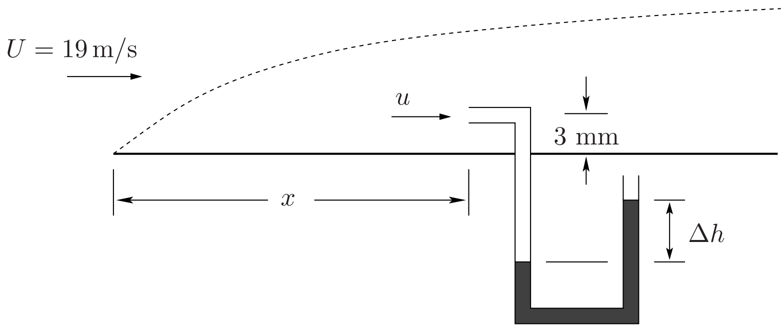

Figure 2: Flöde över en platt platta.

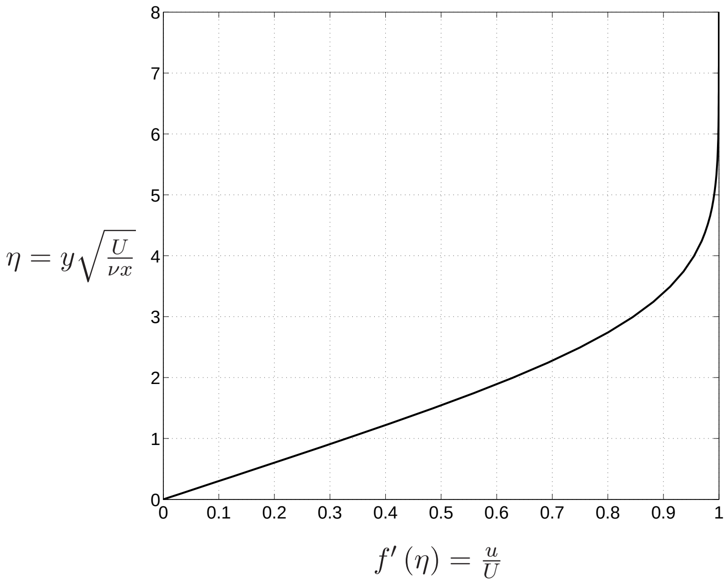

Figure 3: Blasius lösning.

Fluid Mechanics

TME055, 21 December 2007

The use of a regular calculator is permitted.

1 Multiple Choice Questions

(*3 points for each correct answer, -1 point for each incorrect anwser*)

1. The necessity of dealing with *stresses* in *continuum mechanics* is a consequence of

   a the concept is already present in mechanics of individual molecules and is simply retained in mechanics of continuum

   b the Newton's laws can't decribe properly the motion of individual molecules

   c after averaging over the individual molecules, the exchange of momentum is not due to body forces only. There are other mechanisms of exchanging momentum between fluid particles.

   d the need of keeping track of motion of each individual particle

2. Which of the following is *true*

   a in steady flow, particle paths and streamlines never coincide

   b streamlines always end in the fluid

   c streamlines can end on a boundary

   d in unsteady flow, particle paths and streamlines always coincide

3. $\frac{D}{Dt}$ operator used in connection with motion of a fluid-particle represents

   a a Lagrangian representation of the flow

   b the fact that a fluid particle is smaller than an individual molecule

   c the fact that macroscopic changes are not relevant for the motion of a fluid particle

   d the fact that the motion of a fluid particle does not depend on the mean flow

4. Which of the following is *true* for the forces that act on a fluid element

- a body forces are internal forces and do not act throughout the volume
- b contact forces are internal forces which act on a fluid element through its bounding surface
- c body forces are external forces and do not act throughout the volume
- d body forces are external forces which act throughout the volume

5. Which of the following is *not true*

- a Cauchy's equation of motion relates acceleration of a fluid particle to the net force at a point
- b there is no difference between the thermodynamic and mechanical pressure for all fluids (both incompressible and compressible)
- c by definition, the stress tensor is symmetric
- d the formulation of the stress tensor does not depend on the orientation of the surface

6. Which of the following is *true* for the mechanical energy equation

- a pressure contribution is always positive
- b rate of work by viscous dissipation can be of either sign in the equation
- c the sign of the rate of work by viscous dissipation represents the loss of mechanical energy and gain of internal energy
- d the rate of work by body forces can be neglected in the equation

7. When is a fluid motion called barotropic?

- a the density is constant everywhere in the flow field
- b the density is a function of pressure only and vice versa.
- c the density is not constant everywhere, but only along the path of a fluid particle
- d the process must be isothermal and not isentropic or polytropic

2 Open Questions

(a) (*20 pts*) Starting from the Cauchy's equation of motion

$$\rho\frac{Du_i}{Dt} = \rho g_i + \frac{\partial \tau_{ij}}{\partial x_j}$$

- a Derive the mechanical energy equation and discuss the terms obtained. Hint: the Cauchy's equation is to be multiplied with u_i.
- b Use the equation obtained under a and add the continuity equation ($\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0$) multiplied by $\frac{1}{2}\rho u_i^2$. Explain the terms obtained.
- c Discuss the total work of surface forces on a fluid element. Which part of this is not present in the mechanical energy balance? What does this part represent?
- d Using the general form of the Newtonian stress tensor

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij} - \frac{2}{3}\mu\left(\nabla \cdot \vec{u}\right)\delta_{ij}$$

  derive the term representing the deformation work. Hint for the derivation: contracted product of a symmetric and anti-symmetric tensor is zero. Use the strain rate tensor.
- e Discuss the sign of the terms obtained. What do they represent?

(b) (*20 pts*) Starting from the Euler equation

$$\rho\frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \rho g_i$$

- a Derive the Bernoulli equation. Hints: use the fact that the body forces can be expressed as the gradient of a potential function (gz). Furthermore, express the advective term in terms of vorticity and have in mind that the vector product of two vectors ($\vec{u}$ and $\vec{v}$) is expressed in tensor coordinates as $\epsilon_{ijk}e_i u_j v_k$, where ϵ_{ijk} is the alternating tensor and e_i are the unit vectors.
- b Explain the terms obtained.
- c What kind of fluid do we assume here? What are the consequences of that assumption?

d Discuss in detail the special cases of the Bernoulli equation (steady flow, irrotational flow). What is the difference between them?

(c) (*20 pts*) A body of an arbitrary shape, when immersed in a fluid stream, will experience forces and moments from the flow. Let us consider the particular case in which the flow is aligned with one coordinate direction. Starting from the incompressible Navier-Stokes equation in the x-direction,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

a Nondimensionalize this equation for the problem sketched in Figure 1. *Hint:* The body is completely immersed in the flow.

b Identify the dimensionless group(s) appearing in the dimensionless Navier-Stokes equation, and explain their meaning.

c The chimney of an old brick factory located at sea level ($\rho_{air} = 1.23\text{kg/m}^3$, $\mu_{air} = 1.85 \times 10^{-5}\text{kg/m}\cdot\text{s}$) is 2m in diameter and 40m high. A team of engineers needs your help to evaluate if the chimney will have sufficient strength to withstand storm wind loads. Based on a Reynolds number of 10^6 and a drag coefficient (C_D) of 0.3, estimate the drag force (F_D) given by $F_D = \frac{1}{2}C_D\rho U^2 A$.

d What is the estimated wind-induced bending moment M about the bottom of the chimney? *Hint:* Assume the flow is uniform.

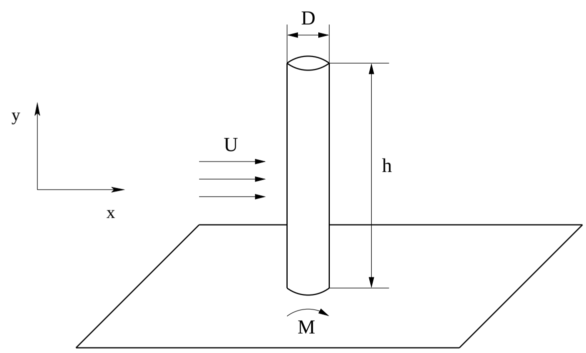

Figure 1: Flow past a chimney.

(d) (*19 pts*) Air at 60°C and 1 atm ($\rho_{air} = 1.06\,\mathrm{kg/m^3}$, $\nu_{air} = 1.86 \times 10^{-5}\,\mathrm{m^2/s}$) flows at 19 m/s past the flat plate as shown in Figure 2. A Pitot tube, placed 3 mm from the wall, develops a manometer height $\Delta h = 10$ mm of Glycerine, $\rho_G = 1260\,\mathrm{kg/m^3}$.

a Calculate the velocity at the downstream position x. *Hint:* Use Bernoulli's equation,

$$\frac{1}{2}q^2 + \int \frac{dp}{\rho} + \mathrm{g}z = \text{constant}$$

b Estimate the downstream position x of the pitot tube. Assume laminar flow, so you can use the Blasius solution given in Figure 3.

c Calculate Re_x. Do we really have laminar flow at this point?

d The shear stress at the wall of the plate can be determined as

$$\tau_{wall} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

What is the shear stress at the wall? *Hint:* use Figure 3 to determine the derivative.

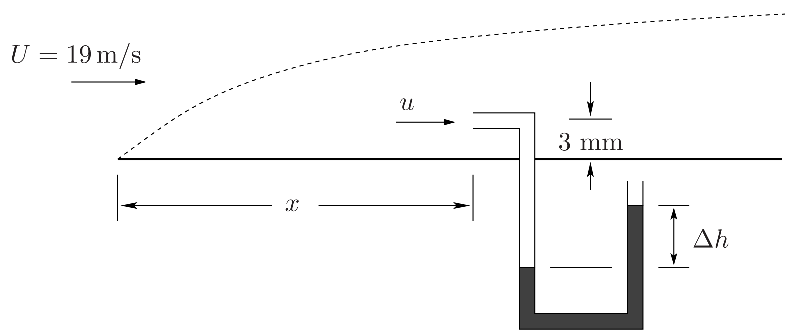

Figure 2: Flow past a flat plate.

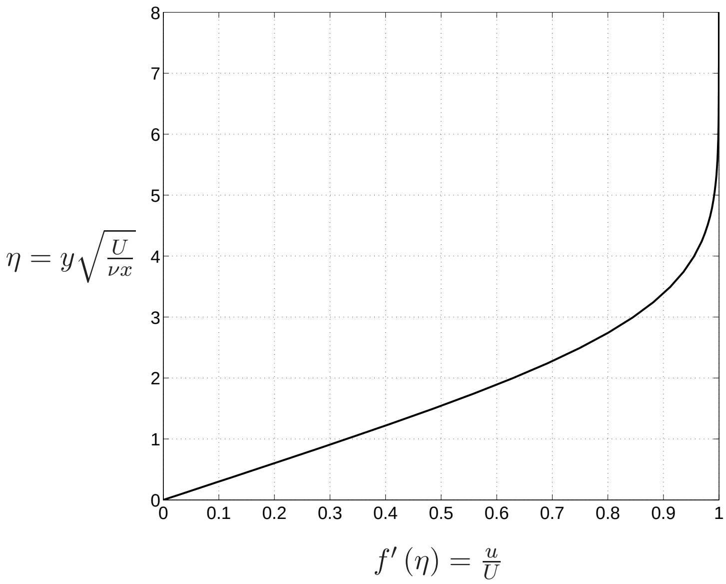

Figure 3: Blasius solution.

Fluid Mechanics
TME055

1 Multiple Choice Questions

1. c
2. c
3. a
4. d
5. b
6. c
7. b

2 Open Questions

1. Cauchy's equation of motion

   a Multiplying by u_i, we obtain

   $$\rho \frac{D}{Dt}\left(\frac{1}{2}u_i^2\right) = \rho u_i \mathrm{g}_i + u_i \frac{\partial \tau_{ij}}{\partial x_j}$$

   where a summation over i is implied in u_i^2. This equation tells us that the rate of increase of kinetic energy at a point (left hand side) equals the rate of work done by body forces (g_i) and the rate of work done by the net surface force ($\frac{\partial \tau_{ij}}{\partial x_j}$).

   b If we add continuity times $\frac{1}{2}\rho u_i^2$ to the above equation, one gets

   $$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u_i^2\right) + \frac{\partial}{\partial x_j}\left(u_j \frac{1}{2}\rho u_i^2\right) = \rho u_i \mathrm{g}_i + u_i \frac{\partial \tau_{ij}}{\partial x_j}$$

   Now the left hand side is the local rate of change of mechanical enegy plus an additional term which is in the form of divergence of kinetic energy flux: $J_j = u_j \frac{1}{2}\rho u_i^2$. The terms on the right hand side are identical to a.

c One can write the total rate of work done by surface forces (increase of kinetic energy per unit volume) as

$$\frac{\partial}{\partial x_j}(u_i\tau_{ij}) = \tau_{ij}\frac{\partial u_i}{\partial x_j} + u_i\frac{\partial \tau_{ij}}{\partial x_j}$$

The first term on the right hand side is not present in the mechanical energy equation and it represents *deformation work*. This contribution goes to internal energy.

d We know the product of a symmetric and an anti-symmetric tensor is zero, so $\tau_{ij}\frac{\partial u_i}{\partial x_j} = \tau_{ij}e_{ij}$. Using the Newtonian stress tensor, we get

$$\tau_{ij}e_{ij} = -pe_{jj} + 2\mu e_{ij}e_{ij} - \frac{2}{3}\mu e_{kk}e_{kk}$$

e We can collect the last two terms in d as follows

$$\tau_{ij}e_{ij} = -pe_{jj} + \phi$$

where $\phi = 2\mu\left(e_{ij} - \frac{1}{3}e_{kk}\delta_{ij}\right)^2$ is obtained after completing the square. The term pe_{jj} represents rate of work by volume expansion and it can be either positive or negative. The term ϕ is always positive and represents rate of viscous dissipation.

2. Euler equation

a If we write the body force in terms of a potential, $\mathrm{g}_i = -\frac{\partial}{\partial x_i}(\mathrm{g}z)$, we get

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_i}(\mathrm{g}z)\,\mathrm{g}_i$$

The second term on the left hand side can be expressed in terms of vorticity

$$u_j\frac{\partial u_i}{\partial x_j} = u_j r_{ij} + \frac{\partial}{\partial x_i}\left(\frac{1}{2}u_j^2\right) = -u_j\epsilon_{ijk}\omega_k + \frac{\partial}{\partial x_i}\left(\frac{1}{2}q^2\right)$$

where the rotation tensor has been expressed in terms of vorticity. Substitution and some rearrangement yields to

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_i}\left(\frac{1}{2}q^2\right) + \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}(\mathrm{g}z)\,\mathrm{g}_i = \epsilon_{ijk}u_j\omega_k$$

With the assumption of *barotropic flow*, we can write the pressure gradient as

$$\frac{1}{\rho}\frac{\partial p}{\partial x_i} = \frac{\partial}{\partial x_i}\int \frac{dp}{\rho}$$

so we get to the Euler equation. If we define the Bernoulli function as

$$B = \frac{1}{2}q^2 + \frac{\partial}{\partial x_i}\int \frac{dp}{\rho} + \mathrm{g}z$$

We finally get the Bernoulli equation

$$\frac{\partial u_i}{\partial t} + \frac{\partial B}{\partial x_i} = \epsilon_{ijk} u_j \omega_k$$

b B represents the total energy per unit mass at a point in the flow, at a given time.

c We assume inviscid and barotropic flow. This is an ideal flow as there is no irreversible work done by viscous forces. Energy is truly conserved.

d For steady flow we get

$$\frac{\partial B}{\partial x_i} = \epsilon_{ijk} u_j \omega_k$$

This equation tells us that surfaces of constant B contain both streamlines and vortex lines. Therefore B is constant along streamlines and vortex lines. For steady and irrotational flow, B is constant everywhere.
In the case of irrotational flow ($\omega_k = 0$), the velocity vector can be written in terms of a velocity potential, $u_i = \frac{\partial \phi}{\partial x_i}$. This leads to

$$\frac{\partial u_i}{\partial t} + \frac{\partial B}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\partial \phi}{\partial t} + \frac{1}{2}q^2 + \frac{\partial}{\partial x_i}\int \frac{dp}{\rho} + \mathrm{g}z\right) = 0$$

or

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}q^2 + \frac{\partial}{\partial x_i}\int \frac{dp}{\rho} + \mathrm{g}z = F(t)$$

where $F(t)$ must be determined from boundayr information.

3. Dimensional analysis

a Using the scaling parameters of Figure 1, we can nondimensionalize the velocity components, pressure, coordinate directions and time as follows:

$$u' = \frac{u}{U} \qquad v' = \frac{v}{U} \qquad w' = \frac{w}{U}$$

$$x' = \frac{x}{D} \qquad y' = \frac{y}{D} \qquad z' = \frac{z}{D}$$

$$t' = \frac{t}{D/U} \qquad p' = \frac{p - p_\infty}{\rho U^2}$$

Where p_∞ is a reference pressure (constant). Then we have,

$$\frac{U^2}{D}\frac{\partial u'}{\partial t'}+\frac{U^2}{D}\left(u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}+w'\frac{\partial u'}{\partial z'}\right) = -\frac{U^2}{D}\frac{\partial p'}{\partial x'}+\frac{\nu U}{D^2}\left(\frac{\partial^2 u'}{\partial x'^2}+\frac{\partial^2 u'}{\partial y'^2}+\frac{\partial^2 w'}{\partial z'^2}\right)$$

or

$$\frac{\partial u'}{\partial t'}+\left(u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}+w'\frac{\partial u'}{\partial z'}\right) = -\frac{\partial p'}{\partial x'}+\frac{\nu}{UD}\left(\frac{\partial^2 u'}{\partial x'^2}+\frac{\partial^2 u'}{\partial y'^2}+\frac{\partial^2 w'}{\partial z'^2}\right)$$

b The term on the right hand side multiplying the three terms between brackets can be identified as the reciprocal of Reynolds number

$$Re = \frac{U\,D}{\nu}$$

Re is the ratio of inertial to viscous forces and provides a criterion for determining dynamic similarity.

c From the estimated Reynolds number, one can get the free-stream

$$U = \frac{10^6 \times \left(1.85 \times 10^{-5}/1.23\right)}{2} = 7.52\text{m/s}$$

and the drag force is

$$F_D = 0.5 \times 0.3 \times 1.23 \times (7.52)^2 \times (2 \times 40) = 834.7\text{N}$$

d If the flow is uniform, the drag force should act at approximately at the chimeny's half-height. So, the bending moment is

$$M = \frac{F_D h}{2} = 834.7 \times 20 = 16694\text{N} \cdot \text{m}$$

4. Flow past a flat plate

a Assuming constant stream pressure, the manometer can be used to estimate the local velocity u at the position of the Pitot inlet:

$$\Delta p = (\rho_G - \rho_{air})\,\mathrm{g}\Delta h = (1260 - 1.06) \times 9.81 \times 0.010 = 124\,\mathrm{Pa}$$

$$u \approx \sqrt{\frac{2\Delta p}{\rho_{air}}} = \sqrt{\frac{2 \times 124}{1.06}} = \boxed{15.3\,\mathrm{m/s}}$$

b The Blasius solution uses $f'(\eta) = \frac{u}{U}$ to determine the position η (see Figure 3)

$$\frac{u}{U} = \frac{15.3}{19} = 0.81$$

which corresponds to $\eta \approx 2.8$ from Figure 3. The downstream position is

$$x = \left(\frac{y}{\eta}\right)^2 \frac{U}{\nu} = \left(\frac{3 \times 10^{-3}}{2.8}\right)^2 \frac{19}{1.86 \times 10^{-5}}$$

$$\boxed{x \approx 1.17\,\mathrm{m}}$$

c We check: $Re_x = 19 \times 1.17/(1.86 \times 10^{-5}) \approx 1.2 \times 10^6$ which can be considered laminar if the flow is very smooth and free of fluctuations that can lead to the growth of instabilities.

d The wall shear stress can be rewritten in terms of boundary layer variables:

$$\tau_{wall} = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \mu \frac{U}{\delta}\frac{df'}{d\eta}\bigg|_{\eta=0}$$

where the boundary layer grows as $\delta \sim \sqrt{\nu x/U}$, and from Figure 3 we can estimate the slope to be $f''(0) \approx 0.37$, so we get

$$\tau_{wall} = \mu \frac{\sqrt{Re_x}}{x} U f''(0) = 1.97 \times 10^{-5} \times \frac{\sqrt{1.2 \times 10^6}}{1.17} \times 19 \times 0.37$$

$$\boxed{\tau_{wall} \approx 0.13\,\mathrm{Pa}}$$