R.Ō. 2 – 13/9-18

1.1

- $\Psi = A e^{-a\left(\frac{mx^2}{\hbar} + it\right)}$

a) $A = ?$, b) $V(x) = ?$, c) $\langle x \rangle, \langle x^2 \rangle, \langle p \rangle, \langle p^2 \rangle = ?$

d) $\sigma_x, \sigma_p, \sigma_x \sigma_p, \geq \frac{\hbar}{2}$?

a) $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1 \Rightarrow A^2 \int_{-\infty}^{\infty} e^{\frac{-2amx^2}{\hbar}} = \left\{ s = \sqrt{\frac{2am}{\hbar}}\, x \Rightarrow dx = \sqrt{\frac{\hbar}{2am}}\, ds \right\} =$

$= A^2 \sqrt{\frac{\hbar}{2am}} \int_{-\infty}^{\infty} e^{-s^2} ds = A^2 \sqrt{\frac{\pi\hbar}{2am}} \Rightarrow A = \left(\frac{2am}{\pi\hbar}\right)^{1/4}$

b) Lös S.E. för $V(x)$!

$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi$

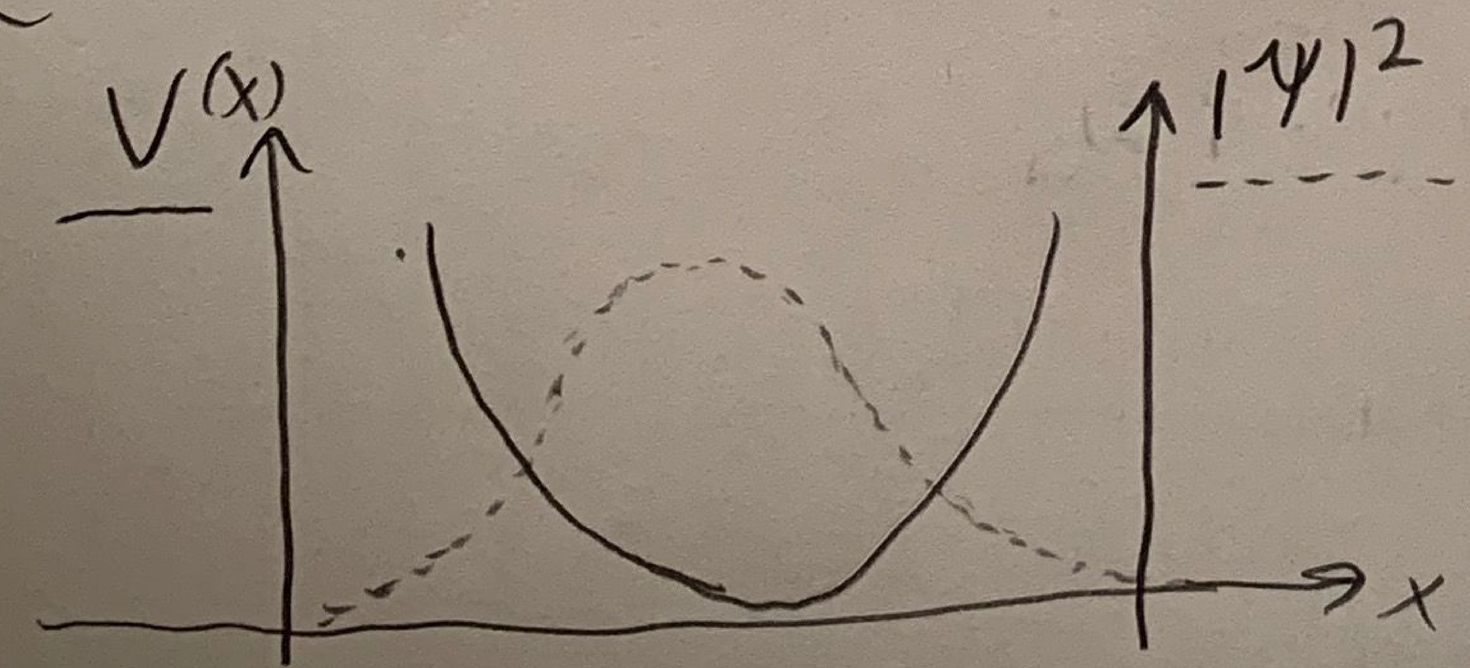
$\frac{\partial \Psi}{\partial t} = -ia\Psi$, $\frac{\partial \Psi}{\partial x} = \frac{-2amx}{\hbar}\Psi \Rightarrow \frac{\partial^2 \Psi}{\partial x^2} = \frac{-2am}{\hbar}(\Psi + \Psi') =$

$= \frac{-2am}{\hbar}\left(1 - \frac{2am}{\hbar}x^2\right)\Psi.$

stoppa in i S.E. $\Rightarrow -ia i\hbar \cancel{\Psi} = -\frac{\hbar^2}{2m}\left(-\frac{2am}{\hbar}\left(1 - \frac{2am}{\hbar}x^2\right)\cancel{\Psi} + V(x)\cancel{\Psi}\right) \Leftrightarrow$

$V(x) = \hbar a - \hbar a\left(1 - \frac{2am}{\hbar}x^2\right) = 2a^2 m x^2.$ ← kvadratisk i position

{Klassiskt! $F = -\frac{\partial V}{\partial x} = -4a^2 m x = -kx$, $k = 4a^2 m$ }



udda fkn på jämnt intervall

c) $\langle x \rangle = \int x|\psi|^2 dx = \text{konst.} \cdot \int_{-\infty}^{\infty} s e^{-s^2} ds = 0$

$\langle x^2 \rangle = \int x^2 |\psi|^2 = A^2 \int x^2 e^{-\frac{2amx^2}{\hbar}} dx = \sqrt{\frac{2am}{\pi\hbar}} \sqrt{\frac{\pi}{\frac{2am}{\hbar}}} \frac{\hbar}{4am} \cdot$

$\langle p \rangle = \int_{-\infty}^{\infty} \psi^* (-i\hbar \frac{\partial}{\partial x}) \psi \, dx = \int_{-\infty}^{\infty} \psi^* \cdot 2iamx \psi \, dx = 2iam \underbrace{\int_{-\infty}^{\infty} x|\psi|^2 dx}_{\text{udda fkn på jämnt intervall}} = 0$

$\hat{p}^2 = \left(-i\hbar \frac{\partial}{\partial x}\right)^2 = \hbar^2 \frac{\partial^2}{\partial x^2}$

$\Rightarrow \langle p^2 \rangle = -\hbar^2 \int_{-\infty}^{\infty} \psi^* \frac{\partial^2 \psi}{\partial x^2} dx = 2am\hbar \int_{-\infty}^{\infty} \left(1 - \frac{2am}{\hbar} x^2\right) |\psi|^2 dx =$

$= 2am\hbar - 4a^2m^2 \langle x^2 \rangle = \ldots = am\hbar.$

d) $\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{\hbar}{4am}} = \frac{1}{2}\sqrt{\frac{\hbar}{am}}$

$\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{am\hbar}$

$\sigma_x \sigma_p = \frac{1}{2} \frac{\sqrt{\hbar}\sqrt{am\hbar}}{\sqrt{am}} = \frac{\hbar}{2}$ ok!

R.Ö 4 – 27/9-18

2.42 . Gaussiskt vågpaket $\Psi(x,0) = Ae^{-ax^2+iqx}$

Sökt: a: A , b: $\Psi(x,t)$, c: $|\Psi|^2$

Lösn: $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$, Lättare i p-space ⇒ F-transformera!

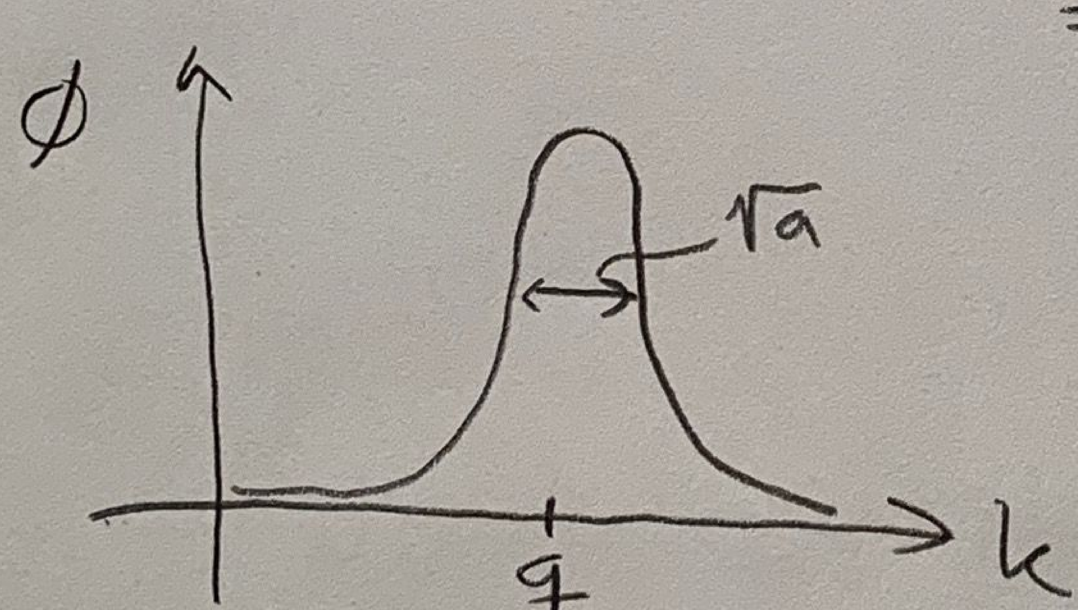
Introducera: $\phi(k,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ikx}\Psi(x,t)\,dx$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ikx}\phi(k,t)\,dk$$

⇒ S.E. i termer av ϕ: $i\hbar\frac{\partial\phi}{\partial t} = \frac{\hbar^2k^2}{2m}\phi \Rightarrow \phi(k,t) = \phi(k,0)e^{-\frac{i\hbar k^2}{2m}t}$

$$\phi(k,0) = \frac{1}{\sqrt{2\pi}}\int dx\, e^{-ikx}\Psi(x,0) = \frac{A}{\sqrt{2\pi}}\int \underbrace{e^{-ax^2-i(k-q)x}}_{=e^{-a\left(x+i\frac{k-q}{2a}\right)^2}e^{-\frac{(k-q)^2}{4a}}}dx =$$

$$= \frac{A}{\sqrt{2\pi}}e^{-\frac{(k-q)^2}{4a}}\underbrace{\int_{-\infty}^{\infty} e^{-a\left(x+i\frac{k-q}{2a}\right)^2}dx}_{=\sqrt{\frac{\pi}{a}}} = \frac{Ae^{-\frac{(k-q)^2}{4a}}}{\sqrt{2a}}$$



släng nu på tidsfaktorn och transformera tillbaka för att få $\Psi(x,t)$

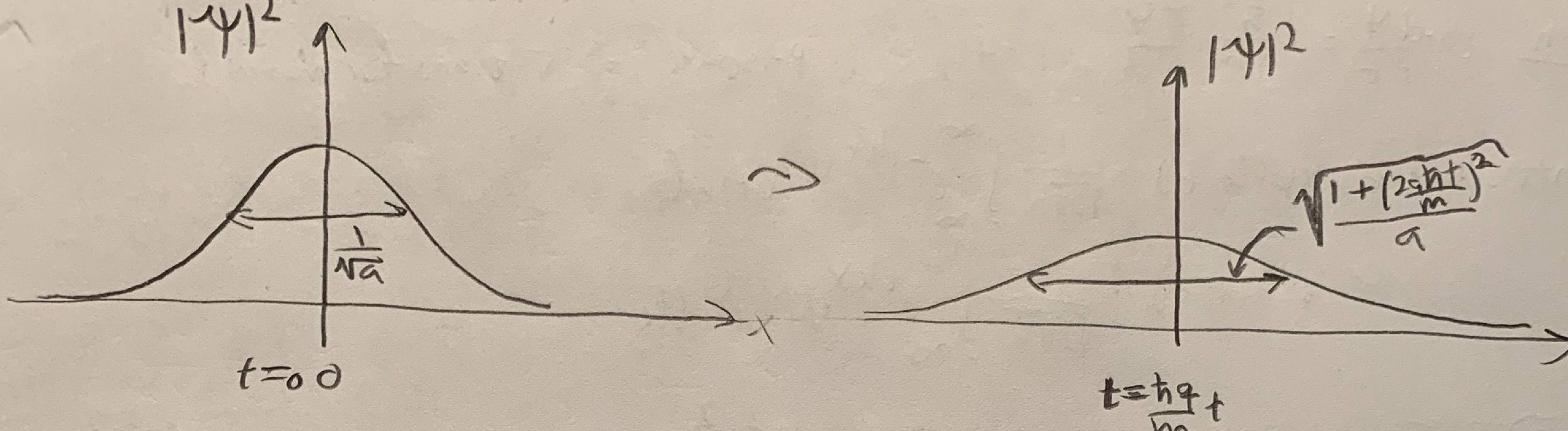
$$\Rightarrow \Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ikx}\phi(k,0)e^{-\frac{i\hbar k^2 t}{2m}}dk = \frac{A}{2\sqrt{\pi}}\int_{-\infty}^{\infty} e^{ikx-\frac{(k-q)^2}{4a}+i\frac{\hbar k^2 t}{2m}}dk =$$

$$= ikx - \frac{(k-q)^2}{4a} - \frac{i\hbar k^2}{2m}t = k^2\left(-i\frac{\hbar t}{2m} - \frac{1}{4a}\right) + k\left(ix - \frac{q}{2a}\right) - \frac{q^2}{4a} = -\underbrace{\left(\frac{1}{4a}+\frac{i\hbar t}{2m}\right)}_{r}\underbrace{\left(k - \frac{ix-\frac{q}{2a}}{\frac{1}{2a}+\frac{i\hbar t}{4m}}\right)^2}_{y^2} -$$

$$\underbrace{-\frac{1}{1+\left(\frac{2a\hbar t}{m}\right)^2}\left(a\left(x-\frac{\hbar q}{m}t\right)^2 - 2i\frac{a^2\hbar t}{m}x^2 - iq\left(x-\frac{\hbar q}{2m}t\right)\right)}$$

$$\Rightarrow \psi(x,t) = \frac{A}{\sqrt{2\pi}} \boxed{\int_{-\infty}^{\infty} e^{-ry^2}\, dk} \cdot \exp\left(-\frac{a}{1+\left(\frac{2a\hbar t}{m}\right)^2}\left(x - \frac{\hbar q}{m} t\right)^2 + i\eta(x,t)\right),$$

där $\eta = -2a\frac{\hbar t}{m}x^2 - q\left(x - \frac{\hbar q}{2m}t\right).$



$1 = \sqrt{\frac{\pi}{r}}$

???

- Operator $\hat{Q} = \frac{\partial^2}{\partial \varphi^2}$, $\varphi \in [0, 2\pi]$

Sökt: När är $\hat{Q}$ hermitsk?

Lösn: $\langle \phi | \hat{Q} \psi \rangle = \langle \hat{Q} \phi | \psi \rangle$ ← Krav för hermitskivitet.

$$\Rightarrow \langle \phi | \hat{Q} \psi \rangle = \int_0^{2\pi} \phi^* \frac{\partial^2}{\partial \varphi^2} \psi \, d\varphi = \left[\phi^* \frac{\partial \psi}{\partial \varphi} \right]_0^{2\pi} - \int_0^{2\pi} \frac{\partial \phi^*}{\partial \varphi} \frac{\partial \psi}{\partial \varphi} d\varphi =$$

$$= \left[\phi^* \frac{\partial \psi}{\partial \varphi} - \frac{\partial \phi^*}{\partial \varphi} \psi \right]_0^{2\pi} + \underbrace{\int \left(\frac{\partial^2 \phi}{\partial \varphi^2} \right)^* \psi \, d\varphi}_{\langle \hat{Q} \phi | \psi \rangle}$$

⇒ första termen måste försvinna för att $\hat{Q}$ ska va hermitsk

$\Rightarrow \left[\phi^* \frac{\partial \psi}{\partial \varphi} - \frac{\partial \phi^*}{\partial \varphi} \psi \right]_0^{2\pi} = 0$. Detta uppfylls om alla $\psi \in \mathcal{H}$ är 2π-periodiska.

Sökt: Egenfunkurna och egenvärdena

Lösn: $\hat{Q} \psi(\varphi) = q \psi(\varphi)$, $q \in \mathbb{C}$

$\frac{\partial^2 \psi}{\partial \varphi^2} = q \psi \Rightarrow \psi = a e^{\sqrt{q} \varphi} + b e^{-\sqrt{q} \varphi}$ (dubbelt degenererade)

Randvillkor: $\psi(\varphi) = \psi(\varphi + 2\pi) \Rightarrow e^{\sqrt{q} \varphi} = e^{\sqrt{q} \varphi + 2\pi \sqrt{q}}$

$\Rightarrow 1 = e^{2\pi \sqrt{q}} \Rightarrow 2\pi \sqrt{q} = i 2\pi n$, $n = 0, 1, \ldots$ (ty $1 = e^{i 2\pi n}$)

$\Rightarrow \underline{q = -n^2}$. $\Rightarrow \underline{\psi = a e^{in\varphi} + b e^{-in\varphi}}$

$\psi(\varphi) = a e^{\sqrt{q} \varphi} + b e^{-\sqrt{q} \varphi} = \psi(\varphi + 2\pi) = a e^{\sqrt{q}(\varphi + 2\pi)} + b e^{-\sqrt{q}(\varphi + 2\pi)}$

$= a e^{\sqrt{q} \varphi} e^{\sqrt{q} 2\pi} + b e^{-\sqrt{q} \varphi} e^{-\sqrt{q} 2\pi} \Leftrightarrow a e^{\sqrt{q} \varphi} \left(1 - e^{2\pi \sqrt{q}} \right) = b e^{-\sqrt{q} \varphi} \left(e^{-\sqrt{q} 2\pi} - 1 \right)$

R.Ö.5 – 4/10-18

3.26 $\cdot\, |\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle$ $\quad \cdot\, |\beta\rangle = i|1\rangle + 2|3\rangle$

a) vad är $\langle\alpha|$, $\langle\beta|$?

lösn: För godtyckligt tillstånd ψ gäller: $|\psi\rangle = \sum_n c_n |n\rangle$

$\Rightarrow \langle\psi| = \sum_n c_n^* \langle n|$

$\Rightarrow \langle\alpha| = -i\langle 1| - 2\langle 2| + i\langle 3|$

$\langle\beta| = -i\langle 1| + 2\langle 3|$

b) $\langle\alpha|\beta\rangle$, $\langle\beta|\alpha\rangle$ = ?

lösn: $\langle\alpha|\beta\rangle = (-i\langle 1| - 2\langle 2| + i\langle 3|)(i|1\rangle + 2|3\rangle) =$

$= (-i)i\underbrace{\langle 1|1\rangle}_{=1} + 2i\underbrace{\langle 3|3\rangle}_{=1} + \ldots \left\{ \leftarrow \text{alla } \langle n|m\rangle = 0 \text{ för } n \neq m \right\} =$

$= 1 + 2i$

P.S.S: $\langle\beta|\alpha\rangle = (-i\langle 1| + 2\langle 3|)(i|1\rangle - 2|2\rangle - i|3\rangle) =$

$= (-i)i\langle 1|1\rangle - 2i\langle 3|3\rangle = 1 - 2i = \langle\alpha|\beta\rangle^*$

c) matrisen av $\hat{X} = |\alpha\rangle\langle\beta|$ = ?

lösn:

$$X = \begin{pmatrix} \langle 1|\hat{X}|1\rangle & \langle 1|\hat{X}|2\rangle & \cdots \\ \langle 2|\hat{X}|1\rangle & \langle 2|\hat{X}|2\rangle & \\ \vdots & & \ddots \end{pmatrix}$$

$$X_{11} = \langle 1|\hat{X}|1\rangle = \langle 1|\left(|\alpha\rangle\langle\beta|\right)|1\rangle = \langle 1|\alpha\rangle\langle\beta|1\rangle =$$

$$= \langle 1|\left(i|1\rangle \cancel{-2|2\rangle} \cancel{-i|3\rangle}\right)\langle\left(-i\langle 1| \cancel{+2\langle 3|}\right)|1\rangle =$$

$$= \langle 1|i\cdot 1\rangle\langle -i\langle 1|1\rangle = i(-i) = 1$$

$$X_{12} = \langle 1|\alpha\rangle\langle\beta|2\rangle = i\cdot 0 = 0$$

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etc. _

$$\Rightarrow X = \begin{pmatrix} 1 & 0 & 2i \\ 2i & 0 & -4 \\ -1 & 0 & -2i \end{pmatrix}$$

Är den hermitsk?

<u>lösn:</u> $X^\dagger = (X^T)^* \neq \quad \neq X \Rightarrow$ Inte hermitsk ty

$$X^\dagger = \begin{pmatrix} 1 & -2i & -1 \\ 0 & 0 & 0 \\ -2i & -4 & 2i \end{pmatrix} \neq X \Rightarrow \text{inte hermitsk}$$

Inses även genom $\hat{X}^\dagger = |\beta\rangle\langle\alpha| \neq \hat{X}$

3.44 $H = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}$

a) $\Psi(t=0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, sökt! $\Psi(t)$

lösn: $i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$. $H\Psi_E = E\Psi_E \Rightarrow i\hbar \frac{\partial \Psi_E}{\partial t} = E\Psi_E \Rightarrow \Psi_E(t) = \Psi_E(0)\, e^{-\frac{i}{\hbar}Et}$

$\Psi(0) = \sum_n c_n \Psi_{E_n} \Rightarrow \Psi(t) = \sum_n c_n e^{-\frac{i}{\hbar}E_n t}\, \Psi_{E_n}$

Lös först egenvärdesproblemet för H

$\Rightarrow \det(H - EI) = 0 \Leftrightarrow \begin{vmatrix} a-E & 0 & b \\ 0 & c-E & 0 \\ b & 0 & a-E \end{vmatrix} = (a-E)^2(c-E) - b^2(c-E) =$

$= (c-E)\left((a-E)^2 - b^2\right) \stackrel{!}{=} 0. \Rightarrow \underline{E_0 = c}$

$(a-E)^2 - b^2 = 0 \Rightarrow \underline{E_{\pm} = a \pm b}$

Egenvektorerna fås genom! $H\Psi = E\Psi \Leftrightarrow (H - EI)\Psi = 0$

Ansätt $\Psi = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

$E = E_0 = c$! $\begin{pmatrix} a-c & 0 & b \\ 0 & 0 & 0 \\ b & 0 & a-c \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} (a-c)\alpha + \beta\gamma = 0 \\ 0 = 0 \\ b\alpha + (a-c)\gamma = 0 \end{cases}$

$\Rightarrow \alpha = \gamma = 0$ är enda lösningen $\Rightarrow \Psi_0 = \begin{pmatrix} 0 \\ \beta \\ 0 \end{pmatrix} \Rightarrow \{\text{normering}\} \Rightarrow \underline{\Psi_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$

$E = E_+ = a+b$: $\begin{pmatrix} -b & 0 & b \\ 0 & c-a-b & 0 \\ b & 0 & -b \end{pmatrix}\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \Rightarrow \begin{cases} -b\alpha + b\gamma = 0 \\ (c-a-b)\beta = 0 \\ b\alpha - b\gamma = 0 \end{cases}$

$\Rightarrow \beta = 0,\ \alpha = \gamma \Rightarrow \psi_+' = \begin{pmatrix}\alpha\\0\\\alpha\end{pmatrix} \Rightarrow \psi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}$

P.s.s. för $E = E_- = a-b$: $\psi_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$

$\Rightarrow$ Egenlösningarna ges av: $E_0 = c$ $\quad E_+ = a+b$ $\quad E_- = a-b$

$\psi_0 = \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \psi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix} \quad \psi_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$

Dessa egenvektorer är ortogonala, ty H är hermitsk

$\Rightarrow \psi(t=0) = \begin{pmatrix}0\\1\\0\end{pmatrix} = \psi_0 \Rightarrow \underline{\psi(t) = e^{-\frac{i}{\hbar}E_0 t}\psi_0 = e^{-\frac{ict}{\hbar}}\begin{pmatrix}0\\1\\0\end{pmatrix}}$

b) $\psi(t=0) = \begin{pmatrix}1\\0\\0\end{pmatrix}$, hitta $\psi(t)$

lösn: $\psi(t=0) = \begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}(\psi_+ + \psi_-)$

$\Rightarrow \psi(t) = \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}E_+ t}\psi_+ + e^{-\frac{i}{\hbar}E_- t}\psi_-\right) =$

$= \frac{e^{-\frac{iat}{\hbar}}}{2}\left[e^{-\frac{ibt}{\hbar}}\begin{pmatrix}1\\0\\1\end{pmatrix} + e^{\frac{ibt}{\hbar}}\begin{pmatrix}1\\0\\-1\end{pmatrix}\right] =$

$= e^{-\frac{iat}{\hbar}}\begin{pmatrix} \frac{e^{-\frac{ibt}{\hbar}} + e^{\frac{ibt}{\hbar}}}{2} \\ 0 \\ \frac{e^{-\frac{ibt}{\hbar}} - e^{\frac{ibt}{\hbar}}}{2} \end{pmatrix} = e^{-\frac{iat}{\hbar}}\begin{pmatrix}\cos\left(\frac{bt}{\hbar}\right) \\ 0 \\ -i\sin\left(\frac{bt}{\hbar}\right)\end{pmatrix}$

3.45 Uttryck $\langle n|\hat{X}|\psi\rangle$ i termer av $c_n = \langle n|\psi\rangle$, där $|n\rangle$ är energiegentillstånd till harmoniska oscillatorn.

Lösn:

Steg 1: $I = \sum_{n=0}^{\infty} |n\rangle\langle n|$

$$\Rightarrow \langle n|\hat{x}|\psi\rangle = \langle n|\hat{x}\Big(\sum_{n'} |n'\rangle\langle n'|\Big)|\psi\rangle = \sum_{n'} \langle n|\hat{x}|n'\rangle \underbrace{\langle n'|\psi\rangle}_{=c_{n'}}$$

$\langle n|\hat{x}|n'\rangle$ löses mha stegoperatorerna $\hat{a}_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} \mp i\hat{p})$

$$\Rightarrow \hat{a}_+|n\rangle = \sqrt{n+1}\,|n+1\rangle, \quad \hat{a}_-|n\rangle = \sqrt{n}\,|n-1\rangle$$

Lös ut $\hat{x}$ ur $\hat{a}_\pm$ ekvationerna $\Rightarrow \hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}_+ + \hat{a}_-)$

$$\Rightarrow \langle n|\hat{x}|n'\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\langle n|\hat{a}_+|n'\rangle + \langle n|\hat{a}_-|n'\rangle\right) =$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\Big(\sqrt{n'+1}\underbrace{\langle n|n'+1\rangle}_{\delta_{n,n'+1}} + \sqrt{n'}\underbrace{\langle n|n'-1\rangle}_{\delta_{n,n'-1}}\Big) \leftarrow \text{definierar hela matrisen } X$$

$$\Rightarrow X = \sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & 1 & 0 & 0 & \cdots 0 \cdots \\ 1 & 0 & \sqrt{2} & 0 & \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \\ \vdots & & & \ddots & \\ 0 & \cdots & \sqrt{n} & 0 & \sqrt{n+1} \\ \vdots & & & & \ddots \end{pmatrix}$$

kroneker deltana kmr "döda" summan

$$\Rightarrow \langle n|\hat{x}|\psi\rangle = \sum_{n'}\sqrt{\frac{\hbar}{2m\omega}}\, c_{n'}\left(\sqrt{n'+1}\,\delta_{n,n'-1} + \sqrt{n'}\,\delta_{n,n'-1}\right) =$$

$$= \underline{\sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\,c_{n-1} + \sqrt{n+1}\,c_{n+1}\right)}$$

R.Ö. 6 – 11/10-18

4.11 $\cdot\; V(r) = \begin{cases} -V_0 & , 0 \le r \le a \\ 0 & , r > a \end{cases}$

Sökt: grundtillståndet ($l=0$). Visa att det kräver $V_0 > \dfrac{\pi^2\hbar^2}{8ma^2}$

Lösn:

$$\hat{H}\psi = E\psi \quad , \quad \hat{H} = \frac{\hat{p}^2}{2m} + V(r)$$

$$\hat{p}^2 = -\hbar^2 \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) - \frac{\hbar^2 \hat{L}^2}{r^2} \qquad \left(\text{vanliga } \hat{p}^2 = \hat{p}_r^2 + \frac{\hat{L}^2}{r^2}\right)$$

Ansätt $\psi(\vec{x}) = \frac{1}{r}\psi_l(r)\, Y_l^m(\theta,\varphi)$

$$\hat{L}^2 Y_l^m = -l(l+1)\, Y_l^m$$

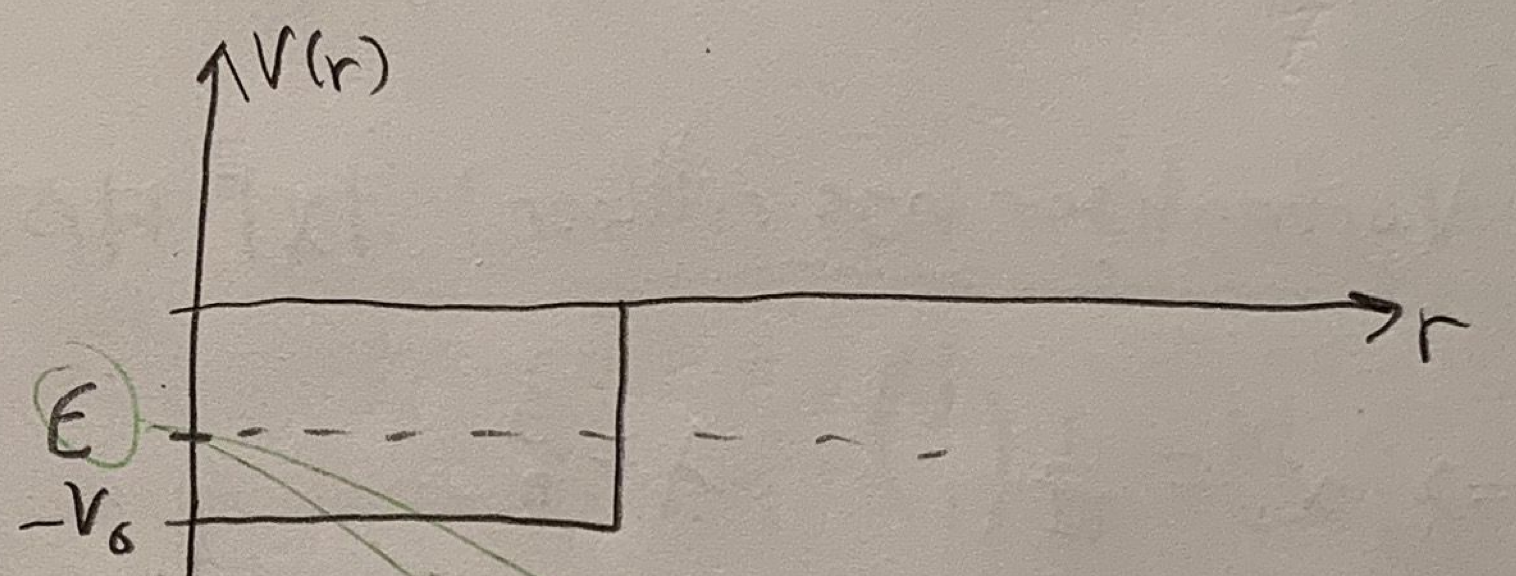
S.E.

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2 \psi_l(r)}{\partial r^2} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\psi_l(r) + V(r)\psi_l(r) = E\psi_l$$

$l=0 \Rightarrow -\frac{\hbar^2}{2m}\psi_0'' + V\psi_0 = E\psi_0$

S.E. reduceras till typ vanliga

1D - S.E.



$$\Rightarrow \psi_0 = \begin{cases} A\sin(kr) + \cancel{B\cos(kr)} & 0 \le r \le a \\ Ce^{\kappa r} + De^{-\kappa r} & r > a \end{cases}$$

$k = \sqrt{\dfrac{2m(E+V_0)}{\hbar^2}}$

$\kappa = \sqrt{\dfrac{2m(-E)}{\hbar^2}}$ ← reell

Randvillkor: $\cdot\; \psi(\infty) < \infty \Rightarrow C = 0$

$\cdot\; \psi(a^+) = \psi(a^-)$ $\Rightarrow A\sin(ka) = De^{-\kappa a}$ (1)

$\cdot\; \psi'(a^+) = \psi'(a^-)$ $\Rightarrow A\cos(ka) = -D\kappa e^{-\kappa a}$ (2)

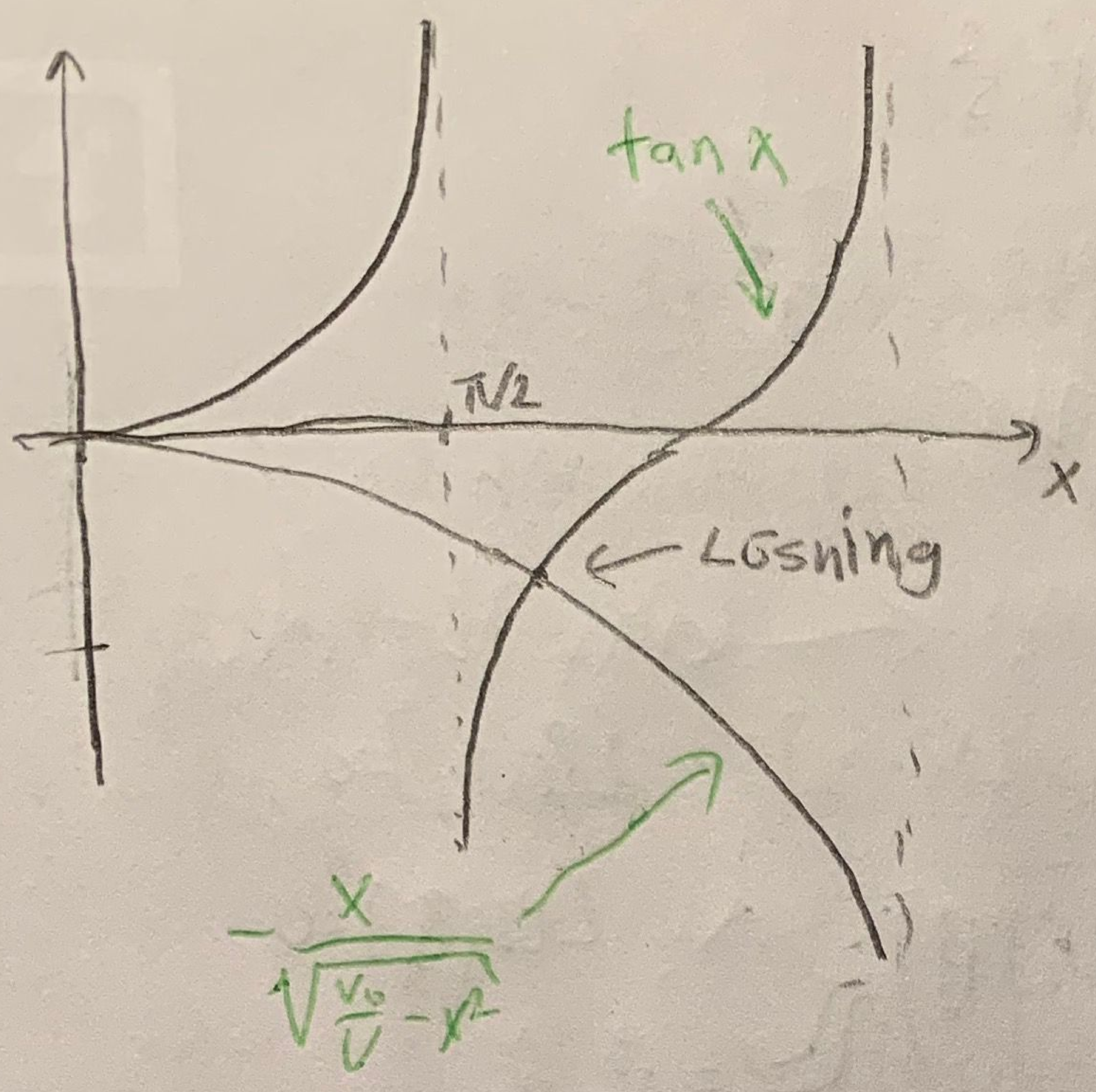
$\cdot\; \psi(0)$ "snäll"

$\frac{B\cos(kr)}{r}$ $\Rightarrow B = 0$

$\frac{(1)}{(2)} \Rightarrow \tan(ka) = -\frac{k}{\kappa}$. Inför $U = \frac{\hbar^2}{2ma^2} \Rightarrow ka = \sqrt{\frac{E+V_0}{U}}$

$\Rightarrow \kappa a = \sqrt{-\frac{E}{U}} = \sqrt{\frac{V_0}{U} - (ka)^2}$

Låt $x = ka \Rightarrow \tan x = -\dfrac{x}{\sqrt{\frac{V_0}{U} - x^2}}$



$\Rightarrow$ Bara lösning för $x_0 > \frac{\pi}{2}$ $\Leftrightarrow$

$V_0 > \frac{\pi^2}{4} U = \frac{\pi^2 \hbar^2}{8ma^2}$. $\boxed{VSV}$

$\psi =$

4.73 $\bullet\ \vec{B} = -\alpha x\hat{i} + (B_0 + \alpha z)\hat{k}$ $\bullet\ \hat{H} = \frac{p^2}{2m} - \gamma\hat{\vec{B}}\cdot\hat{\vec{S}}$

sökt: $\frac{d^2}{dt^2}\langle z\rangle$

lösn: Ehrenfest: $\frac{\partial\langle\hat{z}\rangle}{\partial t} = -\frac{i}{\hbar}\langle[\hat{z},\hat{H}]\rangle$

kommuteringsrelationer: $[x_i, x_j] = 0$, $[x_i, p_j] = i\hbar\delta_{ij}$, $[x_i, S_j] = 0$, $[p_i, S_j] = 0$

$[p_i, p_j] = 0$

$$[\hat{z},\hat{H}] = \frac{1}{2m}[\hat{z}, \underbrace{\hat{p}^2}_{= p_x^2 + p_y^2 + p_z^2}] = \frac{1}{2m}[\hat{z}, p_z^2] = \frac{1}{2m}\left(\hat{p}_z[\hat{z},\hat{p}_z] + [\hat{z},\hat{p}_z]\hat{p}_z\right) = \frac{i\hbar}{m}\hat{p}_z$$

Ty alla andra delar av H kommuterar med $\hat{z}$

$$\Rightarrow \frac{d\langle\hat{z}\rangle}{dt} = \frac{1}{m}\langle p_z\rangle \quad \text{så} \quad \frac{\partial^2\langle\hat{z}\rangle}{\partial t^2} = \frac{1}{m}\frac{d}{dt}\langle p_z\rangle = -\frac{i}{\hbar m}\langle[\hat{p}_z,\hat{H}]\rangle$$

$$[\hat{p}_z,\hat{H}] = [\hat{p}_z, -\gamma\hat{\vec{B}}\cdot\hat{\vec{S}}] = -\gamma[p_z,\hat{\vec{B}}]\hat{\vec{S}}$$

kan ta ut $\hat{\vec{S}}$ ty spinnet kommuterar med $\hat{z}$.

Enligt 3.11: $\bullet\ [\hat{r}, f(\hat{p}_z)] = i\hbar\frac{\partial f}{\partial p_z}(\hat{p}_z)$ $\bullet\ [\hat{p}_z, g(\hat{z})] = -i\hbar\frac{\partial g}{\partial z}(\hat{z})$

$$\Rightarrow [\hat{p}_z, \vec{B}(\vec{x})] = -i\hbar\frac{\partial\vec{B}}{\partial z}(\vec{x}) \Rightarrow [\hat{p}_z,\hat{H}] = \gamma\hbar\frac{\partial\hat{\vec{B}}}{\partial z}\cdot\hat{\vec{S}}$$

$$\frac{\partial\hat{\vec{B}}}{\partial z} = \alpha\hat{k} \Rightarrow \frac{\partial\hat{\vec{B}}}{\partial z}\cdot\hat{\vec{S}} = \alpha S_z \qquad \hat{k}\cdot\hat{\vec{S}} = \hat{S}_z$$

$\alpha\hat{k}\cdot S_x\hat{x} + S_y\hat{y} + S_z\hat{z}$

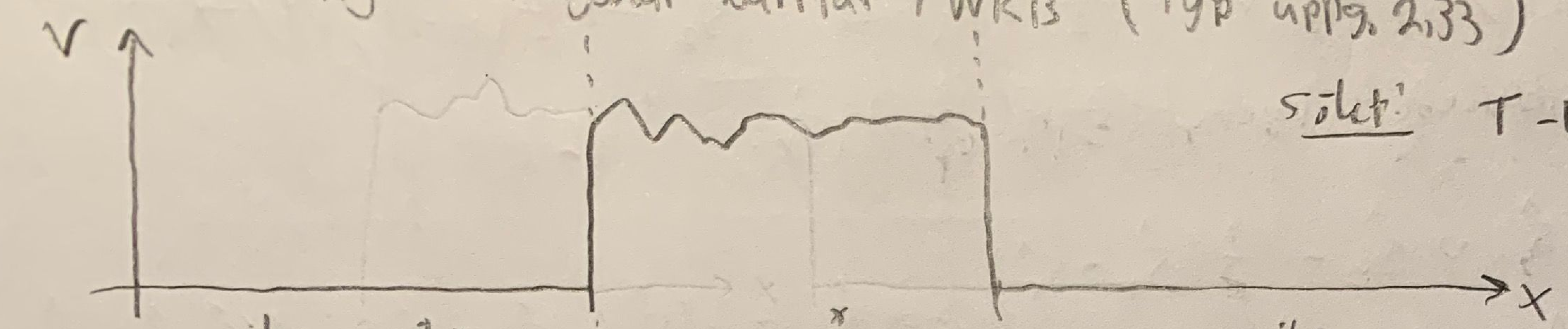
$= \alpha S_z$

$$\Rightarrow \frac{d^2}{dt^2}\langle z\rangle = -\frac{i}{\hbar m}(\gamma i\hbar\alpha)\langle S_z\rangle = -\frac{\gamma\alpha}{m}\langle S_z\rangle$$

R.Ö 7 - 18/10

9.3 spridning rektangulär barriär i WKB (Typ uppg. 2.33)

Sökt: T-koeff.



$\psi(x) = Ae^{ikx} + Be^{-ikx}$ $\quad \psi(x) = \frac{C}{\sqrt{\kappa(x)}} e^{-\int_0^x \kappa(x)dx}$ $\quad \psi(x) = De^{ikx}$

$k = \frac{\sqrt{2mE}}{\hbar}$ $\quad \kappa = \sqrt{2m(V(x)-E)}$

Randvillkor: $\psi(0^-) = \psi(0^+)$, $\psi'(0^-) = \psi'(0^+)$

$\Rightarrow A + B = \frac{C}{\sqrt{\kappa(0)}}$ (1), $ik(A-B) = = C\sqrt{\kappa(0)}$ (2)

$\left\{ \frac{\partial}{\partial x} \frac{C}{\sqrt{\kappa(x)}} e^{-\int_0^x \kappa(x)dx'} \approx -\frac{C}{\sqrt{\kappa(x)}} \kappa(x) e^{-\int \kappa(x)dx} = -C\sqrt{\kappa(x)} e^{-\int \kappa(x)dx} \right\}$

släng den andra termen, ty den är liten

$\Rightarrow \frac{C}{\sqrt{\kappa(a)}} e^{-\int_0^a \kappa(x)dx} = De^{ika}$ (3)

(1) + $\frac{(2)}{ik}$ $\Rightarrow$ $2A = \frac{C}{\sqrt{\kappa(0)}}\left(1 + i\frac{\kappa(0)}{k}\right)$

(3) $\Rightarrow$ $C = \sqrt{\kappa(a)}\, e^{ika} e^{\int_0^a \kappa(x)dx} D$

$\Rightarrow \frac{D}{A} = 2\sqrt{\frac{\kappa(0)}{\kappa(a)}} \cdot \frac{1}{1 + i\frac{\kappa(0)}{k}} e^{-ika} e^{-\int_0^a \kappa(x)dx}$

Transmissionen $T = \left|\frac{D}{A}\right|^2 = \ldots = 4\frac{\kappa(0)}{\kappa(a)} \cdot \frac{1}{1 + \frac{\kappa(0)^2}{k^2}} e^{-2\int_0^a \kappa(x)dx}$

Test: $V(x) = V_0$ $\Rightarrow \kappa(a) = \kappa(0) = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$, $\int_0^a \kappa(x)dx = \kappa a$

$\Rightarrow T = \frac{4E}{V_0} e^{-2\kappa a}$. Jmf. exakt lösning: $T = \frac{1}{1 + \frac{V_0^2}{4E(V_0-E)} \sinh(\kappa a)^2}$ $\rightarrow$

$\kappa a \gg 1$

$\rightarrow \frac{4E}{V_0} \cdot \frac{V_0 - E}{V_0} e^{-2\kappa a}$. stämmer när $V_0 \gg E$. !

4.32

- $S = \frac{1}{2}$ i S_z-basen $\Rightarrow |\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$, där

$\hat{S}_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$, $\hat{S}_z|\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle$ $\Rightarrow$ $\hat{S}_z \to \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$, $|\psi\rangle \to \begin{pmatrix}a\\b\end{pmatrix}$

a) Finn egenvärden och egentillstånd till $\hat{S}_y$.

Lösn:
$$\left.\begin{aligned}\hat{S}_+ &= \hat{S}_x + i\hat{S}_y \Rightarrow \hat{S}_y = \frac{1}{i}(\hat{S}_+ - \hat{S}_x)\\ \hat{S}_- &= \hat{S}_x - i\hat{S}_y \Rightarrow \hat{S}_x = \hat{S}_- + i\hat{S}_y\end{aligned}\right\} \Rightarrow \hat{S}_y = \frac{1}{i}(\hat{S}_+ - (\hat{S}_- + i\hat{S}_y)) = \frac{1}{i}(\hat{S}_+ - \hat{S}_-) - \hat{S}_y \Leftrightarrow$$
$$\Leftrightarrow \hat{S}_y = \frac{1}{2i}(\hat{S}_+ - \hat{S}_-)$$

Egenvärden: $\det(S_y - \lambda I) = \begin{vmatrix} -\lambda & -i\frac{\hbar}{2} \\ i\frac{\hbar}{2} & -\lambda \end{vmatrix} = \lambda^2 - i(-i)\frac{\hbar^2}{4} = \lambda^2 - \frac{\hbar^2}{4} \Rightarrow \begin{cases}\lambda_+ = \frac{\hbar}{2}\\ \lambda_- = -\frac{\hbar}{2}\end{cases}$

eftersom $\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ i den här basen.

Egenvektorer: $\begin{cases}\phi_+ = \begin{pmatrix}x_+\\y_+\end{pmatrix}\\ \phi_- = \begin{pmatrix}x_-\\y_-\end{pmatrix}\end{cases}$

$\Rightarrow 0 = (S_y - \lambda_+ I)\phi_+ = \frac{\hbar}{2}\begin{pmatrix}-1 & -i\\ i & -1\end{pmatrix}\begin{pmatrix}x_+\\y_+\end{pmatrix} \Rightarrow x_+ + iy_+ = 0 \Leftrightarrow y_+ = ix_+$

$\Rightarrow \boxed{\phi_+ = \frac{1}{\sqrt{|x_+|^2 + |y_+|^2}}\begin{pmatrix}x_+\\ix_+\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}}$

P.S.S. $\Rightarrow \boxed{\phi_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}}$

b) För godtyckligt $|\psi\rangle$, vad är möjliga mätvärden och deras sannolikheter?

Lösn: Möjliga resultat är: $\lambda_\pm = \begin{cases}\frac{\hbar}{2}\\ -\frac{\hbar}{2}\end{cases}$

Sannolikheterna ges av: $P(\lambda_+) = |\langle\phi_+|\psi\rangle|^2 = (*)$

Allmänt: $|\psi\rangle = \sum_n c_n|x_n\rangle$, $P_n = |c_n|^2 \Rightarrow \langle x_m|\psi\rangle = \langle x_m|\sum_n c_n|x_n\rangle = \sum_n c_n\langle x_m|x_n\rangle =$
$= \sum_n c_n\delta_{mn} \Leftrightarrow c_n = \langle x_n|\psi\rangle$

$(*) = \frac{1}{2}\left|(1\ -i)\begin{pmatrix}a\\b\end{pmatrix}\right|^2 = \frac{|a-ib|^2}{2} = \frac{|a|^2 + |b|^2 - i(ab^* + a^*b)}{2} = \frac{1}{2} - \mathrm{Im}(ab^*)$

$\Rightarrow P(\lambda_-) = |\langle\phi_-|\psi\rangle|^2 = \frac{1}{2}|a + ib|^2 = \frac{1}{2} + \mathrm{Im}(ab^*)$

c) Gör b) men för $\hat{S}_y^2$

<u>lösn:</u> $\hat{S}_y^2 = \frac{\hbar^2}{4}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{\hbar^2}{4}\begin{pmatrix} (-i)i & 0 \\ 0 & (-i)i \end{pmatrix} = \frac{\hbar^2}{4} \cdot \hat{I}$

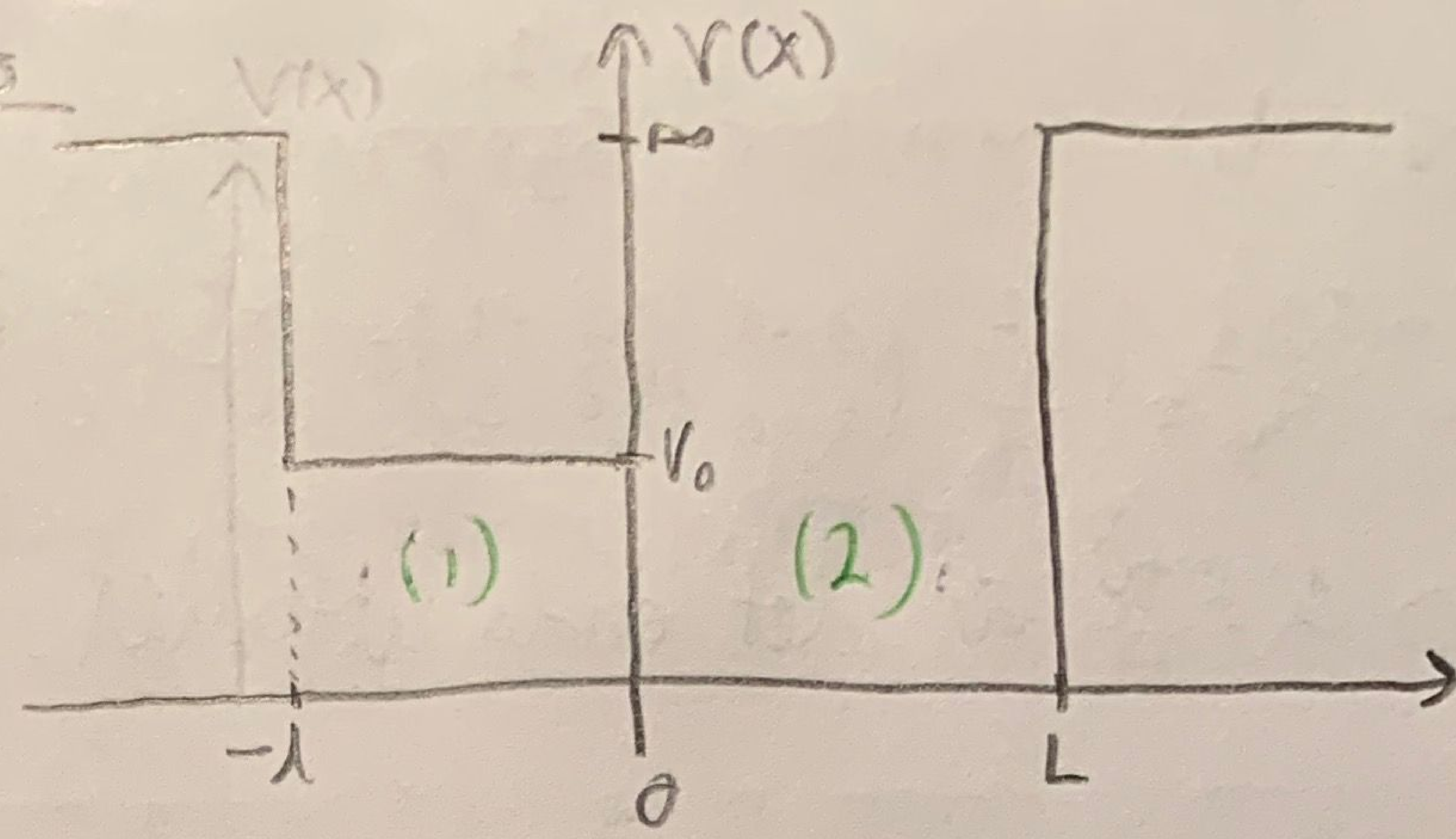
$\Rightarrow \lambda = \frac{\hbar^2}{4}$ är det enda utfallet.

$$\left\{\begin{array}{ll} \hat{S}_+ \chi_- = \hbar \chi_+ & \Rightarrow \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \hbar \\ 0 \end{pmatrix} = \begin{pmatrix} S_{12} \\ S_{22} \end{pmatrix} \Rightarrow S_{12} = \hbar \\ \hat{S}_- \chi_+ = \hbar \chi_- & \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} S_{11} \\ S_{21} \end{pmatrix} \Rightarrow S_{11} = S_{21} = 0 \\ \hat{S}_+ \chi_+ = \hat{S}_- \chi_- = 0 & \end{array}\right\} \Rightarrow S_+ = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

R.Ö. 8 - 22/10

0.1

$V(x) = \begin{cases} \infty, & x < -l \\ V_0, & -l \leq x < 0 \\ 0, & 0 < x \leq L \\ \infty, & L < x \end{cases}$



a) Finn uttrycket för energi egenvärdena

(1) $\psi = ae^{ik_1x} + be^{-ik_1x}$, $k_1 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$

(2) $\psi = ce^{ik_2x} + be^{-ik_2x}$, $k_2 = \frac{\sqrt{2mE}}{\hbar}$

har lösning om $\det(K)=0$

$\psi(-l)=0$

$\psi(0^-)=\psi(0^+)$

$\psi'(0^-)=\psi(0^+)$

$\psi(L)=0$

$\Rightarrow \begin{cases} ae^{-ik_1l} + be^{ik_1l} = 0 \\ a+b = c+d \\ k_1(a-b) = k_2(c-d) \\ ce^{ik_2l} + de^{-ik_2l} = 0 \end{cases}$

$\Rightarrow Kc = 0$, $c = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

$K = \begin{pmatrix} e^{-ik_1l} & e^{ik_1l} & 0 & 0 \\ 1 & 1 & -1 & -1 \\ k_1 & -k_1 & -k_2 & k_2 \\ 0 & 0 & e^{ik_2l} & e^{-ik_2l} \end{pmatrix}$

$\Rightarrow \det(K) = 0 = 4i\left[k_1\cos(k_1l)\sin(k_2l) + k_2\cos(k_2L)\sin(k_1l)\right] \Leftrightarrow$

$\Rightarrow \tan\left(\frac{L\sqrt{2mE}}{\hbar}\right) = -\sqrt{\frac{E}{E-V_0}}\tan\left(\frac{l\sqrt{2m(E-V_0)}}{\hbar}\right)$ ← Detta är ett uttryck för energiegenvärdena

b) Finn l s.a. $E=V_0$

$\tan x \approx x$ (Taylor)

$\lim_{E\to V_0} \frac{\sqrt{E}}{\sqrt{E-V_0}} \tan\left(\frac{\sqrt{2m(E-V_0)}}{\hbar}\right) = \frac{\sqrt{2mE}}{\hbar} \Rightarrow \tan\left(\frac{L\sqrt{2mV_0}}{\hbar}\right) = -\frac{l\sqrt{2mV_0}}{\hbar} \Leftrightarrow$

$l = -\frac{\hbar}{\sqrt{2mV_0}}\tan\left(\frac{L\sqrt{2mV_0}}{\hbar}\right)$ $\lim \frac{\tan x}{x} = 1$

Q2 $\hat{A}$ verkar enligt: $\hat{A}|1\rangle = a|1\rangle + b|2\rangle$

$\hat{A}|2\rangle = c|1\rangle + d|2\rangle$

OBS! operator $\hat{A}$ har en matris A med $A_{ij} = \langle i|\hat{A}|j\rangle$, $i=1,2$, $j=1,2$

a) hitta a,b,c,d s.a. $\hat{A}$ blir hermitesk

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad A_{11} = \langle 1|\hat{A}|1\rangle = a\underbrace{\langle 1|1\rangle}_{=1} + b\underbrace{\langle 1|2\rangle}_{=0} = a$$

$$A_{12} = \langle 1|\hat{A}|2\rangle = c\underbrace{\langle 1|1\rangle}_{=1} + d\underbrace{\langle 1|2\rangle}_{=0} = c$$

P.S.S $\Rightarrow A_{21} = b,\ A_{22} = d \Rightarrow A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

A hermitesk om $A = A^\dagger = (A^*)^T = \begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$$\Rightarrow \begin{cases} a^* = a \Rightarrow a \in \mathbb{R}. \quad d = d^* \Rightarrow d \in \mathbb{R} \\ b^* = c \\ c^* = b \end{cases} \Rightarrow \boxed{A = \begin{pmatrix} a & b \\ b^* & d \end{pmatrix}}$$

b) Vad är möjliga mätresultat om man mäter A

Lösn: Man kan få egenvärdena. Tar fram dessa.

$$\det(A - \lambda I) = 0 = \begin{vmatrix} a-\lambda & b \\ b^* & d-\lambda \end{vmatrix} = (a-\lambda)(d-\lambda) - bb^* =$$

$$= \lambda^2 - (a+d)\lambda + ad - |b|^2 \Rightarrow$$

$$\Rightarrow \boxed{\lambda_\pm = \frac{a+d}{2} \pm \sqrt{\frac{(a+d)^2}{4} - da + |b|^2} = \frac{a+d}{2} \pm \sqrt{\left(\frac{a-d}{2}\right)^2 + |b|^2}}$$

0.3 • Låt $\mathbb{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

a) Bestäm egenvärden till $\mathbb{V}\cdot\hat{L}$

lösn: kalla $\mathbb{V}$-riktningen för z-riktningen $\Rightarrow \mathbb{V}\cdot\hat{L} = |\mathbb{V}|\hat{L}_{"z"}$

men $\hat{L}_z|l,m\rangle = \hbar m|l,m\rangle$, $m = -l, l$

$\Rightarrow \mathbb{V}\cdot\hat{L}$ måste ha egenvärden $|\mathbb{V}|\hbar m$, $m = -l, -l+1, \dots, l-1, l$

b) Lös väntevärdet $\langle \mathbb{V}\cdot\hat{L}\rangle$ i tillståndet $|l,m\rangle$ ← egen tillstånd i z-riktningen

lösn: vi arbetar med $|l,m\rangle$ som satisfierar: $\begin{cases} \hat{L}^2|l,m\rangle = \hbar^2 l(l+1)|l,m\rangle \\ \hat{L}_z|l,m\rangle = \hbar m|l,m\rangle \end{cases}$

dvs $\hat{L}_z = \hat{k}\cdot\hat{L}$ ✓

$\mathbb{V}\cdot\hat{L} = V_x\hat{L}_x + V_y\hat{L}_y + V_z\hat{L}_z$.   stegoperatorerna: $\begin{cases} \hat{L}_+ = \hat{L}_x + i\hat{L}_y \\ \hat{L}_- = \hat{L}_x - i\hat{L}_y \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} L_+|l,m\rangle = \hbar a_{m+1}|l,m+1\rangle \\ L_-|l,m\rangle = \hbar a_m|l,m-1\rangle \end{cases}$, där $a_m = \sqrt{(l+m)(l-m-1)}$

$\Rightarrow \hat{L}_x = \dfrac{\hat{L}_+ + \hat{L}_-}{2}$, $\hat{L}_y = i\dfrac{\hat{L}_- - \hat{L}_+}{2} \Rightarrow \mathbb{V}\cdot\hat{L} = \dfrac{V_x - iV_y}{2}\hat{L}_+ + \dfrac{V_x + iV_y}{2}\hat{L}_- + V_z\hat{L}_z$

och $\mathbb{V}\cdot\hat{L}|l,m\rangle = V_z\hbar m|l,m\rangle + \hbar a_m\dfrac{V_x+iV_y}{2}|l,m\rangle + \hbar a_{m+1}\dfrac{V_x-iV_y}{2}|l,m+1\rangle$

Beräkna väntevärdet på detta tillståndet genom att gångra med en ⟨bra|

$\Rightarrow \boxed{\langle l,m|\mathbb{V}\cdot\hat{L}|l,m\rangle = \hbar m V_z}$ = {ortogonala tillstånd ⇒ bara en term överlever} =

⇒ $\langle l,m|l,m+n\rangle = 0$ om $n \neq 0$

c) Lös variansen $\Delta(\mathbb{V}\cdot\hat{L})^2 = \langle(\mathbb{V}\cdot\hat{L})^2\rangle - \langle\mathbb{V}\cdot\mathbb{L}\rangle^2$ i basen $|l,m\rangle$

lösn: $(\mathbb{V}\cdot\hat{L})^2|l,m\rangle = \mathbb{V}\cdot\hat{L}(\mathbb{V}\cdot\hat{L}|l,m\rangle) = \mathbb{V}\cdot\hat{L}(\hbar m V_z|l,m\rangle + \dots|l,m+1\rangle + \dots) =$

$= \hbar^2\left[\left(V_z^2 m^2 + \dfrac{V_x^2+V_y^2}{4}(a_{m+1}^2 + a_m^2)\right)|l,m\rangle + V_z\dfrac{V_x - iV_y}{2}m a_{m+1}|l,m+1\rangle + \dots|l,m+1\rangle + \dots|l,m+2\rangle\right]$

forts ⇒

$$\Rightarrow \langle \mathbf{V}\cdot\hat{\mathbf{L}}\rangle = \langle l,m|(\mathbf{V}\cdot\hat{\mathbf{L}})^2|l,m\rangle = \hbar^2\left[V_z^2 m^2 + \frac{V_x^2+V_y^2}{4}(a_{m+1}^2 + a_m^2)\right]$$

$$\Rightarrow \Delta(\mathbf{V}\cdot\hat{\mathbf{L}})^2 = \langle(\mathbf{V}\cdot\mathbf{L})^2\rangle - \langle\mathbf{V}\cdot\mathbf{L}\rangle^2 = \hbar^2\frac{V_x^2+V_y^2}{4}(l^2 - m^2 + l)$$

$\underline{0.2}$ $\hat{A}$ verkar enl. $\hat{A}|1\rangle = a|1\rangle + b|2\rangle$, $\hat{A}|2\rangle = c|1\rangle + d|2\rangle$

a) Hitta a,b,c,d s.a. $\hat{A}$ blir hermitesk.

lösn: Hermitesk $\Rightarrow$ $A^\dagger = A \Leftrightarrow \langle\psi|\hat{A}\psi\rangle = \langle\hat{A}\psi|\psi\rangle$

Antag $|1\rangle, |2\rangle$ ON-bas $\Rightarrow$ $A_{ij} = \langle e_i|\hat{A}e_j\rangle$

$$\left.\begin{aligned}
A_{11} &= \langle 1|\hat{A}|1\rangle = \langle 1|(a|1\rangle + b|2\rangle) = a\\
A_{12} &= \langle 1|\hat{A}|2\rangle = \langle 1|(c|1\rangle + d|2\rangle) = c\\
A_{21} &= \langle 2|\hat{A}|1\rangle = \langle 2|b|2\rangle = b\\
A_{22} &= \langle 2|\hat{A}|2\rangle = \langle 2|d|2\rangle = d
\end{aligned}\right\} \Rightarrow A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$A^\dagger = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^* = \begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix} \overset{\text{hermitesk}}{=} A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \Rightarrow \begin{aligned} a^* = a &\Rightarrow a \in \mathbb{R} \\ d^* = d &\Rightarrow d \in \mathbb{R}\end{aligned}$$

$$(b^*)^* = c^* \Leftrightarrow b = c^* \qquad \Rightarrow A = \begin{pmatrix} a & b \\ b^* & d \end{pmatrix}, \ a, d \in \mathbb{R}$$

$$c^* = b$$

b) Vad är möjliga mätresultat vid mätning av $\hat{A}$?

lösn: Egen värdena! $\det(A - \lambda I) = \begin{vmatrix} a-\lambda & b \\ b^* & d-\lambda \end{vmatrix} = (a-\lambda)(d-\lambda) - |b|^2 = 0$

$$ad - a\lambda - d\lambda + \lambda^2 - |b|^2 = 0 \Leftrightarrow \lambda^2 - (a+d)\lambda + ad - |b|^2 = 0 \Rightarrow \lambda = \frac{a+d}{2} \pm \sqrt{\left(\frac{a+d}{2}\right)^2 - ad + |b|^2} =$$

$$= \frac{a+d}{2}\left(1 \pm \sqrt{1 + 2\frac{|b|^2 - ad}{a+d}}\right) \text{ ger } \lambda_+ \text{ och } \lambda_-$$

$$\mathbb{P}(\lambda_+) = \langle e_{\lambda_+}|\underbrace{\psi}_{\text{okänd}}\rangle$$