② | 08-03-13

config	energy	multipl.	notation
AA	$-J$	3	p_0
AB	$+J$	6	p_1

$$p(A)=p(B)=p(C) = p_0+p_1 = \frac{1}{3}$$

constraints:

$$\begin{cases} -3p_0J+6p_1J = u \\ 3p_0+6p_1 = 1 \end{cases}$$

$$s = S_2 - S_1 = S_2 - \ln 3$$

max S_2 : Lagrangian:

$$L = 3p_0 \ln\frac{1}{p_0} + 6p_1 \ln\frac{1}{p_1} + \beta(u+3p_0J-6p_1J) + (\mu-1)(1-3p_0-6p_1)$$

$$\frac{\partial L}{\partial p_0} = 0 \rightarrow -3\ln p_0 + 3\beta J - 3\mu = 0$$

$$p_0 = e^{-\mu+\beta J}$$

$$\frac{\partial L}{\partial p_1} = 0 \rightarrow -6p_1 - 6\beta J - 6\mu = 0$$

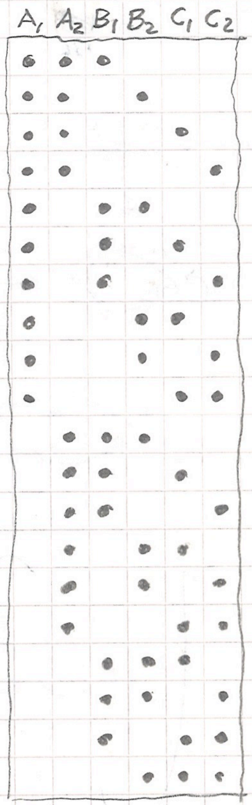
$$p_1 = e^{-\mu-\beta J}$$

normalisation gives $e^{-\mu} = \dfrac{1}{3e^{\beta J}+6e^{-\beta J}}$

and $p_0 = \dfrac{e^{\beta J}}{3e^{\beta J}+6e^{-\beta J}}$, $p_1 = \dfrac{e^{-\beta J}}{3e^{\beta J}+6e^{-\beta J}}$

ITCS exam 2008-03-13

① 3 heavy ● + 3 light ○ balls



$S = \log 20$

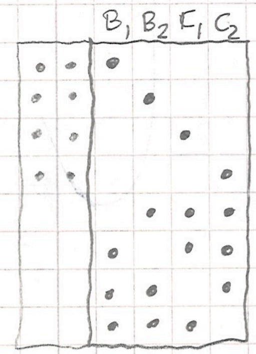
3 measurem. Ideally → $I = \log 27$

I A_1 A_2 → { = 8 cases; 6 cases; 6 cases

II A_1A_2 B_1B_2 → { same

Take worst case of I : $A_1 = A_2$

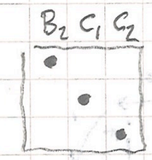
$S = \log 8$



A_1 B_1

{ = 2 cases; \ 3 -"-; / 3 -"-

worst case / : $S = \log 3$



B_2 C_1

→ $S = 0$

08-07-13

④

$$-\frac{dK}{dt} = -\int dx \sum_i \dot{c}_i \left(\ln\frac{c_i}{p_{i0}} + 1\right) = -\int dx \sum_i \dot{c}_i \ln\frac{c_i}{p_{i0}} =$$

since $\sum_i c_i = 1 \to \sum_i \dot{c}_i = 0$

$$= -\int dx \sum_i \ln\frac{c_i}{p_{i0}}\left(D_i \nabla^2 c_i + F_i(\underline{c})\right) =$$

$$= \int dx \sum_i D_i \frac{1}{c_i}(\nabla c_i)^2 - \int dx \sum_i \ln\frac{c_i}{p_{i0}} F_i(\underline{c}) .$$

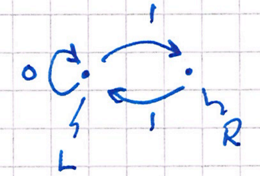
partial integration over 1st term.

QED.

08-03-13

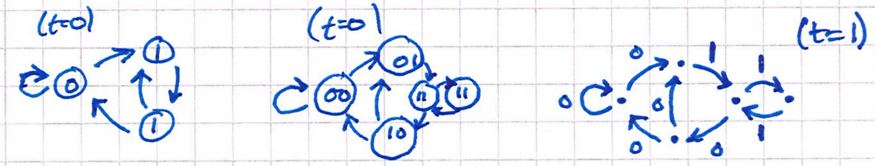
③ R136 : $\left.\begin{matrix}111\\011\end{matrix}\right\} \to 1$ oth $\to 0$

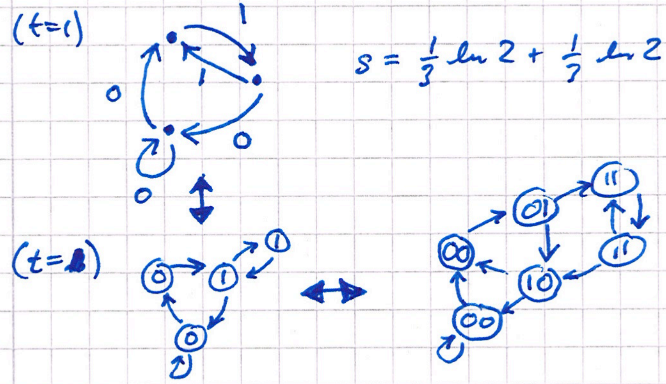
a)



$p(L) = \frac{2}{3}$, $p(R) = \frac{1}{3}$

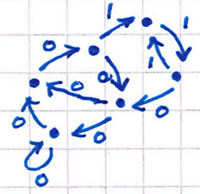
$s(t=0) = \frac{2}{3}\ln 2$





$s = \frac{1}{3}\ln 2 + \frac{1}{3}\ln 2 = \frac{2}{3}\ln 2$

→ (t=2)



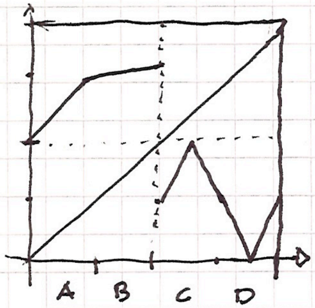
$s(t=2) < s(t=1)$

Here s must be lower as there are several paths of 0's that cannot be distinguished. (Also if you make an example of what happens between t=1 & t=2 you see that information is lost, i.e. s decreases.)

(b) The algorithmic information is constant since there is a finite information needed to produce the state at t+1 from the one at t.

08-03-13

5



$f(\tfrac{1}{2}) = \alpha$

$\frac{3}{4} \le \alpha \le 1$

$A \to C \to B \to D \to A$

$\mu(A) = \mu(B) = \mu(C) = \mu(D) = \frac{1}{4}$

$\lambda = \ln 1 + \frac{1}{4}\ln|f'(B)| + \frac{1}{4}\ln 2 + \frac{1}{4}\ln 2$

$\lambda > 0 \to |f'(B)| > \frac{1}{4}$

$f'(B) = \dfrac{\alpha - 3/4}{1/4} = 4\alpha - 3$

$4\alpha - 3 > \frac{1}{4}$

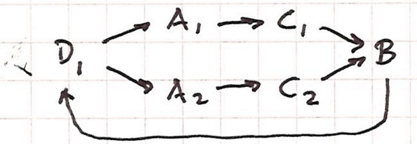
$\underline{\alpha > \frac{13}{16}}$

suppose:

$\alpha = \frac{7}{8}$ $\quad D_1 = \{x : x < \frac{3}{4} \le \frac{7}{8}\}$, $D_2 = D \setminus D_1$

then $A \to C \to B \to D_1$ (but <u>not</u> generating)

split A and C in equal parts :



This is a generating partition, since when symbol sequences get longer they correspond to a shrinking set of start points.

$S_\mu = \mu(D_1) \cdot \ln 2 = \underline{\frac{1}{4}\ln 2}$

$P(A' \text{ at } t+4 \mid A' \text{ at } t) = \frac{1}{2} \to \underline{I = \ln 2}$