Chalmers Tekniska Högskola
Institutionen för signaler och system
Avdelningen för reglerteknik

Tentamen i Reglerteknik SSY310/ERE091

August 28th, 2020 / English version

1. Time: 4 hours. You need to be remotely supervised and ID-checked via Zoom. You need to have a computer/device with camera and microphone.
2. Examiner: Balazs Kulcsar (☎1785, ZOOM, Skype for Business)
3. 20 points can be reached in total (resolution 0.5 points). Table 1 shows the limits for each grade.

Tabell 1: Grades

Points	Grade
≤ 9.5	Failed
$10 \ldots 12.5$	3
$13 \ldots 15.5$	4
$16 \ldots 20$	5

4. Random sets of exam questions may randomly be assigned to students.
5. Handwritten solutions are requested (name and cid on each pages). When done, scan/photograph your solutions and compile it into one pdf document. Upload your file to Canvas (submission site closes 30 min after the examination). If the electronic version of the solution is not readable from the file, it will not be assessed (with 0 point).
6. Cooperation with or external help from other person is prohibited during examination! If signs of cooperations/external help are being discovered, *all exams involved will automatically be disqualified* and we will *automatically* report the *case to Chalmers with a request for suspension at Swedish Higher Education Authority.*
7. We may call students for oral post-check of solutions in the exam period (within a few days after the exam). Then, students will be asked (with short notice) to explain their solutions or answers.
8. All other aids can be used (books, notes, Matlab, etc.).
9. Teacher(s) will online be available. Examination results will be advertised approx 10 days after the exam. Inspection of results via Zoom.

Good luck!

Questions

1. Briefly answer the questions below.

   a) True or false (briefly motivate your answer)? If a system model is linear it is time invariant as well. (**1 point**)

   Definitions of linearity and invariance show that these are different system properties. Linear systems can be time variant as well, so the implication is incorrect.

   b) True or false (briefly motivate your answer)? Input-output and internal stability concepts are identical. (**1 point**)

   Differences: internal stability claims the stability of the state space model, while input-output claims it only for the transfer function related model. If the state space model is minimal, the two properties coincide. Otherwise, internal stability implies input-output one, but not the way around.

   c) True or false (briefly motivate your answer)? Pure time delayed system models are non-minimum phase models. (**1 point**)

   True. a pure time delay means infinitely large phase delay while $\omega \mapsto \infty$.

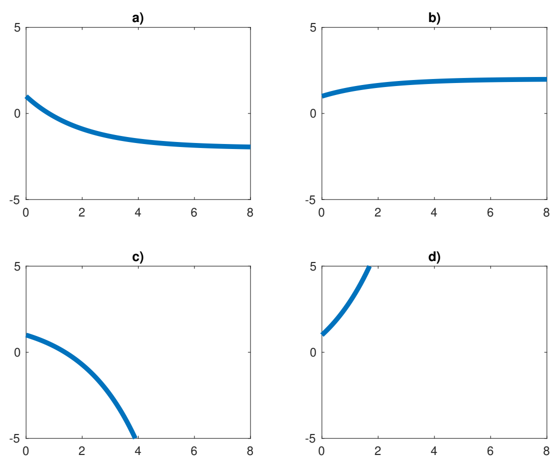
   d) Given a nominal system model $G_n(s)$ (to model the same true/real system model $G(s)$). Given its multiplicative $\Delta_m(s)$ and its additive $\Delta_a(s)$ uncertainty. Show that $\Delta_m(s) = \frac{\Delta_a(s)}{G_n(s)}$. (**1 point**)

   $\Delta_m = \frac{\Delta_a}{G_n} = \frac{G-G_n}{G_n}$ that is the definition of the multiplicative uncertainty.

   e) Given a minimal state space model with finite state dimension. Your task is to design a state feedback controller. Explain the main steps of doing it (no equations need to be solved). (**1 point**)

   Controllability form, eigenvalues of the closed loop system, open loop and closed loop coefficients for the char polynomial, create L, transform back to the original state space coordinates.

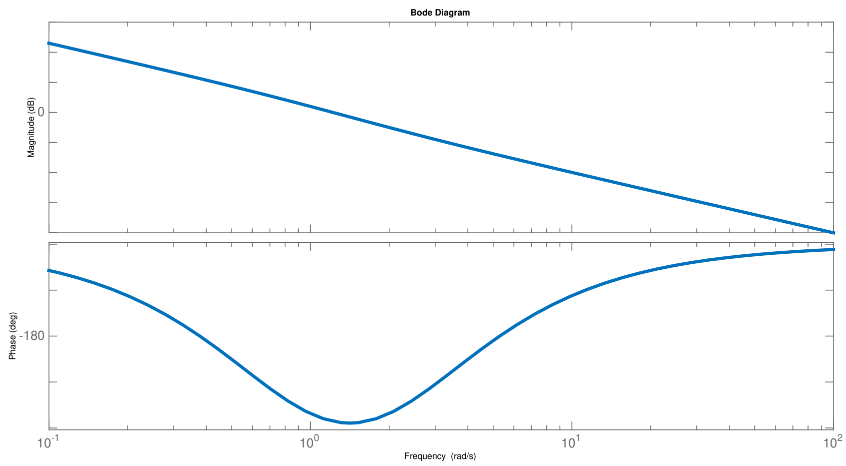
2. In Figure 1 depicts 4 step responses. Given $G(s) = \frac{s+\beta}{s+\alpha}$ (asymptotically stable, but non-minimum phase system). Which of the plots in Fig 1 belongs to the transfer function model? (**2 point**)



Figur 1: Step response plots for $G(s)$

Between a) and b) (stable) β being negative (based on NMP property)shows that the right answer is a).

3. Shown in Figure 2 a loop transfer function $L(i\omega) = K_0 G(i\omega)$ where $L(i\omega)$ has no open loop unstable poles. Is the closed loop (under unity feedback like in Fig. 3) is stable? Motivate your answer. (**2 point**)



Figur 2: Bode plot of the loop transfer function

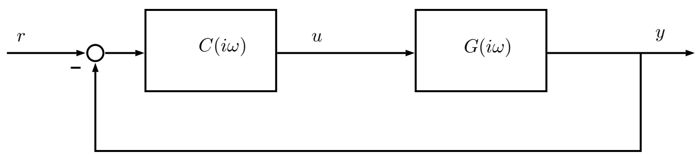
Phase margin at cross over frequency is negative, that implies the closed loop is unstable

4. Shown that the Nyquist plot of $\frac{1}{i\omega+1}$ for positive ω is a semi-circle (radius: $\frac{1}{2}$, center: $(\frac{1}{2}, 0)$). (**2 points**)
   $Im(G(i\omega)) = \frac{1}{1+\omega^2}$, $Re(G(i\omega)) = \frac{-\omega}{1+\omega^2}$. Statement is: $(X-\frac{1}{2})^2 + (Y-0)^2 = (\frac{1}{2})^2 = X^2 + (\frac{1}{2})^2 - 2\frac{1}{2} + Y^2$ is fulfilled because $X^2 - 2\frac{1}{2}X + Y^2 = 0$ with the Im and Re parts substituted in the equation.

5. Given a PI controller $C(i\omega) = K_p(1 + \frac{1}{i\omega})$ and a transfer function for a model described by $G(i\omega) = \frac{6i\omega}{(i\omega+1)((i\omega)^2+3i\omega+2)}$ in a closed loop setup depicted in Figure 3.

   a) Find K_p such that $\varphi_m = 40$ degree. (**2 point**).

   b) What is the gain margin with K_p found in a)? (**1 point**)



Figur 3: Återkopplat blockschema

$K_p \approx 3$, $g_m \approx 11dB$

6. Given a following transfer function,

$$G(s) = \frac{s+1}{(s+2)(s-3)}$$

a) Derive a diagonal state space representation for $G(s)$. Is the diagonal state space model stable? (**1 point**)

b) Given a sampling time $T = 1$, find the discrete time state space representation for the continuous one in a) (**1 point**)

c) Assuming observability is always satisfied, find the observable canonical state space representation. (**0.5 point**)

d) Find the state observer gain, K, as $K = [\kappa_1 \ \ \kappa_2]^T$. First find K that moves the observer's eigenvalues are moved to -1 and -2. (**1.5 point**)

e) Assuming controllability is always satisfied, find the controller canonical state space representation. (**0.5 point**)

f) Find the state feedback gain, L, as $L = [\ell_1 \ \ \ell_2]$. First find K that moves the closed loop system's eigenvalues to -3 and -4. (**1.5 point**)

a)

$$A = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} -\frac{1}{5} \\ \frac{4}{5} \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Unstable, one of the eigenvalues has positive real part.

b)

$$A_d = \begin{bmatrix} 0.13 & 0 \\ 0 & 20.1 \end{bmatrix}, B_d = \begin{bmatrix} -0.1 \\ 5.1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix}$$

c)

$$A_o = \begin{bmatrix} 1 & 1 \\ 6 & 0 \end{bmatrix}, B_o = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix}$$

d) $K = [3 \ 1]^T$

e)

$$A_c = \begin{bmatrix} 1 & 6 \\ 1 & 0 \end{bmatrix}, B_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix}$$

f) $L = [8 \ 18]$