# Exam in <br> SSY305 Communication Systems 

## Department of Electrical Engineering

Exam date: March 14, 2024

## Teaching Staff

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Material Allowed material:

- Chalmers-approved calculator.
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your handwritten notes. Both sides of the page can be used. Photocopies, printouts, other students' notes, or any other material is not allowed.
- A paper-based dictionary without added notes (electronic dictionaries are not allowed).

Students are required to solve the exam problems individually. Cooperation, in any form or with anyone, is strictly forbidden.

Grading A correct, precise, and well-motivated solution gives 12 points per problem at most. An incorrect answer or unclear, incomplete, or poorly motivated solutions give point reductions to a minimum of 0 points. No fractional points are awarded. Answers in any other language than Swedish or English are ignored.

Solutions Are made available at the earliest on March 15 on the course web page.
Results Exam results are posted on Canvas no later than March 28. The grading reviews will be on April 2 according to a process explained on the course webpage.

Grades The final grade of the course will be decided by the project (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). The project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade.

| Total Score | $0-39$ | $40-68$ | $69-79$ | $\geq 80$ |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Fail | 3 | 4 | 5 |

## Table over the Q-function

|  | Q(x) |  | Q(x) |  | Q(x) |  | $Q(x)$ |  | Q(x) |  | Q(x) |  | Q(x) |  | $Q(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | $5.0000 \mathrm{E}-01$ | 0.76 | $2.2360 \mathrm{E}-01$ | 1.52 | $6.4260 \mathrm{E}-02$ | 2.28 | $1.1300 \mathrm{E}-02$ | 3.04 | $1.1830 \mathrm{E}-03$ | 3.80 | $7.2350 \mathrm{E}-05$ | 4.56 | $2.5580 \mathrm{E}-06$ | 5.32 | 5.1880 E |
| 0.01 | $4.9600 \mathrm{E}-01$ | 0.77 | $2.2060 \mathrm{E}-0$ | 1.5 | $6.3010 \mathrm{E}-02$ | 2.29 | 1.1 | 3.05 | $1.1440 \mathrm{E}-03$ | 3.81 | $6.9480 \mathrm{E}-05$ | 4.57 | $2.4390 \mathrm{E}-06$ | 3 |  |
| 0.02 | 4.9200 | 0. | $2.1770 \mathrm{E}-01$ |  | $6.1780 \mathrm{E}-02$ | 2. | $1.0720 \mathrm{E}-02$ | 3.0 | $1.1070 \mathrm{E}-03$ | 3.8 | 5 | 8 | 6 | 5.34 | $4.6470 \mathrm{E}-08$ |
| 0.03 | $4.8800 \mathrm{E}-01$ | 0.79 | $2.1480 \mathrm{E}-01$ | 1.55 | 6.0 | 2.3 | 1.0 | 3.07 | $1.0700 \mathrm{E}-03$ |  | 5 | 4.59 | $2.2160 \mathrm{E}-06$ | 5.35 | $4.3980 \mathrm{E}-08$ |
| 0.04 | $4.8400 \mathrm{E}-01$ | 0.8 | 2.1 | 1.5 | 5.938 | 2.3 | 1.01 | 3.08 | 1.03 | 3.8 | 6.1 | 4.60 | $2.1120 \mathrm{E}-06$ | 5.36 | $4.1610 \mathrm{E}-08$ |
| 0.05 | 4.8010 | 0.81 | 2.0900 | 1.57 | 5.82 | 2.33 | 9.9030 | 3.09 | 1.00 | 3.85 | $5.9060 \mathrm{E}-05$ | 4.61 | 30 | 5.3 | $3.9370 \mathrm{E}-08$ |
| . 06 | 4.761 | 0.82 | 2.061 | 1.58 | 5.7 | 2.34 | 9.6 | 3.10 | 9.6760E-04 | 3.86 | $5.6690 \mathrm{E}-05$ | 4.6 | 6 | 8 | $3.7240 \mathrm{E}-08$ |
| 0.07 | 4.72 | 0.83 | 2.0330E-01 | 1.59 | 5. | 2. | 9. | 3.11 | $9.3540 \mathrm{E}-04$ | 3.87 | 5. | 4.63 | 1.8280E-06 | 5.39 | $3.5230 \mathrm{E}-08$ |
| 0.08 | 4.6 | 0.84 | 2.0 | 1.60 | 5. | 2.3 | 9.1 | 3.12 | $9.0430 \mathrm{E}-04$ | 3.8 | $5.2230 \mathrm{E}-05$ | 4.64 | 1.7420E-06 | 5.40 | 3.3 |
| 0.09 | $4.6410 \mathrm{E}-01$ | 0.85 | 770 | 1.61 | $5.3700 \mathrm{E}-02$ | . 3 | 8.8940 | 3.1 | 7400E-04 | 3.89 | $5.0120 \mathrm{E}-05$ | 4.6 | 6600E-06 | 5.4 | 3.1510E-08 |
| 0.10 | 4.6020 | 0.86 | 1.9490E-01 | 1.62 | 5. | 2.3 | 8.6 | 3.14 |  | 3.90 | 5 | 6 | 6 | 5.42 | $2.9800 \mathrm{E}-08$ |
| 0.11 | 4.5620 | 0.87 | $1.9220 \mathrm{E}-01$ | 1.63 | 5. | 2.39 | 8. | 3.15 | 8.1640E-04 | 3.9 |  | 4.67 | 1.5060E-06 | 5.43 | $2.8180 \mathrm{E}-08$ |
| 0.12 | 4. | 0.88 | 1.8940E-0 | 1.64 | 5. | 2.40 | 8. | 3.16 | 7.888 | 3.9 | 4. | 4.68 | $1.4340 \mathrm{E}-06$ | 5.44 | 2.6 |
| 0.13 | $4.4830 \mathrm{E}-01$ | 0.8 | 1.8670E-01 | 1.65 | 4.9470 | 2.41 | 7.9760 | 3.17 | 6220 | 3.93 | $4.2470 \mathrm{E}-05$ | 4.69 | 3660E-06 | 5.45 | $2.5180 \mathrm{E}-08$ |
|  | 4.44 | 0 |  | 1. | 4.8460E-02 | 2.42 | $7.7600 \mathrm{E}-03$ | 3. | $7.3640 \mathrm{E}-04$ | 3.94 | 5 | 0 | 1.3010E-06 |  | $2.3810 \mathrm{E}-08$ |
| 0.15 | 4.40 | 0 | $1.8140 \mathrm{E}-01$ | 1.67 | 4. | 2.43 | 7. | 3. | $7.1140 \mathrm{E}-04$ | 3.9 | 3.9 | 4.71 | 1.2390E-06 | 5.47 | $2.2500 \mathrm{E}-08$ |
| 0.16 | 4.364 | 0.92 | $1.7880 \mathrm{E}-01$ | 1.68 | 4. | 2.44 | 7. | 3. | 6. | 3.96 | $3.7470 \mathrm{E}-05$ | 4.72 | $1.1790 \mathrm{E}-06$ | 5.48 | $2.1270 \mathrm{E}-08$ |
| 0.17 | 4.3250 | 0.93 | 1.7620E-01 | 1.69 | $4.5510 \mathrm{E}-02$ | 2.4 | 7.1430E-03 | 3.21 | 6.63 | 3.97 | $3.5940 \mathrm{E}-05$ | 4.73 | 1230E-06 | 9 | $2.0100 \mathrm{E}-08$ |
| 0.18 | 4.286 | 0 | 1.7360E-01 |  | $4.4570 \mathrm{E}-02$ | 2.46 |  | 3. |  | 3.9 | $3.4460 \mathrm{E}-05$ | 4.74 | 6 | 5.50 |  |
| 0.19 | 4.247 | 0.95 | 1.7110E-01 | 1.71 | 4.363 | 2.47 | 6. | 3.23 | $6.1900 \mathrm{E}-04$ | 3.9 | $3.3040 \mathrm{E}-05$ | 4.75 | $1.0170 \mathrm{E}-06$ | 5.51 | 1.7 |
| 0.20 | 4.2070 | 0. | 1.6850 | 1. | 4.2720 | 2.48 | 6.5690 | 3.24 | 5.9760 | 4.00 | $3.1670 \mathrm{E}-05$ | 4.76 | 9.6800 | 5.52 | 1.6950E-08 |
| 0.21 | 4.168 | 0 | 1.6600 | 1. | 4.1820E-02 | 2.4 | $6.3870 \mathrm{E}-03$ | 3.25 |  | 4.01 | 3.0 | 7 | $9.2110 \mathrm{E}-07$ | 5.53 | $1.6010 \mathrm{E}-08$ |
| 0.22 | 4.1290 | 0.9 | 1.6350E-01 | 1.7 | 4.0930E-02 | 2.50 |  | 3.2 |  | 4.02 | $2.9100 \mathrm{E}-05$ | 4.78 | $8.7650 \mathrm{E}-07$ |  | $1.5120 \mathrm{E}-08$ |
| 0.23 | 4.0900 | 0.99 | 1.6110 | 1.75 | 4.0060E-02 | 2.51 | 6.0 | 3. | 5.3 | 4.03 | 2.78 | 4.79 | 8.33 | 5.55 | 1.4 |
| 0.24 | 4.0520 | 1.00 | 1.5 | 1.76 | 3.9200 | 2.52 | 5.868 | 3.2 | 5.19 | 4.04 | $2.6730 \mathrm{E}-05$ | 4.80 | 9330E | 5.56 | .3490E-08 |
| 0.25 | 4.0130 | 1.0 | 1.5620 | 1. | . 83 | 2.5 | 5.7030 | 3. | 5.0090E-04 | 5 | 2.5 | 4.8 | $7.5470 \mathrm{E}-07$ | 5.57 | $1.2740 \mathrm{E}-08$ |
| 0.26 | 3.97 |  | 1.5390E-01 | 1.7 | $3.7540 \mathrm{E}-02$ |  |  | 3. | $4.8340 \mathrm{E}-04$ | 4.06 | 2. | 4.82 | $7.1780 \mathrm{E}-07$ | 5.58 | $1.2030 \mathrm{E}-08$ |
| 0.27 | 3.9360 | 1.03 | 1.5150 | 1.79 | 3. | 2.5 | 5.3 | 3.3 | 665 | 4.07 | $2.3510 \mathrm{E}-05$ |  | 6.8270 | 5.59 | $1.1350 \mathrm{E}-08$ |
| 0.28 | 3.897 | 1.04 | 20 | 1.80 | 3. | 2.56 | $5.2340 \mathrm{E}-03$ | 3.32 | 501 | 4.08 | $2.2520 \mathrm{E}-05$ | 4.84 | $6.4920 \mathrm{E}-07$ | . 6 | 08 |
| 0.29 | 3.859 | 1.05 | 1.4690 | 1.8 | 3.5150 | 2.5 | 5.0850 | 3.33 | $4.3420 \mathrm{E}-04$ | 4.09 | 2.1 | 4.8 | $6.1730 \mathrm{E}-07$ | 5.61 | $1.0120 \mathrm{E}-08$ |
| 0.30 | 3.8 |  | $1.4460 \mathrm{E}-01$ | 1.8 | $3.4380 \mathrm{E}-02$ | 2.5 | $4.9400 \mathrm{E}-03$ | 3.34 | $4.1890 \mathrm{E}-04$ | 4.10 | $2.0660 \mathrm{E}-05$ |  | $5.8690 \mathrm{E}-07$ | 5.62 | 9.5 |
| 0.31 | 3.783 | 1.07 | $1.4230 \mathrm{E}-01$ | 1.83 | 3.3620 | 2.59 | $4.7990 \mathrm{E}-03$ | 3. | 4. | 4.11 | $1.9780 \mathrm{E}-05$ |  | 5.5800 | 5.63 | 9.0100E-09 |
| 0.32 | 3.745 | 1. | 1.4010 | 1.8 | 3.2880E-02 | 2.6 | $4.6610 \mathrm{E}-03$ | 3.36 | 3.8970E-04 | 4.12 | $1.8940 \mathrm{E}-05$ | 4.88 | $5.3040 \mathrm{E}-07$ | 5.64 | .5030E-09 |
| 0.33 | 3.70 | 1. | 1.3790 | 1.8 | 3.2160 | 2.6 | 4.52 | 3.3 | $3.7580 \mathrm{E}-04$ | 4.13 | $1.8140 \mathrm{E}-05$ | 4.89 | 5.0420E-07 | 5.65 | $8.0220 \mathrm{E}-09$ |
|  | 3.6 |  |  |  |  | 2.62 |  | 3.38 |  |  |  | 4.90 | 7 |  |  |
| 0.35 | $3.6320 \mathrm{E}-01$ |  | 35 | 1.87 | 3. | 2.63 | $4.2690 \mathrm{E}-03$ | 3.39 | 3.4950 | 4.15 | $1.6620 \mathrm{E}-05$ | 4. | 5540E-07 | 5.67 | .1400E-09 |
| 0.36 | 3.5940 | 1. | 1.31 | 1. | . 005 | 2.6 | 4.1450 | 3.40 | 369 | 4.16 | 1.59 | 4.9 | 327 | 5.68 | .7350E-09 |
| 0.37 | 3.5570 |  | 1.2920 |  | 2.938 |  | 4.0250 |  | $3.2480 \mathrm{E}-04$ | 4.17 | 1.52 | 4. | $4.1110 \mathrm{E}-07$ | 5.69 | $6.3520 \mathrm{E}-09$ |
|  | 3.52 |  |  |  | 2. | 2.66 |  |  | 3.1 | - | $1.4580 \mathrm{E}-05$ |  | $3.9060 \mathrm{E}-07$ | 5.7 | 5.9900E-09 |
| 0.39 | 3.4830 | 1.1 | $1.2510 \mathrm{E}-01$ | 1.91 | 2.8070 | 2.67 | $3.7930 \mathrm{E}-03$ | 3.43 | 3.0180E-04 | - | $1.3950 \mathrm{E}-05$ | 4.95 | $3.7110 \mathrm{E}-07$ | 5.71 | 09 |
| 0.40 | 3.4460 | 1.1 | 1.230 | 1.92 | $2.7430 \mathrm{E}-02$ | 2.6 | 3.6 |  | 2.90 | - | 1.3350 | 4. | 3.5250 | 5.72 | 5.32 |
|  | 3.4090 |  | 1.2100E-01 | 1.93 | $2.6800 \mathrm{E}-02$ | 2.6 |  |  | 2.8 |  | $1.2770 \mathrm{E}-05$ |  | , |  | $5.0220 \mathrm{E}-09$ |
|  | 3.3 | 1.18 | $1.1900 \mathrm{E}-01$ |  | 2. | 2.70 | 3. | 3.46 | $2.7010 \mathrm{E}-04$ | 4.22 | 20E-05 | 4.98 | $3.1790 \mathrm{E}-07$ | 5.74 | 09 |
|  | 3.3360 | 1.19 | 1.1700 | 1.9 | 2.5590 E | 2.7 | 3.3640 | 3.47 | 2.6 | 4.23 | $1.1680 \mathrm{E}-05$ | - | 3.0190 | 5.75 | 09 |
|  | 3.3000 | 1. | 1.1510 | 1.9 | 2.500 | 2.72 | 3.2 | 3. | 2.50 | 4.24 | - | 5.00 | 2.8670 | 5.76 | . 2 |
|  | 3.2 |  |  |  |  | 2.7 |  |  | 2. |  | $1.0690 \mathrm{E}-05$ |  | , | 5.77 | $3.9640 \mathrm{E}-09$ |
|  | 3.2 |  | 1.1120 | 1.98 | 2.3850 E |  | $3.0720 \mathrm{E}-03$ | 3.50 | 2.3 |  | 5 | 5.02 | $2.5840 \mathrm{E}-07$ | 5.78 | 09 |
|  | 3.1920 | 1.23 | 93 | 1.99 | . 33 | 2.7 | 2.98 | 3.51 | 2.241 | 4.27 | $9.7740 \mathrm{E}-06$ | 5.03 | 2.4520 | 5.79 | 3.5 |
|  | 3.1560 |  | 1.0750 E | 2.0 | 2.2750 E | 2.7 | 2.8 | 3.52 | 2.15 | 4.28 | $9.3450 \mathrm{E}-06$ | 5. | $2.3280 \mathrm{E}-07$ | 5.80 | 3. |
|  | 3.1210 | 1.25 | $1.0560 \mathrm{E}-01$ | 2. | 2. | 2. | 2.8 |  | 2.0 |  | $8.9340 \mathrm{E}-06$ | 5. | $2.2090 \mathrm{E}-07$ | 5.81 | $3.1240 \mathrm{E}-09$ |
|  | 3.0850 | 1.26 | 1.0380E-01 | 2.02 | 2.1690 | 2.7 | $2.7180 \mathrm{E}-03$ |  | . 001 | 4.30 | $8.5400 \mathrm{E}-06$ | 5.06 | - | 5.82 | -09 |
|  | 3.0500 | 1. | 1.0200 | 2.03 | . 1 | 2.7 | 2.6350 | 3. | 1.9260E-04 | 4.31 | 8.16 | 5.07 | 9890 | 5.83 | 2.7 |
|  | 3.01 | 1.2 | 1.0030E-01 | 2.0 | 2. | 2.8 | 2. | 3.56 | $1.8540 \mathrm{E}-04$ | 4.32 | $7.8010 \mathrm{E}-06$ | 8 | 1.8870E-07 |  | $2.6100 \mathrm{E}-09$ |
| . | 2.9810 | 1.29 | 9.8530 | . 0 | 2.0180 | 2.8 | $2.4770 \mathrm{E}-03$ | 3.5 | 50 | 4.33 | 4550 | 5.0 | 7900E-07 | 5.8 | $2.4580 \mathrm{E}-09$ |
|  | 2.9460 | 1.30 | $9.6800 \mathrm{E}-02$ | 2.0 | 1.9700 | 2.8 | . 40 | 3.5 | 1.7180E-04 | 4.3 | 7.1240 | 5. | 6980 | 5.86 | $2.3140 \mathrm{E}-09$ |
|  | 2.91 | 1.3 | 9.5100 | 2.07 | 1.9 | 2.83 | 2.32 | 3.5 | $1.6530 \mathrm{E}-04$ | 4. | 6.807 | 5. | 6110 | 5.87 | 2.1 |
|  | 2.8 | 1.32 | 9.3 | 2. | 1. | 2.8 | 2. | 3.6 | 1.5 |  | 6.5 | 5. | 1.5280E-07 | 5.88 | $2.0510 \mathrm{E}-09$ |
| 0.57 | 2.8430E-01 | 1. | 9.1760 | 2.09 | 1.831 | 2.85 | . 1860 | 3.6 | 1.5310 E | 4.37 | $6.2120 \mathrm{E}-06$ | 5. | $4490 \mathrm{E}-07$ | 5.8 | 1.9310E-09 |
| 0.58 | 2.8100 | 1. | 9.0120E-02 | 2.10 | 1.78 | 2.86 | 2.1180 | 3.6 | 1.47 | 4.3 | 5.9340 | 5. | 1.3740 | 5.90 | 1.8180E-09 |
|  | 2.7 | 1.35 | 8.8510E-02 | 2.1 | 1.7430 | 2.8 | 2.0520 | 3. | $1.4170 \mathrm{E}-04$ | 4.39 | 5.6 | 5. | 3020 | 5.91 | 1.7 |
| 0.60 | $2.7430 \mathrm{E}-01$ | 1. | 8.6910 | 2.1 | 1.7000 | 2.8 | 1.988 | 3.6 | 1.3630 | 4. | 5.4130 | 5.16 | $1.2350 \mathrm{E}-07$ | 5.9 | 1.6100E-09 |
|  | $2.7090 \mathrm{E}-01$ | 1.3 | 8.5340 | 2.13 | 1.6590 | 2.89 | 260 | 3.65 | 1.3110 | 4.4 | 5.1690 | 5.1 | $1.1700 \mathrm{E}-07$ | 5.93 | 1.5150E-09 |
| 0.62 | 2.676 | 1.38 | . 37 | 2.1 | 1.61 | 2.90 | 1.8660E-03 | 3.6 | 1.26 | 4.4 | 4.9350 | 5.1 | 1090 | 5.94 | $1.4250 \mathrm{E}-09$ |
| 0.63 | 2.6 | 1.39 | 8. | 2.15 | 1.5 | 2.91 | 1. | 3.6 | 1.2 | 4.43 | $4.7120 \mathrm{E}-06$ | 5. |  | 5.95 | $1.3410 \mathrm{E}-09$ |
| 0.64 | $2.6110 \mathrm{E}-01$ | 1.40 | 8.0760 | 2.16 | 1.5390E | 2.9 | 1.7500 | 3.68 | $1.1660 \mathrm{E}-0$ | 4.4 | 4.4980E-06 | 5.20 | $9.9640 \mathrm{E}-08$ | 5.96 | $1.2610 \mathrm{E}-09$ |
| 0.65 | $2.5780 \mathrm{E}-01$ | 1.4 | 9270 | 2.1 | 1.5000 | 2.9 | .6950 | 3.6 | 1.1210 | 4.4 | 4.2940 | 5.2 | $9.4420 \mathrm{E}-08$ | 5.9 | $1.1860 \mathrm{E}-09$ |
| 0.66 | 2.5460 |  | 800 | . 18 | 1.4630 | 2.94 | 641 | 3.70 | 1.0780 | 4.46 | 4.0980 | 5.2 | $8.9460 \mathrm{E}-08$ | 5.9 | $1.1160 \mathrm{E}-09$ |
| 0.67 | 2.5140 E | 1.43 | 360 | 2.19 | 1. | 2.95 | 1.589 | 3. | 1.0360 | 4. | 3.91 | 5.23 | 8.4760 | 5. | $1.0490 \mathrm{E}-09$ |
| 68 | $2.4830 \mathrm{E}-01$ | 1.44 | 4930 | 2.20 | 1.3900 E | 2.9 | 1.5380 | 3.7 | $9.9610 \mathrm{E}-0$ | 4.48 | $3.7320 \mathrm{E}-06$ | 5.24 | $8.0290 \mathrm{E}-08$ | 6.00 | $9.8660 \mathrm{E}-10$ |
| . 69 | $2.4510 \mathrm{E}-01$ | 1. | 3530E | 2.2 | 1.3550E | 2.9 | 1.4890 | 3.7 | 9.5740 | 4.4 | 3.5610 | 5.2 | 7.6050 E | 6.0 | 9.2760 |
| 0.70 | $2.4200 \mathrm{E}-01$ | 1.46 | 2150 | 2.22 | .3210 | 2.9 | $1.4410 \mathrm{E}-03$ | 3.7 | 9.2010 | 4.50 | 3.3980 | 5.26 | $7.2030 \mathrm{E}-08$ | 6.0 | $8.7210 \mathrm{E}-10$ |
| . 71 | $2.3890 \mathrm{E}-01$ | 1. | 7.0780 | 2.23 | 1.2870E | 2.99 | 1.3950 E | 3.7 | 8.8420E-05 | 4.51 | $3.2410 \mathrm{E}-06$ | 5.27 | $6.8210 \mathrm{E}-08$ | 6.03 | $8.1980 \mathrm{E}-10$ |
| 72 | $2.3580 \mathrm{E}-01$ | 1.48 | $6.9440 \mathrm{E}-0$ | 2.24 | 1.2550E-02 | 3.00 | $1.3500 \mathrm{E}-03$ | 3.76 | 8.4960E-05 | 4.52 | $3.0920 \mathrm{E}-06$ | 5.28 | $6.4590 \mathrm{E}-08$ | 6.04 | 7.7060E-10 |
| 73 | $2.3270 \mathrm{E}-01$ | 1.49 | 6.8110 | 2.25 | $1.2220 \mathrm{E}-02$ | 3.01 | 1.3060 | 3.77 | $8.1620 \mathrm{E}-05$ | 4.53 | $2.9490 \mathrm{E}-06$ | 5.29 | $6.1160 \mathrm{E}-08$ | 6.05 | 7.2420 |
| 0.74 | $2.2960 \mathrm{E}-01$ | 1.50 | 6.6810E-02 | 2.26 | $1.1910 \mathrm{E}-02$ | . 02 | 1.2640 E | 3.78 | $7.8410 \mathrm{E}-05$ | 4.54 | $2.8130 \mathrm{E}-06$ | 5.30 | $5.7900 \mathrm{E}-08$ | 6.06 | $6.8060 \mathrm{E}-10$ |
| 0.75 | $2.2660 \mathrm{E}-01$ | 1.51 | 6.5520 E | 2.27 | $1.1600 \mathrm{E}-02$ | 3.03 | $1.2230 \mathrm{E}-03$ | 3.79 | $7.5320 \mathrm{E}-05$ | 4.55 | $2.6820 \mathrm{E}-06$ | 5.31 | $5.4810 \mathrm{E}-08$ | 6.07 | $6.3960 \mathrm{E}-1$ |

1. Consider the following transmission system


The physical layer signal alternatives are $\left\{s_{1}(t), s_{2}(t), s_{3}(t), s_{4}(t)\right\}$, where $s_{1}(t)$ and $s_{2}(t)$ are defined in the figure below and where $s_{3}(t)=-s_{1}(t)$ and $s_{4}(t)=-s_{2}(t)$.


(a) What is the maximum data rate that can be transmitted without ISI? (4p)
(b) What is the average symbol energy, the average bit energy, and the transmit power when transmitting at the rate in Part (a)? (4p)
(c) Suppose the received signal is $r(t)=s_{m}(t)+n(t)$, where $r(t)$ is defined in the figure below, $n(t)$ is the noise component, and $m$ is the transmitted symbol. What decision on $m$ would a minimum-distance receiver make? Motivate. (4p)


## Solution Problem 1

(a) In order to compute the data rate $R_{b}$ we need to apply the following:

$$
R_{b}=\frac{\log _{2}(M)}{T_{s}}
$$

where $T_{s}$ is the symbol duration and $M$ is the number of symbols. In our case, we have:

$$
T_{s}=1 \mu s, M=4 \rightarrow R_{b}=\frac{\log _{2}(4)}{1 \cdot 10^{-6}}=2 \mathrm{Mbit} / \mathrm{s}
$$

(b) The energy of a symbol $s_{i}$ can be computed as:

$$
E_{s_{i}}=\int_{0}^{T}\left|s_{i}(t)\right|^{2} d t \rightarrow \int_{0}^{10^{-6}} \frac{1}{4} d t=\frac{1}{4} 10^{-6} \mathrm{~J} \quad \forall i=1 \ldots 4
$$

The average symbol energy can be computed as follows:

$$
E_{s}=\sum_{i=1}^{4} p_{i} \cdot E_{s_{i}}=\frac{1}{4} 10^{-6} \mathrm{~J}
$$

where $p_{i}=\frac{1}{4}$ is the probability that symbol $s_{i}$ is transmitted. We can then compute the value of the average energy per bit as:

$$
E_{b}=\frac{E_{s}}{\log _{2}(M)}=\frac{1}{8} 10^{-6} \mathrm{~J}
$$

The average transmit power can be computed as:

$$
P=\frac{E_{s}}{T}=0.25 \mathrm{~W}
$$

(c) A minimum distance receiver computes the distances between the received signal and all signal alternatives. At the end, the signal alternative with the smallest (squared) distance is chosen.

$$
m^{*}=\arg \min _{m}\left(\left\|r(t)-s_{m}(t)\right\|\right)^{2}=\arg \min _{m} \int_{-\infty}^{\infty}\left|r(t)-s_{m}(t)\right|^{2} d t
$$

We need to compute the distance squared between $r(t)$ and each $s_{m}(t)$ :

$$
\begin{aligned}
& d_{1}^{2}=\int_{-\infty}^{\infty}\left|r(t)-s_{1}(t)\right|^{2} d t=\int_{0}^{T / 2}(1 / 4+1 / 2)^{2} d t+\int_{T / 2}^{T}(2-1 / 2)^{2} d t=\frac{45}{32} 10^{-6} \\
& d_{2}^{2}=\int_{-\infty}^{\infty}\left|r(t)-s_{2}(t)\right|^{2} d t=\int_{0}^{T / 2}(1 / 4+1 / 2)^{2} d t+\int_{T / 2}^{T}(2+1 / 2)^{2} d t=\frac{109}{32} 10^{-6} \\
& d_{3}^{2}=\int_{-\infty}^{\infty}\left|r(t)-s_{2}(t)\right|^{2} d t=\int_{0}^{T / 2}(1 / 4-1 / 2)^{2} d t+\int_{T / 2}^{T}(2+1 / 2)^{2} d t=\frac{101}{32} 10^{-6} \\
& d_{4}^{2}=\int_{-\infty}^{\infty}\left|r(t)-s_{2}(t)\right|^{2} d t=\int_{0}^{T / 2}(1 / 4-1 / 2)^{2} d t+\int_{T / 2}^{T}(2-1 / 2)^{2} d t=\frac{37}{32} 10^{-6}
\end{aligned}
$$

Comparing the results, $d_{4}^{2}$ has the smallest value. As a result, the minimum distance receiver will pick $s_{4}(t)$.
2. Consider a LAN with four hosts, ST-A, ST-B, ST-C, and ST-D, as in the figure below. Suppose that $d=100 \mathrm{~m}$ and that medium propagation speed is $c=2 c_{0} / 3$, where $c_{0}=$ $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ is the speed of light in vacuum. The medium bit rate is $R=100 \mathrm{Mbit} / \mathrm{s}$.


The link layer uses a modified version of the Stop-and-Wait ARQ protocol to provide a reliable, in-sequence packet transmission service for the network layer. In this version of Stop-and-Wait ARQ, a receiver will not send an ACK back after receiving a data frame with transmission errors.
The link layer information frames are $n_{f}$ bits long, including a 26 byte header and 32 CRC parity bits. An ACK frame is 74 bytes long. The medium introduces independent bit errors with probability $p$.
We assume that error detection is perfect (i.e., the receiver link layer does not accept erroneous frames). The receiver and transmitter processing times are negligible. The ARQ protocol ignores any information frames or ACK frames with detected errors.
To share the channel, nodes use persistent-CSMA with sensing time $4 \mu$ s. For simplicity, we will assume that only host ST-C has data to send, i.e., there will be no collisions on the medium. We will also assume that ACK frames are always error-free.
(a) How should the frame length and timeout be chosen such that the bit rate experienced by the network layer is $80 \%$ of $R$ when $p=0$ ? ( 2 p )
(b) Assuming $p=10^{-5}$, what is the experienced bit rate for the network layer for the frame length and timeout calculated in Part (a)? (4p)
(c) Assuming $p=10^{-5}$ and a timeout double the one calculated in Part (a). The frame length is the same as calculated in Part (a). What is the experienced bit rate for the network layer? ( 4 p )
(d) Repeat part (c) with $p=10^{-4}$. What can be said about the importance of choosing the time out? (2p)

## Solution Problem 2

(a) In the absence of transmission errors in the medium, the effective rate experienced by the network layer is defined as

$$
R_{e f f}=\frac{n_{f}-n_{0}}{t_{0}}
$$

where $n_{f}$ is the number of bits on the link layer PDU and $n_{0}=26 \cdot 8+32=240$ bits. The value of $t_{0}$ represents the turnaround time, and it can be computed as follows:

$$
t_{0}=2 t_{\text {sens }}+2 t_{\text {prop }}+t_{f}+t_{a}=2 \cdot\left(t_{\text {sens }}+t_{\text {prop }}\right)+\frac{n_{f}}{R}+\frac{n_{a}}{R}
$$

where $t_{\text {sens }}$ is the sensing time of the CSMA protocol (i.e., we need to sense the channel before sending the data frame and also before sending an ACK frame), $t_{\text {prop }}$ is the propagation delay. At the same time, $t_{f}$ and $t_{a}$ are the time required to transmit a data and an ACK frame, respectively.
Since it is required to have $R_{e f f}=0.8 \cdot R$, then

$$
\begin{gathered}
0.8 \cdot R=\frac{n_{f}-n_{0}}{t_{0}} \rightarrow n_{f}=0.8 \cdot R \cdot\left[2 \cdot\left(t_{\text {sens }}+t_{\text {prop }}\right)+\frac{n_{f}}{R}+\frac{n_{a}}{R}\right]+n_{0} \rightarrow \\
n_{f}=4 \cdot R \cdot\left[2 \cdot\left(t_{\text {sens }}+t_{\text {prop }}\right)+\frac{n_{a}}{R}\right]+\frac{n_{0}}{0.2}
\end{gathered}
$$

Plugging in the values of the known quantities, we obtain:

$$
t_{\text {prop }}=\frac{2 \cdot d}{c}=1 \mu \mathrm{~s}, t_{\text {sens }}=4 \mu \mathrm{~s}, n_{a}=74 \cdot 8=592 \mathrm{bits} \rightarrow n_{f}=7568 \mathrm{bits}
$$

When computing $t_{\text {prop }}$, we need to assume the worst case regarding the transmission distance, i.e., $2 \cdot d$.
Knowing the value of $n_{f}$, we can derive also the value of $t_{0}$ as

$$
t_{0}=2 \cdot\left(t_{\text {sens }}+t_{\text {prop }}\right)+\frac{n_{f}}{R}+\frac{n_{a}}{R}=91.6 \mu \mathrm{~s}
$$

Now that we know the value of $t_{0}$, we can set the timeout $t_{\text {out }}$ to the same value to avoid unnecessary retransmissions.

$$
t_{\text {out }}=t_{0}=91.6 \mu \mathrm{~s}
$$

(b) In the presence of transmission errors in the medium, the effective rate experienced by the network layer is defined as

$$
R_{e f f}=\frac{n_{f}-n_{0}}{E\left[t_{s w}\right]}
$$

where

$$
E\left[t_{s w}\right]=t_{0}+t_{\text {out }} \cdot \frac{P_{f}}{1-P_{f}}
$$

The frame error probability $P_{f}$ can be computed from the value of the bit error probability $p=10^{-5}$ as follows

$$
P_{f}=1-(1-p)^{n_{f}} \rightarrow P_{f}=0.073
$$

If $t_{\text {out }}=t_{0}$

$$
E\left[t_{s w}\right]=\frac{t_{0}}{1-P_{f}} \rightarrow R_{e f f}=\frac{n_{f}-n_{0}}{t_{0}} \cdot\left(1-P_{f}\right)=74.17 \mathrm{Mbit} / \mathrm{s}
$$

(c) When $t_{\text {out }}=2 \cdot t_{0}$ we have that

$$
E\left[t_{s w}\right]=t_{0}+t_{\text {out }} \cdot \frac{P_{f}}{1-P_{f}}=t_{0}+2 \cdot t_{0} \cdot \frac{P_{f}}{1-P_{f}}=\frac{t_{0} \cdot\left(1+P_{f}\right)}{1-P_{f}}
$$

as a result

$$
R_{e f f}=\frac{n_{f}-n_{0}}{t_{0} \cdot\left(1+P_{f}\right)} \cdot\left(1-P_{f}\right)=69.13 \mathrm{Mbit} / \mathrm{s}
$$

(d) When $p=10^{-4}$

$$
P_{f}=1-(1-p)^{n_{f}} \rightarrow P_{f}=0.5309
$$

and

$$
R_{e f f}=\frac{n_{f}-n_{0}}{t_{0} \cdot\left(1+P_{f}\right)} \cdot\left(1-P_{f}\right)=24.52 \mathrm{Mbit} / \mathrm{s}
$$

Looking at the result from Part (b) and (c), we see how increasing the value of $t_{\text {out }}$ an impact on the effective rate, i.e., from $74.17 \mathrm{Mbit} / \mathrm{s}$ when $t_{\text {out }}=t_{0}$ to $69.13 \mathrm{Mbit} / \mathrm{s}$ when $t_{\text {out }}=2 \cdot t_{0}$ This is because when transmission errors occur, the ARQ must wait longer to re-transmit a frame. This is even more evident when the medium introduces errors more frequently ( $p=10^{-4}$ causing even more frequent retransmissions. This confirms the need to set the time out as close to the turnaround time as possible.
3. Consider a LAN with $M=100$ attached stations. The protocol stack consists of a physical layer that provides a data rate $R=100 \mathrm{Mbit} / \mathrm{s}$, a network layer, and a link layer. We assume that the physical layer is quite robust: in the absence of collisions on the medium, we can neglect frame errors. The LAN medium length is $d=1000 \mathrm{~m}$, and the propagation speed is $c=2 \times 10^{8} \mathrm{~m} / \mathrm{s}$. We assume that the M stations can be located anywhere over the LAN. The network layer PDUs have size 1000 bytes, and the link layer header and trailer are together 24 bytes.
(a) Suppose that the MAC layer is TDMA with slot duration $T_{\mathrm{s}}$. We design the slot time such that (i) the bits transmitted in a slot can carry an entire network layer PDU, (ii) no collisions can occur at any receiving station, and (iii) maximize the data rate for the stations. What is the (average) data rate $R_{u}$ in bit/s that the network layer entity in a station can experience? Assume that the station always has data to transmit. (3p)
(b) Let $R_{L A N, u}$ be the aggregated, useful data rate on the LAN, i.e., the average number of network PDU bits from all stations transmitted on the LAN per second. Suppose that $N$ of the $M$ stations have data to transmit. What is the normalized throughput, $\rho=R_{L A N, u} / R$, for the TDMA scheme in Part (a) when $N=M$ and $N=M / 10$, respectively? (2p) Comment on the results. (1p)
(c) Repeat Part (a) and (b) when $R=1$ Gbit/s. Comment on the results. (2p)
(d) Suppose we replace the TDMA scheme with a simple reservation scheme. The reservation messages are 24 byte long and are transmitted in minislots of duration $T_{\mathrm{ms}}$, which is designed such that (i) the bits transmitted in a slot can carry a whole reservation message, (ii) no collisions can occur at any receiving station, and (iii) to maximize the data rate for the stations. A cycle starts with the $M$ stations sending one reservation message each. Then, the $N$ stations that have data to transmit send a single network PDU each, which completes the cycle. What is the highest data rate $R_{u}$ in bit/s that the network layer entity in a station can experience when $N=M$ and $N=M / 10$, respectively? Assume $R=1$ Gbit/s. (3p)
(e) Comment on the results in Part (d) vs. the ones in Part (c). (1p)

## Solution Problem 3

(a) The duration of a TDMA slot duration is equal to

$$
T_{s}=t_{\text {prop }}+T_{f}
$$

where $t_{\text {prop }}$ is the maximum propagation delay on the LAN, and $T_{f}$ is the transmission time of a link layer PDU. The size of a network layer PDU is $n_{P D U}=1000 \cdot 8$ bits, while the overhead bits are $n_{0}=24 \cdot 8$. We can then compute:

$$
t_{\text {prop }}=\frac{d}{c}=\frac{1000}{2 \cdot 10^{8}}=5 \mu s, T_{f}=\frac{n_{P D U}+n_{0}}{R}=81.92 \mu s \rightarrow T_{s}=86.92 \mu \mathrm{~s}
$$

The rate experienced by the network layer at each station can be expressed as:

$$
R_{u}=\frac{n_{P D U}}{M \cdot T_{s}}=0.92 \mathrm{Mbit} / \mathrm{s}
$$

(b) When N stations are transmitting the aggregated rate of the LAN (i.e., $R_{L A N, u}$ ) can be computed as:

$$
R_{L A N, u}=\frac{N \cdot n_{P D U}}{M \cdot T_{s}}
$$

When $\mathrm{N}=\mathrm{M}$

$$
R_{L A N, u}=\frac{n_{P D U}}{T_{s}}=92.04 \mathrm{Mbit} / \mathrm{s} \rightarrow \rho=\frac{R_{L A N, u}}{R}=\frac{n_{P D U}}{R \cdot T_{s}}=0.92
$$

When $\mathrm{N}=\mathrm{M} / 10$

$$
R_{L A N, u}=\frac{n_{P D U}}{10 \cdot T_{s}}=9.204 \mathrm{Mbit} / \mathrm{s} \rightarrow \rho=\frac{R_{L A N, u}}{R}=\frac{n_{P D U}}{R \cdot 10 \cdot T_{s}}=0.092
$$

We can observe how the number of stations transmitting traffic impacts the value of $\rho$. The lower the value of N , the lower the value of the normalized throughput of the system. This is because we have fewer empty data slots, which are wasted bandwidth.
(c) When the rate increases to $R=1 \mathrm{Gbit} / \mathrm{s}$, the value of the propagation delay does not change, but the values of $T_{f}$ and $T_{s}$ change as follows:

$$
T_{f}=\frac{n_{P D U}+n_{0}}{R}=8.192 \mu s \rightarrow T_{s}=13.192 \mu s
$$

As a result, we have

$$
R_{u}=\frac{n_{P D U}}{M \cdot T_{s}}=6.06 \mathrm{Mbit} / \mathrm{s}
$$

The rate experienced by the network layer at each station increases. This is because, with a higher LAN rate, each TDMA slot $T_{s}$ is shorter, allowing each station to transmit more frequently. The impact of the new value of R on $R_{L A N, u}$ is as follows. When $\mathrm{N}=\mathrm{M}$

$$
R_{L A N, u}=\frac{n_{P D U}}{T_{s}}=606.4 \mathrm{Mbit} / \mathrm{s} \rightarrow \rho=\frac{R_{L A N, u}}{R}=\frac{n_{P D U}}{R \cdot T_{s}}=0.606
$$

When $\mathrm{N}=\mathrm{M} / 10$

$$
R_{L A N, u}=\frac{n_{P D U}}{10 \cdot T_{s}}=60.64 \mathrm{Mbit} / \mathrm{s} \rightarrow \rho=\frac{R_{L A N, u}}{R}=\frac{n_{P D U}}{R \cdot 10 \cdot T_{s}}=0.0606
$$

In both cases, the normalized throughput $\rho$ decreases compared to the values obtained in Part (b). This is because, with a higher LAN rate, the propagation delay is no longer negligible compared to the time required to transmit a link layer PDU $T_{f}$ (i.e., the value of the delay bandwidth product increases). This is overhead time in which each station occupies resources in the LAN without transmitting any data.
(d) The duration of a reservation minislot $T_{m s}$ can be computed as:

$$
T_{m s}=t_{\text {prop }}+T_{\text {res }}
$$

where $t_{\text {prop }}$ is the maximum propagation delay on the LAN, and $T_{\text {res }}$ is the transmission time of reservation message of $n_{\text {res }}=24.8$ bits. We can then compute:

$$
t_{\text {prop }}=\frac{d}{c}=5 \mu \mathrm{~s}, T_{\text {res }}=\frac{n_{\text {res }}}{R}=0.192 \mu \mathrm{~s} \rightarrow T_{\text {res }}=5.192 \mu \mathrm{~s}
$$

For the data slots, the values of $T_{f}$ and $T_{s}$ are the same as the ones computed in Part (c):

$$
T_{f}=\frac{n_{P D U}+n_{0}}{R}=8.192 \mu s \rightarrow T_{s}=13.192 \mu \mathrm{~s}
$$

The data rate $R_{u}$ experienced by the network layer entity in a station can be computed as:

$$
R_{u}=\frac{n_{P D U}}{M \cdot T_{\text {res }}+N \cdot T_{s}}
$$

When $\mathrm{N}=\mathrm{M}$

$$
R_{u}=\frac{n_{P D U}}{M \cdot\left(T_{\text {res }}+T_{s}\right)}=4.35 \mathrm{Mbit} / \mathrm{s}
$$

When $\mathrm{N}=\mathrm{M} / 10$

$$
R_{u}=\frac{n_{P D U}}{M \cdot\left(T_{\text {res }}+0.1 \cdot T_{s}\right)}=12.29 \mathrm{Mbit} / \mathrm{s}
$$

(e) Comparing the results just computed with the value of $R_{u}=6.06 \mathrm{Mbit} / \mathrm{s}$ obtained in Part (c), we can derive the following. When all stations in the LAN are transmitting, a TDMA scheme is more efficient than a reservation scheme (i.e., 6.06 Mbit/s vs. $4.35 \mathrm{Mbit} / \mathrm{s})$. Data slots are allocated beforehand to all stations, saving the reservation overhead (i.e., $M \cdot T_{\text {res }}$ ) that a reservation scheme needs to coordinate the transmission. On the other hand, when only a few stations have data to transmit, a TDMA scheme is less efficient than a reservation scheme. In the former, each station can transmit only every $M \cdot T_{s}$ seconds, regardless of how many M stations are active. In the latter, the active stations get to transmit more often (i.e., $T_{\text {res }}+0.1 \cdot T_{s}=6.5 \mu \mathrm{~s}$ vs. $\left.T_{s}=13.192 \mu \mathrm{~s}\right)$.
4. (a) Explain the difference between CSMA with collision detection (CD) and CSMA with collision avoidance (CA). Which method would you recommend for use in wireless systems? Motivate. (3p)
(b) What is the purpose of the demodulator and decoder blocks in Shannon's communication model? (3p)
(c) What is the difference between a physical address, a network address, and a domain name? Give examples of each of them. (3p)
(d) Define the security goals of integrity and authentication. Which one is also able to provide the security goal of confidentiality inherently? (3p)
(a) In CSMA-CD, collisions are detected by monitoring the channel while transmitting frames. The current frame transmission is aborted if traffic from another source is detected during the contention period. In CSMA-CA, the channel is not monitored while a frame's transmission occurs. Collisions are declared in the absence of ACKs. Using full duplex communication is impossible in a wireless channel, so CD is not an option. Only CSMA-CA can be used when choosing between CSMA-CD and CSMA-CA.
(b) The purpose of the demodulator block is to convert the received analog signal to decisions on the transmitted bits. The channel decoder takes the output of the demodulator (i.e., bits). It exploits redundancy (e.g., parity bit, CRC bits, etc.) to estimate the sequence of information bits that were input to the channel encoder. This means that error detection, correction, and control are tasks of the channel decoder
(c) A physical address is a unique identifier assigned to a network interface card (NIC) to help transfer data packets to their intended destination within the same local network. Example: 6 bytes Ethernet addresses to identify each NIC on a LAN. This address is globally unique, meaning no two NICs have the same physical address. The first 24 bits identify the NIC manufacturer; the second 24 bits are the serial number. A network address is used by network-layer protocols, e.g., to route packets independently of the physical connection and topology of the network. It is also used to identify a network interface card uniquely. A node can have multiple network addresses, e.g., if it is connected to several networks. Example of network address: IP addresses. Physical and network addresses are difficult for humans to remember. Domain names are symbolic names that are unique and easy to remember. They are independent of physical location. Example: www.chalmers.se: chalmers.se is the domain name, and www is the host name.
(d) The goal of integrity is to guarantee that messages have not been altered in transmission by a third party. The goal of authentication is to ensure that a receiver can verify that a received message is from the claimed sender. None of these two methods can provide intrinsically a confidential exchange of information. To accomplish the goal of confidentiality, they need to be combined with an encryption method.

