Exam in
SSY305 Communication Systems

Department of Electrical Engineering

Exam date: August 18, 2023

Teaching Staff

Paolo Monti (examiner), 031-772 6027
Morteza Barzegar Astanjin, 070-798 2924

Material Allowed material:

- Chalmers-approved calculator.
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photocopies, printouts, other students' notes, or any other material is not allowed.
- A paper-based dictionary without added notes (electronic dictionaries are not allowed).

Students are required to solve the exam problems individually. Cooperation, in any form or with anyone, is strictly forbidden.

Grading A correct, precise, and well-motivated solution gives at most 12 points per problem. An incorrect answer, unclear, incomplete, or poorly motivated solutions give point reductions to a minimum of 0 points. No fractional points are awarded. Answers in any other language than Swedish or English are ignored.

Solutions Are made available at the earliest on **August 18** EOB on the course web page.

Results Exam results are posted on Canvas no later than **August 31**. The grading review process will be explained on the course webpage.

Grades The final grade of the course will be decided by the project (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). The project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade.

Total Score	0–39	40–68	69–79	≥ 80
Grade	Fail	3	4	5

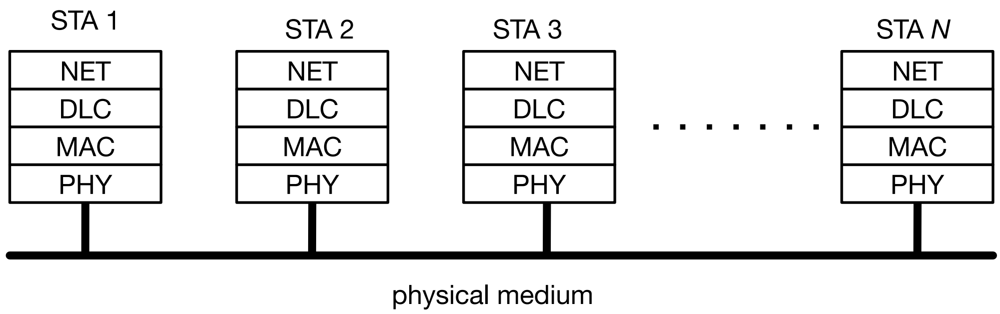
PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED BASED ON THEIR DIFFICULTY
Good luck!

Table over the Q-function

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
0.00	5.0000E-01	0.76	2.2360E-01	1.52	6.4260E-02	2.28	1.1300E-02	3.04	1.1830E-03	3.80	7.2350E-05	4.56	2.5580E-06	5.32	5.1880E-08
0.01	4.9600E-01	0.77	2.2060E-01	1.53	6.3010E-02	2.29	1.1010E-02	3.05	1.1440E-03	3.81	6.9480E-05	4.57	2.4390E-06	5.33	4.9110E-08
0.02	4.9200E-01	0.78	2.1770E-01	1.54	6.1780E-02	2.30	1.0720E-02	3.06	1.1070E-03	3.82	6.6730E-05	4.58	2.3250E-06	5.34	4.6470E-08
0.03	4.8800E-01	0.79	2.1480E-01	1.55	6.0570E-02	2.31	1.0440E-02	3.07	1.0700E-03	3.83	6.4070E-05	4.59	2.2160E-06	5.35	4.3980E-08
0.04	4.8400E-01	0.80	2.1190E-01	1.56	5.9380E-02	2.32	1.0170E-02	3.08	1.0350E-03	3.84	6.1520E-05	4.60	2.1120E-06	5.36	4.1610E-08
0.05	4.8010E-01	0.81	2.0900E-01	1.57	5.8210E-02	2.33	9.9030E-03	3.09	1.0010E-03	3.85	5.9060E-05	4.61	2.0130E-06	5.37	3.9370E-08
0.06	4.7610E-01	0.82	2.0610E-01	1.58	5.7050E-02	2.34	9.6420E-03	3.10	9.6760E-04	3.86	5.6690E-05	4.62	1.9190E-06	5.38	3.7240E-08
0.07	4.7210E-01	0.83	2.0330E-01	1.59	5.5920E-02	2.35	9.3870E-03	3.11	9.3540E-04	3.87	5.4420E-05	4.63	1.8280E-06	5.39	3.5230E-08
0.08	4.6810E-01	0.84	2.0050E-01	1.60	5.4800E-02	2.36	9.1370E-03	3.12	9.0430E-04	3.88	5.2230E-05	4.64	1.7420E-06	5.40	3.3320E-08
0.09	4.6410E-01	0.85	1.9770E-01	1.61	5.3700E-02	2.37	8.8940E-03	3.13	8.7400E-04	3.89	5.0120E-05	4.65	1.6600E-06	5.41	3.1510E-08
0.10	4.6020E-01	0.86	1.9490E-01	1.62	5.2620E-02	2.38	8.6560E-03	3.14	8.4470E-04	3.90	4.8100E-05	4.66	1.5810E-06	5.42	2.9800E-08
0.11	4.5620E-01	0.87	1.9220E-01	1.63	5.1550E-02	2.39	8.4240E-03	3.15	8.1640E-04	3.91	4.6150E-05	4.67	1.5060E-06	5.43	2.8180E-08
0.12	4.5220E-01	0.88	1.8940E-01	1.64	5.0500E-02	2.40	8.1980E-03	3.16	7.8880E-04	3.92	4.4270E-05	4.68	1.4340E-06	5.44	2.6640E-08
0.13	4.4830E-01	0.89	1.8670E-01	1.65	4.9470E-02	2.41	7.9760E-03	3.17	7.6220E-04	3.93	4.2470E-05	4.69	1.3660E-06	5.45	2.5180E-08
0.14	4.4430E-01	0.90	1.8410E-01	1.66	4.8460E-02	2.42	7.7600E-03	3.18	7.3640E-04	3.94	4.0740E-05	4.70	1.3010E-06	5.46	2.3810E-08
0.15	4.4040E-01	0.91	1.8140E-01	1.67	4.7460E-02	2.43	7.5490E-03	3.19	7.1140E-04	3.95	3.9080E-05	4.71	1.2390E-06	5.47	2.2500E-08
0.16	4.3640E-01	0.92	1.7880E-01	1.68	4.6480E-02	2.44	7.3440E-03	3.20	6.8710E-04	3.96	3.7470E-05	4.72	1.1790E-06	5.48	2.1270E-08
0.17	4.3250E-01	0.93	1.7620E-01	1.69	4.5510E-02	2.45	7.1430E-03	3.21	6.6370E-04	3.97	3.5940E-05	4.73	1.1230E-06	5.49	2.0100E-08
0.18	4.2860E-01	0.94	1.7360E-01	1.70	4.4570E-02	2.46	6.9470E-03	3.22	6.4100E-04	3.98	3.4460E-05	4.74	1.0690E-06	5.50	1.8990E-08
0.19	4.2470E-01	0.95	1.7110E-01	1.71	4.3630E-02	2.47	6.7560E-03	3.23	6.1900E-04	3.99	3.3040E-05	4.75	1.0170E-06	5.51	1.7940E-08
0.20	4.2070E-01	0.96	1.6850E-01	1.72	4.2720E-02	2.48	6.5690E-03	3.24	5.9760E-04	4.00	3.1670E-05	4.76	9.6800E-07	5.52	1.6950E-08
0.21	4.1680E-01	0.97	1.6600E-01	1.73	4.1820E-02	2.49	6.3870E-03	3.25	5.7700E-04	4.01	3.0360E-05	4.77	9.2110E-07	5.53	1.6010E-08
0.22	4.1290E-01	0.98	1.6350E-01	1.74	4.0930E-02	2.50	6.2100E-03	3.26	5.5710E-04	4.02	2.9100E-05	4.78	8.7650E-07	5.54	1.5120E-08
0.23	4.0900E-01	0.99	1.6110E-01	1.75	4.0060E-02	2.51	6.0370E-03	3.27	5.3770E-04	4.03	2.7890E-05	4.79	8.3390E-07	5.55	1.4280E-08
0.24	4.0520E-01	1.00	1.5870E-01	1.76	3.9200E-02	2.52	5.8680E-03	3.28	5.1900E-04	4.04	2.6730E-05	4.80	7.9330E-07	5.56	1.3490E-08
0.25	4.0130E-01	1.01	1.5620E-01	1.77	3.8360E-02	2.53	5.7030E-03	3.29	5.0090E-04	4.05	2.5610E-05	4.81	7.5470E-07	5.57	1.2740E-08
0.26	3.9740E-01	1.02	1.5390E-01	1.78	3.7540E-02	2.54	5.5430E-03	3.30	4.8340E-04	4.06	2.4540E-05	4.82	7.1780E-07	5.58	1.2030E-08
0.27	3.9360E-01	1.03	1.5150E-01	1.79	3.6730E-02	2.55	5.3860E-03	3.31	4.6650E-04	4.07	2.3510E-05	4.83	6.8270E-07	5.59	1.1350E-08
0.28	3.8970E-01	1.04	1.4920E-01	1.80	3.5930E-02	2.56	5.2340E-03	3.32	4.5010E-04	4.08	2.2520E-05	4.84	6.4920E-07	5.60	1.0720E-08
0.29	3.8590E-01	1.05	1.4690E-01	1.81	3.5150E-02	2.57	5.0850E-03	3.33	4.3420E-04	4.09	2.1570E-05	4.85	6.1730E-07	5.61	1.0120E-08
0.30	3.8210E-01	1.06	1.4460E-01	1.82	3.4380E-02	2.58	4.9400E-03	3.34	4.1890E-04	4.10	2.0660E-05	4.86	5.8690E-07	5.62	9.5480E-09
0.31	3.7830E-01	1.07	1.4230E-01	1.83	3.3620E-02	2.59	4.7990E-03	3.35	4.0410E-04	4.11	1.9780E-05	4.87	5.5800E-07	5.63	9.0100E-09
0.32	3.7450E-01	1.08	1.4010E-01	1.84	3.2880E-02	2.60	4.6610E-03	3.36	3.8970E-04	4.12	1.8940E-05	4.88	5.3040E-07	5.64	8.5030E-09
0.33	3.7070E-01	1.09	1.3790E-01	1.85	3.2160E-02	2.61	4.5270E-03	3.37	3.7580E-04	4.13	1.8140E-05	4.89	5.0420E-07	5.65	8.0220E-09
0.34	3.6690E-01	1.10	1.3570E-01	1.86	3.1440E-02	2.62	4.3960E-03	3.38	3.6240E-04	4.14	1.7370E-05	4.90	4.7920E-07	5.66	7.5690E-09
0.35	3.6320E-01	1.11	1.3350E-01	1.87	3.0740E-02	2.63	4.2690E-03	3.39	3.4950E-04	4.15	1.6620E-05	4.91	4.5540E-07	5.67	7.1400E-09
0.36	3.5940E-01	1.12	1.3140E-01	1.88	3.0050E-02	2.64	4.1450E-03	3.40	3.3690E-04	4.16	1.5910E-05	4.92	4.3270E-07	5.68	6.7350E-09
0.37	3.5570E-01	1.13	1.2920E-01	1.89	2.9380E-02	2.65	4.0250E-03	3.41	3.2480E-04	4.17	1.5230E-05	4.93	4.1110E-07	5.69	6.3520E-09
0.38	3.5200E-01	1.14	1.2710E-01	1.90	2.8720E-02	2.66	3.9070E-03	3.42	3.1310E-04	4.18	1.4580E-05	4.94	3.9060E-07	5.70	5.9900E-09
0.39	3.4830E-01	1.15	1.2510E-01	1.91	2.8070E-02	2.67	3.7930E-03	3.43	3.0180E-04	4.19	1.3950E-05	4.95	3.7110E-07	5.71	5.6490E-09
0.40	3.4460E-01	1.16	1.2300E-01	1.92	2.7430E-02	2.68	3.6810E-03	3.44	2.9090E-04	4.20	1.3350E-05	4.96	3.5250E-07	5.72	5.3260E-09
0.41	3.4090E-01	1.17	1.2100E-01	1.93	2.6800E-02	2.69	3.5730E-03	3.45	2.8030E-04	4.21	1.2770E-05	4.97	3.3480E-07	5.73	5.0220E-09
0.42	3.3720E-01	1.18	1.1900E-01	1.94	2.6190E-02	2.70	3.4670E-03	3.46	2.7010E-04	4.22	1.2220E-05	4.98	3.1790E-07	5.74	4.7340E-09
0.43	3.3360E-01	1.19	1.1700E-01	1.95	2.5590E-02	2.71	3.3640E-03	3.47	2.6020E-04	4.23	1.1680E-05	4.99	3.0190E-07	5.75	4.4620E-09
0.44	3.3000E-01	1.20	1.1510E-01	1.96	2.5000E-02	2.72	3.2640E-03	3.48	2.5070E-04	4.24	1.1180E-05	5.00	2.8670E-07	5.76	4.2060E-09
0.45	3.2640E-01	1.21	1.1310E-01	1.97	2.4420E-02	2.73	3.1670E-03	3.49	2.4150E-04	4.25	1.0690E-05	5.01	2.7220E-07	5.77	3.9640E-09
0.46	3.2280E-01	1.22	1.1120E-01	1.98	2.3850E-02	2.74	3.0720E-03	3.50	2.3260E-04	4.26	1.0220E-05	5.02	2.5840E-07	5.78	3.7350E-09
0.47	3.1920E-01	1.23	1.0930E-01	1.99	2.3300E-02	2.75	2.9800E-03	3.51	2.2410E-04	4.27	9.7740E-06	5.03	2.4520E-07	5.79	3.5190E-09
0.48	3.1560E-01	1.24	1.0750E-01	2.00	2.2750E-02	2.76	2.8900E-03	3.52	2.1580E-04	4.28	9.3450E-06	5.04	2.3280E-07	5.80	3.3160E-09
0.49	3.1210E-01	1.25	1.0560E-01	2.01	2.2220E-02	2.77	2.8030E-03	3.53	2.0780E-04	4.29	8.9340E-06	5.05	2.2090E-07	5.81	3.1240E-09
0.50	3.0850E-01	1.26	1.0380E-01	2.02	2.1690E-02	2.78	2.7180E-03	3.54	2.0010E-04	4.30	8.5400E-06	5.06	2.0960E-07	5.82	2.9420E-09
0.51	3.0500E-01	1.27	1.0200E-01	2.03	2.1180E-02	2.79	2.6350E-03	3.55	1.9260E-04	4.31	8.1630E-06	5.07	1.9890E-07	5.83	2.7710E-09
0.52	3.0150E-01	1.28	1.0030E-01	2.04	2.0680E-02	2.80	2.5550E-03	3.56	1.8540E-04	4.32	7.8010E-06	5.08	1.8870E-07	5.84	2.6100E-09
0.53	2.9810E-01	1.29	9.8530E-02	2.05	2.0180E-02	2.81	2.4770E-03	3.57	1.7850E-04	4.33	7.4550E-06	5.09	1.7900E-07	5.85	2.4580E-09
0.54	2.9460E-01	1.30	9.6800E-02	2.06	1.9700E-02	2.82	2.4010E-03	3.58	1.7180E-04	4.34	7.1240E-06	5.10	1.6980E-07	5.86	2.3140E-09
0.55	2.9120E-01	1.31	9.5100E-02	2.07	1.9230E-02	2.83	2.3270E-03	3.59	1.6530E-04	4.35	6.8070E-06	5.11	1.6110E-07	5.87	2.1790E-09
0.56	2.8770E-01	1.32	9.3420E-02	2.08	1.8760E-02	2.84	2.2560E-03	3.60	1.5910E-04	4.36	6.5030E-06	5.12	1.5280E-07	5.88	2.0510E-09
0.57	2.8430E-01	1.33	9.1760E-02	2.09	1.8310E-02	2.85	2.1860E-03	3.61	1.5310E-04	4.37	6.2120E-06	5.13	1.4490E-07	5.89	1.9310E-09
0.58	2.8100E-01	1.34	9.0120E-02	2.10	1.7860E-02	2.86	2.1180E-03	3.62	1.4730E-04	4.38	5.9340E-06	5.14	1.3740E-07	5.90	1.8180E-09
0.59	2.7760E-01	1.35	8.8510E-02	2.11	1.7430E-02	2.87	2.0520E-03	3.63	1.4170E-04	4.39	5.6680E-06	5.15	1.3020E-07	5.91	1.7110E-09
0.60	2.7430E-01	1.36	8.6910E-02	2.12	1.7000E-02	2.88	1.9880E-03	3.64	1.3630E-04	4.40	5.4130E-06	5.16	1.2350E-07	5.92	1.6100E-09
0.61	2.7090E-01	1.37	8.5340E-02	2.13	1.6590E-02	2.89	1.9260E-03	3.65	1.3110E-04	4.41	5.1690E-06	5.17	1.1700E-07	5.93	1.5150E-09
0.62	2.6760E-01	1.38	8.3790E-02	2.14	1.6180E-02	2.90	1.8660E-03	3.66	1.2610E-04	4.42	4.9350E-06	5.18	1.1090E-07	5.94	1.4250E-09
0.63	2.6430E-01	1.39	8.2260E-02	2.15	1.5780E-02	2.91	1.8070E-03	3.67	1.2130E-04	4.43	4.7120E-06	5.19	1.0510E-07	5.95	1.3410E-09
0.64	2.6110E-01	1.40	8.0760E-02	2.16	1.5390E-02	2.92	1.7500E-03	3.68	1.1660E-04	4.44	4.4980E-06	5.20	9.9640E-08	5.96	1.2610E-09
0.65	2.5780E-01	1.41	7.9270E-02	2.17	1.5000E-02	2.93	1.6950E-03	3.69	1.1210E-04	4.45	4.2940E-06	5.21	9.4420E-08	5.97	1.1860E-09
0.66	2.5460E-01	1.42	7.7800E-02	2.18	1.4630E-02	2.94	1.6410E-03	3.70	1.0780E-04	4.46	4.0980E-06	5.22	8.9460E-08	5.98	1.1160E-09
0.67	2.5140E-01	1.43	7.6360E-02	2.19	1.4260E-02	2.95	1.5890E-03	3.71	1.0360E-04	4.47	3.9110E-06	5.23	8.4760E-08	5.99	1.0490E-09
0.68	2.4830E-01	1.44	7.4930E-02	2.20	1.3900E-02	2.96	1.5380E-03	3.72	9.9610E-05	4.48	3.7320E-06	5.24	8.0290E-08	6.00	9.8660E-10
0.69	2.4510E-01	1.45	7.3530E-02	2.21	1.3550E-02	2.97	1.4890E-03	3.73	9.5740E-05	4.49	3.5610E-06	5.25	7.6050E-08	6.01	9.2760E-10
0.70	2.4200E-01	1.46	7.2150E-02	2.22	1.3210E-02	2.98	1.4410E-03	3.74	9.2010E-05	4.50	3.3980E-06	5.26	7.2030E-08	6.02	8.7210E-10
0.71	2.3890E-01	1.47	7.0780E-02	2.23	1.2870E-02	2.99	1.3950E-03	3.75	8.8420E-05	4.51	3.2410E-06	5.27	6.8210E-08	6.03	8.1980E-10
0.72	2.3580E-01	1.48	6.9440E-02	2.24	1.2550E-02	3.00	1.3500E-03	3.76	8.4960E-05	4.52	3.0920E-06	5.28	6.4590E-08	6.04	7.7060E-10
0.73	2.3270E-01	1.49	6.8110E-02	2.25	1.2220E-02	3.01	1.3060E-03	3.77	8.1620E-05	4.53	2.9490E-06	5.29	6.1160E-08	6.05	7.2420E-10
0.74	2.2960E-01	1.50	6.6810E-02	2.26	1.1910E-02	3.02	1.2640E-03	3.78	7.8410E-05	4.54	2.8130E-06	5.30	5.7900E-08	6.06	6.8060E-10
0.75	2.2660E-01	1.51	6.5520E-02	2.27	1.1600E-02	3.03	1.2230E-03	3.79	7.5320E-05	4.55	2.6820E-06	5.31	5.4810E-08	6.07	6.3960E-10

1. Consider a network of $N = 100$ stations that are attached to a common physical medium. The medium access control method is a simple reservation protocol where communication is done in cycles. A cycle consists of N minislots (reservation slots) followed by M data slots. Here, M is the number of stations that have data to send.

   The network layer (NET) is located on top of the data link layer. The data link layer consists of the data link control (DLC) sublayer just above the medium access control (MAC) sublayer, which in turn is just above the physical (PHY) layer, see the figure below.



   Suppose the data link (DL) layer header and trailer require in total 25 byte. The DL header is added by the DLC before sending the data to the MAC sublayer. The MAC layer does not add any header bits to the data frames received from the DLC. The reservation messages (created by the MAC sublayer) are 25 byte long. The physical layer provides the data rate $R_{\text{PHY}} = 100\,\text{Mbit/s}$ to the MAC sublayer. For simplicity, we assume that the physical layer is error-free. Let $M = 10$.

   Let us assume that all stations always have data to transmit. Let R_{DL} be the effective data rate of the DL layer service, i.e., the data rate experienced by the network layer.

   (a) Compute R_{DL} when the DL service data unit (SDU) is 400 byte long. Assume that the data slot duration is chosen to maximize the effective data rate. (3p)

   (b) Suppose the data slot duration is as in Part (a). What is R_{DL} when the DL SDU is 800 byte? Assume that the DLC split the DL SDU into a number of smaller fragments before pushing the data down to the MAC. (3p)

   (c) Suppose the data slot duration is as in Part (a). What is R_{DL} when the DL SDU is 200 byte? Assume that the DLC pads the DL SDU with zero bytes before pushing the data down to the MAC. (3p)

   (d) Give a general expression for R_{DL} when the DL SDU size is n_{DSDU} byte long. Assume that the DLC does fragmentation and zero-padding as needed and that the data slot duration is as in Part (a). (3p)

Solution

(a) Let T_r be the slot duration of one reservation frame, and T_D be the slot duration of one data frame. Assuming that the propagation and processing times are negligible, the effective data rate is maximized if the slot duration is equal to the transmission time of a frame, hence

$$T_\mathrm{r} = \frac{n_\mathrm{r}}{R_\mathrm{PHY}}$$
$$T_\mathrm{D} = \frac{n_\mathrm{o} + n_\mathrm{DSDU}}{R_\mathrm{PHY}},$$

where n_o is the length of the header and trailer in bits, and n_r and n_DSDU are the length of the reservation message and DL SDU in bits, respectively. Substituting with the numerical values, we get

$$T_\mathrm{r} = \frac{25 \times 8}{10^8} = 2\,\mu\mathrm{s}.$$
$$T_\mathrm{D} = \frac{425 \times 8}{10^8} = 34\,\mu\mathrm{s}.$$

To compute the effective data rate experienced by the NET layer of a particular station, we take into account that the station can transmit one data frame each cycle of N reservation slots and M data slots, hence

$$R_\mathrm{DL} = \frac{400 \times 8}{NT_\mathrm{r} + MT_\mathrm{D}} = 5.926\,\mathrm{Mbit/s}.$$

(b) In this case the station needs to send two data frames. This can be done in two transmission cycles, hence

$$R_\mathrm{DL}^{(b)} = \frac{800 \times 8}{2(NT_\mathrm{r} + MT_\mathrm{D})} = 5.926\,\mathrm{Mbit/s}.$$

(c) In this case only one transmission cycle is needed. However, since the data slot time is optimized for data frames that are 400 bytes long, a lower data rate is experienced

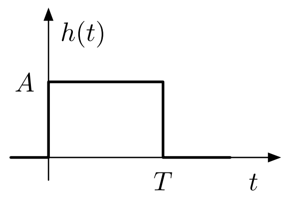
$$R_\mathrm{DL}^{(c)} = \frac{200 \times 8}{NT_\mathrm{r} + MT_\mathrm{D}} = 2.963\,\mathrm{Mbit/s}.$$

(d) In general, we need $\lceil \frac{n_\mathrm{DSDU}}{(400\times 8)} \rceil$ cycles to transmit all the data, hence

$$R_\mathrm{DL} = \frac{n_\mathrm{DSDU}}{\lceil \frac{n_\mathrm{DSDU}}{(400\times 8)} \rceil (NT_\mathrm{r} + MT_\mathrm{D})}.$$

Note that $R_\mathrm{DL}^\mathrm{max} = 5.926\,\mathrm{Mbit/s}$.

2. Consider transmission of 1500 byte long packets over an AWGN channel with noise power spectral density $N_0/2$ using M-ary PAM with the pulse shape $h(t)$ (depicted below) and where $N_0 = 1.1 \times 10^{-20}$ W/Hz. The bits that make up the packets are assumed to be independent and equally likely. The signal constellation is $\mathcal{A} = \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$, the data rate is 100 Mbit/s, and the transmission should be ISI-free.



(a) What is the smallest A that is required to achieve the bit error probability $P_b = 10^{-3}$ for $M = 2$? (4p)

(b) What is the smallest A that is required to achieve the packet error probability $P_p = 10^{-1}$ for $M = 2$? (4p)

(c) Repeat Part (b) for $M = 4$. (4p)

Hint: the symbol error probability for M-ary PAM is

$$P_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6}{M^2-1}\frac{E_s}{N_0}}\right)$$

where E_s is the symbol energy.

Solution Problem 2

(a) We have for $M = 2$

$$P_b = P_e = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

where the symbol energy E_s is given by

$$\begin{aligned}E_s &= E_a E_h.\\ E_a &= \frac{1}{|\mathcal{A}|}\sum_{i=1}^{M}|a_i|^2.\\ E_h &= \int_{-\infty}^{+\infty}|h(t)|^2 dt = A^2 T.\end{aligned}$$

For $M = 2$, $E_a = 1$. The symbol duration T can be deduced from the data rate. Since we are using a rectangular pulse

$$T = \frac{\log_2(M)}{R} = 10^{-8}\,\text{s}$$

From the Q-function table, we can get

$$Q(x) \leq 10^{-3} \longrightarrow x \geq 3.10.$$

Then,

$$\begin{aligned}x^2 &= \frac{2E_s}{N_0} = \frac{2A^2T}{N_0}\\ A &= \sqrt{\frac{x^2 N_0}{2T}}\\ A &= 2.3\,\mu\text{v}\end{aligned}$$

(b) Since symbol errors are independent for PAM over a baseband AWGN channel, the packet error probability is $P_p = 1-(1-P_e)^{n_s}$, where P_e is the symbol error probability and n_s is the number of symbols in the packet. Cleary, $n_s = 8n_b/\log_2(M)$, where n_b is the number of bytes in the packet and $\log_2(M)$ is the number of bits per symbol. Putting everything together, we can compute the symbol error probability as

$$\begin{aligned}P_p &= 1-(1-P_e)^{n_s} \Rightarrow\\ P_e &= 1-(1-P_p)^{\log_2(M)/(8n_b)} \qquad (1)\\ &= 8.78\times10^{-6}\end{aligned}$$

Then, we continue in similar steps as in (a). From the Q-function table we get

$$\begin{aligned}Q(x) \leq 8.78\times10^{-6} &\longrightarrow x \geq 4.30.\\ A &= \sqrt{\frac{x^2 N_0}{2T}}\\ &= 3.19\,\mu\text{v}.\end{aligned}$$

(c) We use (1) to compute the symbol error probability needed to achieve $P_p = 10^{-1}$.

$$P_e = 1.7560 \times 10^{-5}$$

In the case $M = 4$

$$P_e = \frac{3}{2} Q\left(\sqrt{\frac{2}{5}\frac{E_s}{N_0}}\right)$$

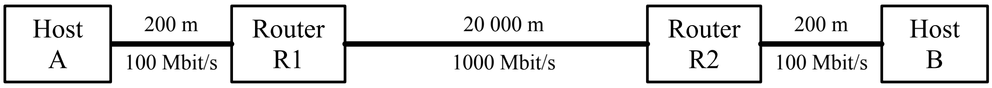
Using the Q-function table, we get

$$Q(x) \leq \frac{2}{3} 1.7560 \times 10^{-5} \longrightarrow x \geq 4.23.$$

For $\mathcal{A} = \{-3, -1, 1, 3\}$, we have that $E_a = 5$ and $E_s = 5A^2T$, where $T = \log_2(M)/R = 2.10^{-8}$. Then,

$$\begin{aligned} x^2 &= \frac{2}{5}\frac{E_s}{N_0} = \frac{2}{5}\frac{5A^2T}{N_0} = \frac{2A^2T}{N_0} \\ A &= \sqrt{\frac{N_0}{2T} x^2} \\ &= 2.22\,\mu\text{v}. \end{aligned}$$

3. Consider a network consisting of four nodes, the two end-nodes (hosts) A and B and an intermediate routers R1 and R2. The distances between the nodes and the link data rates are defined in the figure below. The propagation speed on all links is $c = 2 \times 10^8$ m/s.



The hosts communicate using a Stop-And-Wait protocol, where the data packets are 1250 byte long, including a 50-byte header, and the ACK packets are 50 byte long. The routers are store-and-forward routers. That is, an incoming packet is completely received before it is forwarded to the output port. Processing times in hosts and routers are assumed to be negligible.

(a) Determine the value of the time-out such that (i) no unnecessary retransmissions occur and (ii) the effective data rate of the ARQ protocol is maximized. (4p)

(b) What is the effective data rate assuming error-free transmission? (2p)

(c) Suppose R1 drops packets with probability 0.1 and R2 drops packets with probability 0.2. Assume that the packet drops are independent of each other. Note that both data packets and ACK packets are subject to packet drops. What is the effective rate of the ARQ protocol? (4p)

(d) Suppose your boss is not happy with the effective data rate in Part (c). She asks you to suggest another ARQ protocol to improve the effective data rate. What protocol would you suggest? Your boss wants a good motivation for your suggestion, not just an answer. (2p)

Hint: Suppose the success probability of trial is p, then the average number of independent trials needed until a successful trial is

$$\sum_{k=1}^{\infty} kp(1-p)^{k-1} = \frac{1}{p}.$$

Solution Problem 3

(a) Let $\tau_{\text{prop},1}$ be the propagation delay between Host A/B and R1/R2, and let $\tau_{\text{prop},2}$ be the propagation delay between R1 and R2. Then,

$$\tau_{\text{prop},1} = \frac{200}{c} = 1\mu\text{s}$$
$$\tau_{\text{prop},2} = \frac{20000}{c} = 100\mu\text{s}$$

Let n_d and n_a be the length of data packet and ACK packet in bits, respectively. The round-trip-time (RTT) for the stop and wait protocol can be computed as

$$t_0 = \frac{n_\text{d}}{10^8} + \tau_{\text{prop},1} + \frac{n_\text{d}}{10^9} + \tau_{\text{prop},2} + \frac{n_\text{d}}{10^8} + \tau_{\text{prop},1} \tag{2}$$
$$+ \frac{n_\text{a}}{10^8} + \tau_{\text{prop},1} + \frac{n_\text{a}}{10^9} + \tau_{\text{prop},2} + \frac{n_\text{a}}{10^8} + \tau_{\text{prop},1} = 0.423\,\text{ms} \tag{3}$$

To avoid unnecessary retransmissions and maximize the effective data rate of the protocol we choose the time out to be equal to the RTT, that is

$$t_\text{out} = t_0 = 0.423\,\text{ms}.$$

(b) Let n_o be the length of the header. The effective data rate in absence of errors can be computed as

$$R_\text{sw}^0 = \frac{n_\text{d} - n_\text{o}}{t_0} = 22.7\,\text{Mbit/s}.$$

(c) Let p be the probability that a transmission trial is successful. This can be expressed as

$$p = \Pr\{\text{"Packet frame is not dropped by R1"} \cap \text{"Packet frame is not dropped by R2"} \cap \text{"ACK frame is not dropped by R2"} \cap \text{"ACK frame is not dropped by R1"}\}$$

Under the assumption that packets drops in R1 and R2 are independent and the assumption that data packets drops and ACK packets drops are independent, p is given by

$$p = \Pr\{\text{"Packet frame is not dropped by R1"}\} \times \Pr\{\text{"Packet frame is not dropped by R2"}\} \times \Pr\{\text{"ACK frame is not dropped by R2"}\} \times \Pr\{\text{"ACK frame is not dropped by R1"}\}$$

$$p = (1 - 0.1)^2(1 - 0.2)^2 = 0.5184.$$

Then we can readily compute the effective data rate taking into account that the average number of retransmissions needed until a frame is successfully transmitted is $\frac{1}{p}$ (check the hint), as

$$R_\text{sw} = \frac{n_\text{d} - n_\text{o}}{\frac{1}{p} t_\text{out}} = 11.8\,\text{Mbit/s}.$$

(d) We can use either Go-Back-N or Selective Repeat protocols to improve the performance. In error-free conditions, Go-Back-N (with a sufficiently large send window and full-duplex links) is able to transmit and deliver data continuously. That is, the effective rate is close to the data rate of the bottleneck links along the path, which in this case is 100 Mbit/s. However, all packets in the send window needs to be retransmitted in case of an unsuccessful transmission attempt. Since the probability for success is low in this example, it is better to use Selective Repeat, which is less sensitive to unsuccessful transmissions compared to Go-Back-N.

4. (a) Why is a hierarchical addressing scheme preferable to a flat addressing scheme for routing? (3p)

   (b) Consider an Ethernet LAN consisting of a number of hosts with full-duplex connections to an Ethernet switch. Explain why the Ethernet CSMA/CD medium access method is not needed for this setup. In this setting which event can still trigger a frame loss? (3p)

   (c) Explain what piggy-backing is and what the advantage of it is. (3p)

   (d) Define the security goal "integrity". (1p)

   (e) Suppose Alice wants to communicate confidentially with Bob using asymmetric cryptography. Which key does Alice use to encrypt her messages to Bob? Choose from Alice's private key, Alice's public key, Bob's private key, or Bob's public key. Motivate. (2p)

   (a) With a flat addressing scheme, a router need to maintain a table with as many rows as there are addresses. This table becomes very large as the network grows. With a hierarchical addressing scheme, all addresses with a certain prefix can share an entry in the table. For instance, suppose that all packets with the prefix 192, i.e., addresses of the form 192.XXX.XXX.XXX, should be forwarded to port 2, then this requires only one entry in the routing table.

   (b) Since all the hosts are connected to the switch with a full duplex, point-to-point link, collisions can not occur. Hence there is no need to have a collision detection mechanism. In such a setting, the only event that may still result in a frame loss is the buffer overflowing at the switch.

   (c) When we use an ARQ protocol, the transmitting node (e.g., Node A)sends information frames to the receiving node (e.g., Node B) while ACK frames go in the opposite direction. Now, suppose Node A and Node B transmit data frames in both directions. Then ACK frames also go in both directions. We can then embed the ACK-frame payloads into data frames (i.e., let the ACK-frame payload piggy-back onto the data frame). This will save communication resources since it reduces the overhead needed to transmit explicit ACK frames (addresses, etc.)

   (d) The goal of integrity is to guarantee that a message has not been altered in transmission by a third party.

   (e) Alice will use Bobâs public key to encrypt messages to Bob, and Bob will use his private key to decrypt the message. The public keys are not secret. Hence these cannot be used for decrypting messages that are supposed to be confidential (since anyone can use the public key). Alice cannot use her public key to encrypt since no one, including Bob, should know her private key. Hence, the only possibility is for Alice to encrypt with Bobâs public key (which is known to Alice). To summarize, everyone can encrypt messages to Bob, but since only Bob knows the Bobâs private, only Bob can decrypt the message.