Exam in SSY305 Communication Systems

Department of Electrical Engineering

date: June 10, 2021, 14:00–18:00

Teaching Staff

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Material All material is allowed on this exam. Students are required to solve the exam problems individually. Cooperation, in any form or with anyone, is strictly forbidden.

Grading A correct, clear and well-motivated solution gives at most 12 points per problem.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. No fractional points are awarded.

Answers in any other language than Swedish or English are ignored.

Submission Exam problem solutions should be solved on paper as in a normal exam.

- Ensure that each paper is clearly marked with your name, exam problem number, and the page number.
- Scan or photograph your solutions.
- Name your image files `Problem_YY_Page_XX`. Example: `Problem_01_Page_02.jpg`.
- If you want, you can combine images for the same problem into a single document (e.g., Word or PDF) named `Problem_YY`.
- Submit your solutions by uploading the image files or documents via Canvas before **June 10, 18:30**.

Solutions Are made available at the earliest on June 11 at 19:00 on the course web page.

Results Exam results are posted on Canvas no later than June 18, 2021. The grading reviews will have to be done remotely according to a process that will be explained in the course webpage.

Grades The final grade of the course will be decided by the project (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). The project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade.

Total Score	0–39	40–68	69–79	≥ 80
Grade	Fail	3	4	5

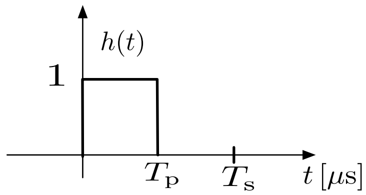
PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED BASED ON THEIR DIFFICULTY

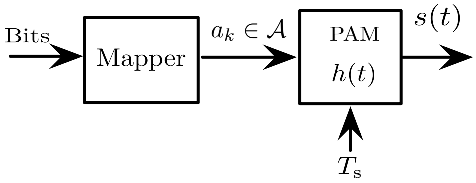
Good luck!

Table over the Q-function

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
0.00	5.0000E-01	0.76	2.2360E-01	1.52	6.4260E-02	2.28	1.1300E-02	3.04	1.1830E-03	3.80	7.2350E-05	4.56	2.5580E-06	5.32	5.1880E-08
0.01	4.9600E-01	0.77	2.2060E-01	1.53	6.3010E-02	2.29	1.1010E-02	3.05	1.1440E-03	3.81	6.9480E-05	4.57	2.4390E-06	5.33	4.9110E-08
0.02	4.9200E-01	0.78	2.1770E-01	1.54	6.1780E-02	2.30	1.0720E-02	3.06	1.1070E-03	3.82	6.6730E-05	4.58	2.3250E-06	5.34	4.6470E-08
0.03	4.8800E-01	0.79	2.1480E-01	1.55	6.0570E-02	2.31	1.0440E-02	3.07	1.0700E-03	3.83	6.4070E-05	4.59	2.2160E-06	5.35	4.3980E-08
0.04	4.8400E-01	0.80	2.1190E-01	1.56	5.9380E-02	2.32	1.0170E-02	3.08	1.0350E-03	3.84	6.1520E-05	4.60	2.1120E-06	5.36	4.1610E-08
0.05	4.8010E-01	0.81	2.0900E-01	1.57	5.8210E-02	2.33	9.9030E-03	3.09	1.0010E-03	3.85	5.9060E-05	4.61	2.0130E-06	5.37	3.9370E-08
0.06	4.7610E-01	0.82	2.0610E-01	1.58	5.7050E-02	2.34	9.6420E-03	3.10	9.6760E-04	3.86	5.6690E-05	4.62	1.9190E-06	5.38	3.7240E-08
0.07	4.7210E-01	0.83	2.0330E-01	1.59	5.5920E-02	2.35	9.3870E-03	3.11	9.3540E-04	3.87	5.4420E-05	4.63	1.8280E-06	5.39	3.5230E-08
0.08	4.6810E-01	0.84	2.0050E-01	1.60	5.4800E-02	2.36	9.1370E-03	3.12	9.0430E-04	3.88	5.2230E-05	4.64	1.7420E-06	5.40	3.3320E-08
0.09	4.6410E-01	0.85	1.9770E-01	1.61	5.3700E-02	2.37	8.8940E-03	3.13	8.7400E-04	3.89	5.0120E-05	4.65	1.6600E-06	5.41	3.1510E-08
0.10	4.6020E-01	0.86	1.9490E-01	1.62	5.2620E-02	2.38	8.6560E-03	3.14	8.4470E-04	3.90	4.8100E-05	4.66	1.5810E-06	5.42	2.9800E-08
0.11	4.5620E-01	0.87	1.9220E-01	1.63	5.1550E-02	2.39	8.4240E-03	3.15	8.1640E-04	3.91	4.6150E-05	4.67	1.5060E-06	5.43	2.8180E-08
0.12	4.5220E-01	0.88	1.8940E-01	1.64	5.0500E-02	2.40	8.1980E-03	3.16	7.8880E-04	3.92	4.4270E-05	4.68	1.4340E-06	5.44	2.6640E-08
0.13	4.4830E-01	0.89	1.8670E-01	1.65	4.9470E-02	2.41	7.9760E-03	3.17	7.6220E-04	3.93	4.2470E-05	4.69	1.3660E-06	5.45	2.5180E-08
0.14	4.4430E-01	0.90	1.8410E-01	1.66	4.8460E-02	2.42	7.7600E-03	3.18	7.3640E-04	3.94	4.0740E-05	4.70	1.3010E-06	5.46	2.3810E-08
0.15	4.4040E-01	0.91	1.8140E-01	1.67	4.7460E-02	2.43	7.5490E-03	3.19	7.1140E-04	3.95	3.9080E-05	4.71	1.2390E-06	5.47	2.2500E-08
0.16	4.3640E-01	0.92	1.7880E-01	1.68	4.6480E-02	2.44	7.3440E-03	3.20	6.8710E-04	3.96	3.7470E-05	4.72	1.1790E-06	5.48	2.1270E-08
0.17	4.3250E-01	0.93	1.7620E-01	1.69	4.5510E-02	2.45	7.1430E-03	3.21	6.6370E-04	3.97	3.5940E-05	4.73	1.1230E-06	5.49	2.0100E-08
0.18	4.2860E-01	0.94	1.7360E-01	1.70	4.4570E-02	2.46	6.9470E-03	3.22	6.4100E-04	3.98	3.4460E-05	4.74	1.0690E-06	5.50	1.8990E-08
0.19	4.2470E-01	0.95	1.7110E-01	1.71	4.3630E-02	2.47	6.7560E-03	3.23	6.1900E-04	3.99	3.3040E-05	4.75	1.0170E-06	5.51	1.7940E-08
0.20	4.2070E-01	0.96	1.6850E-01	1.72	4.2720E-02	2.48	6.5690E-03	3.24	5.9760E-04	4.00	3.1670E-05	4.76	9.6800E-07	5.52	1.6950E-08
0.21	4.1680E-01	0.97	1.6600E-01	1.73	4.1820E-02	2.49	6.3870E-03	3.25	5.7700E-04	4.01	3.0360E-05	4.77	9.2110E-07	5.53	1.6010E-08
0.22	4.1290E-01	0.98	1.6350E-01	1.74	4.0930E-02	2.50	6.2100E-03	3.26	5.5710E-04	4.02	2.9100E-05	4.78	8.7650E-07	5.54	1.5120E-08
0.23	4.0900E-01	0.99	1.6110E-01	1.75	4.0060E-02	2.51	6.0370E-03	3.27	5.3770E-04	4.03	2.7890E-05	4.79	8.3390E-07	5.55	1.4280E-08
0.24	4.0520E-01	1.00	1.5870E-01	1.76	3.9200E-02	2.52	5.8680E-03	3.28	5.1900E-04	4.04	2.6730E-05	4.80	7.9330E-07	5.56	1.3490E-08
0.25	4.0130E-01	1.01	1.5620E-01	1.77	3.8360E-02	2.53	5.7030E-03	3.29	5.0090E-04	4.05	2.5610E-05	4.81	7.5470E-07	5.57	1.2740E-08
0.26	3.9740E-01	1.02	1.5390E-01	1.78	3.7540E-02	2.54	5.5430E-03	3.30	4.8340E-04	4.06	2.4540E-05	4.82	7.1780E-07	5.58	1.2030E-08
0.27	3.9360E-01	1.03	1.5150E-01	1.79	3.6730E-02	2.55	5.3860E-03	3.31	4.6650E-04	4.07	2.3510E-05	4.83	6.8270E-07	5.59	1.1350E-08
0.28	3.8970E-01	1.04	1.4920E-01	1.80	3.5930E-02	2.56	5.2340E-03	3.32	4.5010E-04	4.08	2.2520E-05	4.84	6.4920E-07	5.60	1.0720E-08
0.29	3.8590E-01	1.05	1.4690E-01	1.81	3.5150E-02	2.57	5.0850E-03	3.33	4.3420E-04	4.09	2.1570E-05	4.85	6.1730E-07	5.61	1.0120E-08
0.30	3.8210E-01	1.06	1.4460E-01	1.82	3.4380E-02	2.58	4.9400E-03	3.34	4.1890E-04	4.10	2.0660E-05	4.86	5.8690E-07	5.62	9.5480E-09
0.31	3.7830E-01	1.07	1.4230E-01	1.83	3.3620E-02	2.59	4.7990E-03	3.35	4.0410E-04	4.11	1.9780E-05	4.87	5.5800E-07	5.63	9.0100E-09
0.32	3.7450E-01	1.08	1.4010E-01	1.84	3.2880E-02	2.60	4.6610E-03	3.36	3.8970E-04	4.12	1.8940E-05	4.88	5.3040E-07	5.64	8.5030E-09
0.33	3.7070E-01	1.09	1.3790E-01	1.85	3.2160E-02	2.61	4.5270E-03	3.37	3.7580E-04	4.13	1.8140E-05	4.89	5.0420E-07	5.65	8.0220E-09
0.34	3.6690E-01	1.10	1.3570E-01	1.86	3.1440E-02	2.62	4.3960E-03	3.38	3.6240E-04	4.14	1.7370E-05	4.90	4.7920E-07	5.66	7.5690E-09
0.35	3.6320E-01	1.11	1.3350E-01	1.87	3.0740E-02	2.63	4.2690E-03	3.39	3.4950E-04	4.15	1.6620E-05	4.91	4.5540E-07	5.67	7.1400E-09
0.36	3.5940E-01	1.12	1.3140E-01	1.88	3.0050E-02	2.64	4.1450E-03	3.40	3.3690E-04	4.16	1.5910E-05	4.92	4.3270E-07	5.68	6.7350E-09
0.37	3.5570E-01	1.13	1.2920E-01	1.89	2.9380E-02	2.65	4.0250E-03	3.41	3.2480E-04	4.17	1.5230E-05	4.93	4.1110E-07	5.69	6.3520E-09
0.38	3.5200E-01	1.14	1.2710E-01	1.90	2.8720E-02	2.66	3.9070E-03	3.42	3.1310E-04	4.18	1.4580E-05	4.94	3.9060E-07	5.70	5.9900E-09
0.39	3.4830E-01	1.15	1.2510E-01	1.91	2.8070E-02	2.67	3.7930E-03	3.43	3.0180E-04	4.19	1.3950E-05	4.95	3.7110E-07	5.71	5.6490E-09
0.40	3.4460E-01	1.16	1.2300E-01	1.92	2.7430E-02	2.68	3.6810E-03	3.44	2.9090E-04	4.20	1.3350E-05	4.96	3.5250E-07	5.72	5.3260E-09
0.41	3.4090E-01	1.17	1.2100E-01	1.93	2.6800E-02	2.69	3.5730E-03	3.45	2.8030E-04	4.21	1.2770E-05	4.97	3.3480E-07	5.73	5.0220E-09
0.42	3.3720E-01	1.18	1.1900E-01	1.94	2.6190E-02	2.70	3.4670E-03	3.46	2.7010E-04	4.22	1.2220E-05	4.98	3.1790E-07	5.74	4.7340E-09
0.43	3.3360E-01	1.19	1.1700E-01	1.95	2.5590E-02	2.71	3.3640E-03	3.47	2.6020E-04	4.23	1.1680E-05	4.99	3.0190E-07	5.75	4.4620E-09
0.44	3.3000E-01	1.20	1.1510E-01	1.96	2.5000E-02	2.72	3.2640E-03	3.48	2.5070E-04	4.24	1.1180E-05	5.00	2.8670E-07	5.76	4.2060E-09
0.45	3.2640E-01	1.21	1.1310E-01	1.97	2.4420E-02	2.73	3.1670E-03	3.49	2.4150E-04	4.25	1.0690E-05	5.01	2.7220E-07	5.77	3.9640E-09
0.46	3.2280E-01	1.22	1.1120E-01	1.98	2.3850E-02	2.74	3.0720E-03	3.50	2.3260E-04	4.26	1.0220E-05	5.02	2.5840E-07	5.78	3.7350E-09
0.47	3.1920E-01	1.23	1.0930E-01	1.99	2.3300E-02	2.75	2.9800E-03	3.51	2.2410E-04	4.27	9.7740E-06	5.03	2.4520E-07	5.79	3.5190E-09
0.48	3.1560E-01	1.24	1.0750E-01	2.00	2.2750E-02	2.76	2.8900E-03	3.52	2.1580E-04	4.28	9.3450E-06	5.04	2.3280E-07	5.80	3.3160E-09
0.49	3.1210E-01	1.25	1.0560E-01	2.01	2.2220E-02	2.77	2.8030E-03	3.53	2.0780E-04	4.29	8.9340E-06	5.05	2.2090E-07	5.81	3.1240E-09
0.50	3.0850E-01	1.26	1.0380E-01	2.02	2.1690E-02	2.78	2.7180E-03	3.54	2.0010E-04	4.30	8.5400E-06	5.06	2.0960E-07	5.82	2.9420E-09
0.51	3.0500E-01	1.27	1.0200E-01	2.03	2.1180E-02	2.79	2.6350E-03	3.55	1.9260E-04	4.31	8.1630E-06	5.07	1.9890E-07	5.83	2.7710E-09
0.52	3.0150E-01	1.28	1.0030E-01	2.04	2.0680E-02	2.80	2.5550E-03	3.56	1.8540E-04	4.32	7.8010E-06	5.08	1.8870E-07	5.84	2.6100E-09
0.53	2.9810E-01	1.29	9.8530E-02	2.05	2.0180E-02	2.81	2.4770E-03	3.57	1.7850E-04	4.33	7.4550E-06	5.09	1.7900E-07	5.85	2.4580E-09
0.54	2.9460E-01	1.30	9.6800E-02	2.06	1.9700E-02	2.82	2.4010E-03	3.58	1.7180E-04	4.34	7.1240E-06	5.10	1.6980E-07	5.86	2.3140E-09
0.55	2.9120E-01	1.31	9.5100E-02	2.07	1.9230E-02	2.83	2.3270E-03	3.59	1.6530E-04	4.35	6.8070E-06	5.11	1.6110E-07	5.87	2.1790E-09
0.56	2.8770E-01	1.32	9.3420E-02	2.08	1.8760E-02	2.84	2.2560E-03	3.60	1.5910E-04	4.36	6.5030E-06	5.12	1.5280E-07	5.88	2.0510E-09
0.57	2.8430E-01	1.33	9.1760E-02	2.09	1.8310E-02	2.85	2.1860E-03	3.61	1.5310E-04	4.37	6.2120E-06	5.13	1.4490E-07	5.89	1.9310E-09
0.58	2.8100E-01	1.34	9.0120E-02	2.10	1.7860E-02	2.86	2.1180E-03	3.62	1.4730E-04	4.38	5.9340E-06	5.14	1.3740E-07	5.90	1.8180E-09
0.59	2.7760E-01	1.35	8.8510E-02	2.11	1.7430E-02	2.87	2.0520E-03	3.63	1.4170E-04	4.39	5.6680E-06	5.15	1.3020E-07	5.91	1.7110E-09
0.60	2.7430E-01	1.36	8.6910E-02	2.12	1.7000E-02	2.88	1.9880E-03	3.64	1.3630E-04	4.40	5.4130E-06	5.16	1.2350E-07	5.92	1.6100E-09
0.61	2.7090E-01	1.37	8.5340E-02	2.13	1.6590E-02	2.89	1.9260E-03	3.65	1.3110E-04	4.41	5.1690E-06	5.17	1.1700E-07	5.93	1.5150E-09
0.62	2.6760E-01	1.38	8.3790E-02	2.14	1.6180E-02	2.90	1.8660E-03	3.66	1.2610E-04	4.42	4.9350E-06	5.18	1.1090E-07	5.94	1.4250E-09
0.63	2.6430E-01	1.39	8.2260E-02	2.15	1.5780E-02	2.91	1.8070E-03	3.67	1.2130E-04	4.43	4.7120E-06	5.19	1.0510E-07	5.95	1.3410E-09
0.64	2.6110E-01	1.40	8.0760E-02	2.16	1.5390E-02	2.92	1.7500E-03	3.68	1.1660E-04	4.44	4.4980E-06	5.20	9.9640E-08	5.96	1.2610E-09
0.65	2.5780E-01	1.41	7.9270E-02	2.17	1.5000E-02	2.93	1.6950E-03	3.69	1.1210E-04	4.45	4.2940E-06	5.21	9.4420E-08	5.97	1.1860E-09
0.66	2.5460E-01	1.42	7.7800E-02	2.18	1.4630E-02	2.94	1.6410E-03	3.70	1.0780E-04	4.46	4.0980E-06	5.22	8.9460E-08	5.98	1.1160E-09
0.67	2.5140E-01	1.43	7.6360E-02	2.19	1.4260E-02	2.95	1.5890E-03	3.71	1.0360E-04	4.47	3.9110E-06	5.23	8.4760E-08	5.99	1.0490E-09
0.68	2.4830E-01	1.44	7.4930E-02	2.20	1.3900E-02	2.96	1.5380E-03	3.72	9.9610E-05	4.48	3.7320E-06	5.24	8.0290E-08	6.00	9.8660E-10
0.69	2.4510E-01	1.45	7.3530E-02	2.21	1.3550E-02	2.97	1.4890E-03	3.73	9.5740E-05	4.49	3.5610E-06	5.25	7.6050E-08	6.01	9.2760E-10
0.70	2.4200E-01	1.46	7.2150E-02	2.22	1.3210E-02	2.98	1.4410E-03	3.74	9.2010E-05	4.50	3.3980E-06	5.26	7.2030E-08	6.02	8.7210E-10
0.71	2.3890E-01	1.47	7.0780E-02	2.23	1.2870E-02	2.99	1.3950E-03	3.75	8.8420E-05	4.51	3.2410E-06	5.27	6.8210E-08	6.03	8.1980E-10
0.72	2.3580E-01	1.48	6.9440E-02	2.24	1.2550E-02	3.00	1.3500E-03	3.76	8.4960E-05	4.52	3.0920E-06	5.28	6.4590E-08	6.04	7.7060E-10
0.73	2.3270E-01	1.49	6.8110E-02	2.25	1.2220E-02	3.01	1.3060E-03	3.77	8.1620E-05	4.53	2.9490E-06	5.29	6.1160E-08	6.05	7.2420E-10
0.74	2.2960E-01	1.50	6.6810E-02	2.26	1.1910E-02	3.02	1.2640E-03	3.78	7.8410E-05	4.54	2.8130E-06	5.30	5.7900E-08	6.06	6.8060E-10
0.75	2.2660E-01	1.51	6.5520E-02	2.27	1.1600E-02	3.03	1.2230E-03	3.79	7.5320E-05	4.55	2.6820E-06	5.31	5.4810E-08	6.07	6.3960E-10

1. Consider a standard PAM modulation with symbol duration $T_s = 1$ [μs], constellation $\mathcal{A} = \{-3c, -c, c, 3c\}$, and transmit pulse $h(t)$ which has a duration $T_p = 0.5$ [μs]. The pulse and a block diagram of the PAM transmitter are depicted below.

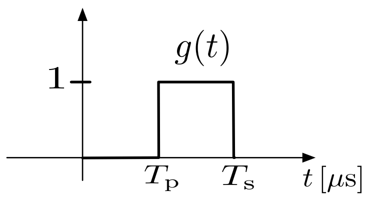




   (a) What is the data rate of the system ($T_s = 1$ [μs])? Is it the maximum data rate that can be achieved without introducing inter symbol interference? Motivate. (3p)

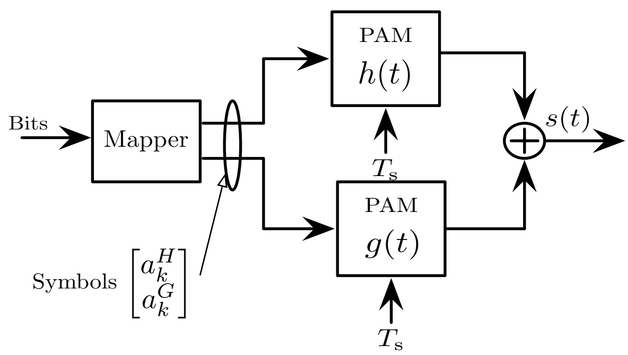
   (b) Assuming equally likely transmitted symbols, compute the average symbol energy and transmit power of the standard PAM transmitter. (3p)

   (c) Consider now a variant PAM transmitter that uses two pulses, the pulse $h(t)$ and another pulse $g(t)$, with symbol duration $T_s = 1$ [μs]. The pulse $g(t)$ and the block diagram of the variant PAM system are shown below.



Mapper

Bits	Symbols $\begin{bmatrix} a_k^H \\ a_k^G \end{bmatrix}$
00	$\begin{bmatrix} -c \\ -c \end{bmatrix}$
01	$\begin{bmatrix} -c \\ c \end{bmatrix}$
10	$\begin{bmatrix} c \\ -c \end{bmatrix}$
11	$\begin{bmatrix} c \\ c \end{bmatrix}$



      i. Depict the four different signal alternatives used by the variant PAM transmitter? (2p)

      ii. Compute the average symbol energy and transmit power of the variant PAM transmitter and compare them with the results found in (b). What do you conclude? (4p)
      Hint: In your conclusions, take into account the data rate achieved by the standard and variant PAM systems.

(a) There are 4 different constellation points, thus $M = 4$. The data rate is given by

$$R_{\mathrm{b}} = \frac{\log_2(M)}{T_{\mathrm{s}}} = 2 \text{ [Mbit/s]}.$$

This is not the maximum data rate that can be used in the system. Since $h(t)$ is time-limited pulse, we can ensure ISI-free transmission with $T_{\mathrm{s}} \geq T_{\mathrm{p}}$. Thus the maximum data rate correspond to using the smallest possible symbol duration $T_{\mathrm{s,min}} = T_{\mathrm{p}}$, that is

$$R_{\mathrm{b,max}} = \frac{\log_2(M)}{T_{\mathrm{s,min}}} = \frac{2}{T_{\mathrm{p}}} = 4 \text{ [Mbit/s]}.$$

(b) We can compute the average symbol energy of the 4-PAM transmission following $E_{\mathrm{s}} = E_h \times E_a$, where E_h is the energy of the pulse $h(t)$ and E_a is the average constellation energy.

$$E_h = \int_{-\infty}^{+\infty} |h(t)|^2 dt = \int_0^{T_{\mathrm{p}}} 1 dt = T_{\mathrm{p}}.$$

$$E_a = \sum_{k=1}^{k=4} \Pr\{a = a_k\}|a_k|^2 = \frac{1}{4}(9c^2 + c^2 + c^2 + 9c^2) = 5c^2.$$

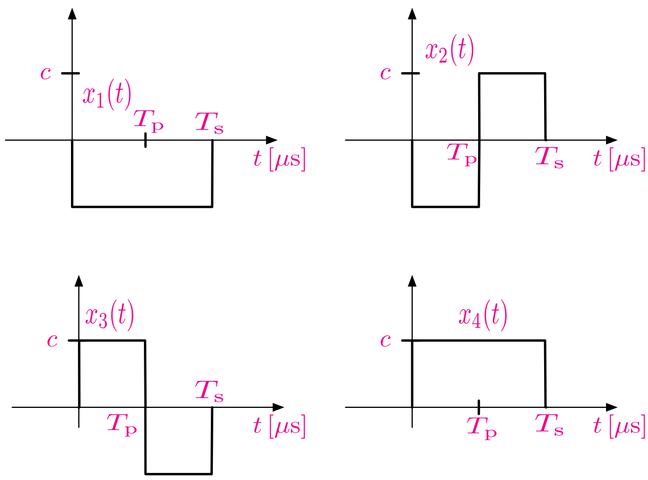
$$E_{\mathrm{s}} = E_h \times E_a = T_{\mathrm{p}} \times 5c^2 = 2.5c^2 \text{ } [\mu\text{J}].$$

The transmitted power can be straightforwardly computed as

$$P = \frac{E_{\mathrm{s}}}{T_{\mathrm{s}}} = \frac{2.5c^2 \text{ } \mu\text{J}}{1 \text{ } \mu s} = 2.5c^2 \text{ [w]}.$$

(c) The signal alternatives are shown below.

$$x(t) \in \{-ch(t) - cg(t), -ch(t) + cg(t), ch(t) - cg(t), ch(t) + cg(t)\}$$



(d) Let E_{x_k} be the energy of signal alternative $x_k(t)$, $k = 1, 2, 3, 4$.

$$E_{x_k} = \int_{-\infty}^{+\infty} |x_k(t)|^2 dt$$

$$= \int_0^{T_{\mathrm{s}}} c^2 dt = c^2 T_{\mathrm{s}} = c^2 \text{ } [\mu\text{J}], \quad k = 1, 2, 3, 4.$$

That is, all signal alternatives (pulses) have the same average energy. Then, we can compute the average symbol energy as follows.

$$\begin{aligned} E'_{\text{s}} &= \sum_{k=1}^{4} \Pr\{x = x_k\} E_{x_k} \\ &= \sum_{k=1}^{4} \Pr\{x = x_k\} c^2 \\ &= c^2 \underbrace{\sum_{k=1}^{4} \Pr\{x = x_k\}}_{=1} \\ &= c^2 \ [\mu\text{J}]. \end{aligned}$$

Note: we can also assume that the different signal alternatives are equally likely, that is $\Pr\{x = x_k\} = 1/4$, $k = 1, 2, 3, 4$, when computing the average symbol energy.

The average power is given by

$$P' = E'_{\text{s}}/T_{\text{s}} = c^2 \ [\text{w}]$$

Note that the data rate of the system is given by $R_{\text{b,var}} = \log_2 M'/T_{\text{s}} = 2\text{Mbit/s}$ ($M' = 4$ is the number of signal alternatives used by the variant PAM system).

In comparison to the 4-PAM system in b), we see that the variant PAM system ensures the same data rate at lower energy consumption. Thus, the variant PAM system is more energy efficient.

2. Consider two stations (i.e., A and B) connected over a physical medium of length d=200 [m] and with propagation speed 2×10^8 [m/s]. The data link layer uses a Stop-And-Wait ARQ protocol, while the medium access control is based on Aloha. Data frames (n_f) consists of 4000 bit while ACK frames (n_a) are 1000 bit long. The data rate on the medium is 1 [Gbit/s]. The protocol use binary state variables S_l and R_n, where S_l is the sequence number of the last transmitted frame and R_n is the sequence number of the frame that the receiver expects next. They are initialized as $S_\mathrm{l} = 1$ and $R_\mathrm{n} = 0$. Assume that A is the only station transmitting data and that the frame processing time is negligible.

   (a) Draw a timing diagram until three consecutive data frames are received correctly and acknowledged. The timeout is set to $t_{out} = 8 \cdot t_f$, where t_f is the data frame duration. Assume that no data frame or ACK frame errors occur. (2p)

   (b) Repeat part (a) assuming that the medium access control is now based on a persistent Carrier Sensing Multiple Access (CSMA) protocol with sensing time t_s, where $t_s = 0.25 \cdot t_f$. (2p)

   (c) Repeat part (a) assuming the CSMA MAC protocol described in part (b) and that the first transmitted ACK frame suffers a frame error. All other frame transmissions are successful. (2p)

   (d) Assume the CSMA MAC protocol described in part (b). Compute the effective rate R_{eff} experienced by the network layer at station A when no data or ACK frame errors occur and when the overhead of the Stop-And-Wait ARQ protocol (i.e., n_0) is equal to 200 bits. (2p)

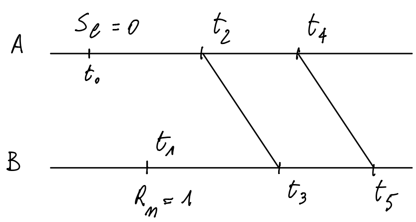
   (e) Assume now that the medium introduces independent bit errors with probability $p = 10^{-4}$ that affects the information frames only (i.e., ACK frames are error-free). Compute the effective rate R_{eff} experienced by the network layer at station A for the frame length, timeout, and MAC layer protocols used while solving part (b). (4p)

The timing diagram should have two time axis, one for the transmitter events and one for the receiver events. On the axes, it should be noted when:

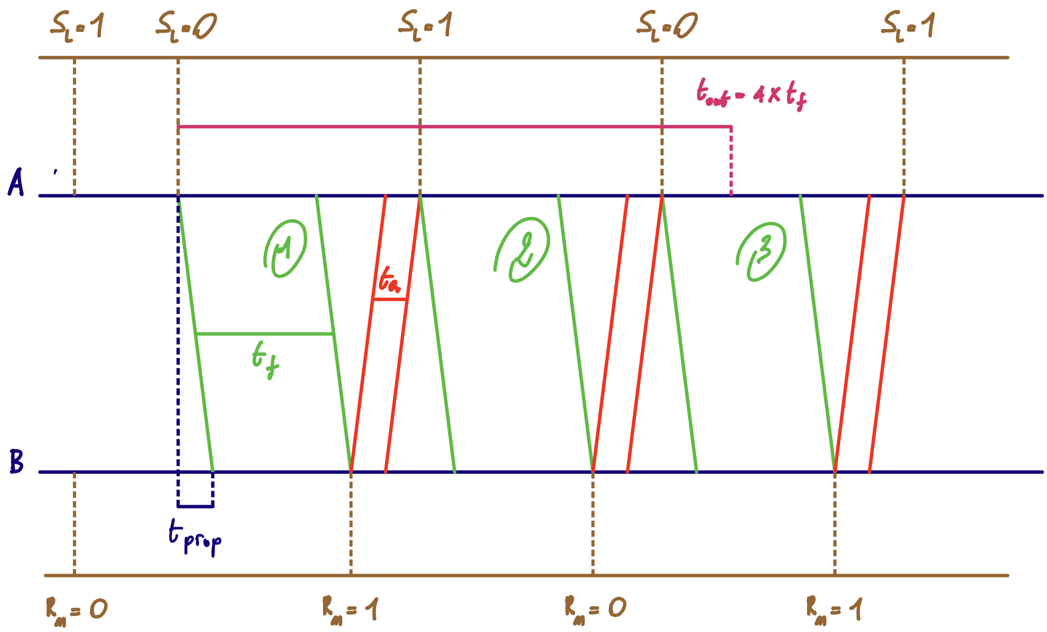
- data frame and ACK frame transmission/reception start
- sensing time of the persistent CSMA MAC protocol starting and expiration points,
- timeout timer starting and expiration points,
- state variables changes

Make sure to label the axis and draw carefully. See below for a (hypothetical) example that illustrates events in the table below.

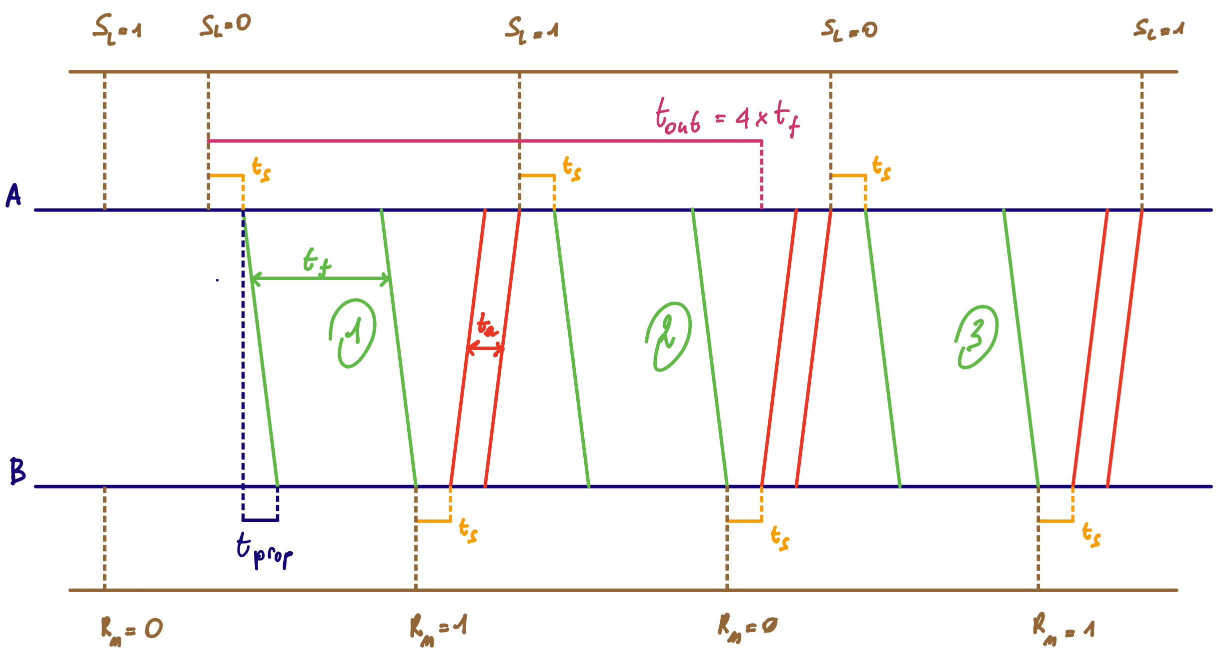
Time	Event
t_0	S_l change from 1 to 0
t_1	R_n change from 0 to 1
t_2	Data frame transmission starts
t_3	Data frame reception starts
t_4	Data frame transmission ends
t_5	Data frame reception ends



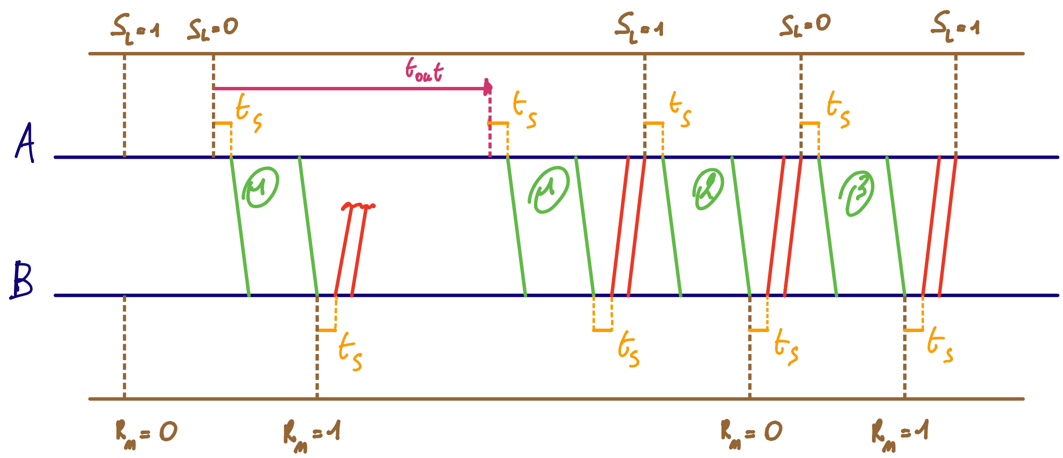
(a) The time diagram representing three consecutive data frames transmitted and received correctly and acknowledged when Aloha is used at the MAC layer.



(b) The time diagram representing three consecutive data frames transmitted and received correctly and acknowledged when a persistent CSMA with $t_s = 0.25\, t_f$ is used at the MAC layer



(c) The time diagram when the first transmitted ACK frame suffers a frame error and all other frame transmissions are successful. A persistent CSMA with $t_s = 0.25\, t_f$ is used at the MAC layer.



(d) In the absence of data frame or ACK frame errors R_{eff} can be computed as follows:

$$R_{eff} = \frac{n_f - n_0}{t_0} \tag{1}$$

Looking at the solution of part (b), the turn around time t_0 can be computed as:

$$t_0 = 2 \cdot t_s + 2 \cdot t_{prop} + t_f + t_a \tag{2}$$

where

$$t_{prop} = c/d = 1 \cdot 10^{-6} \text{ [s]},$$
$$t_f = n_f/R = 4 \cdot 10^{-6} \text{ [s]},$$
$$t_a = n_a/R = 1 \cdot 10^{-6} \text{ [s]},$$
$$t_s = 0.25 \cdot t_f = 1 \cdot 10^{-6} \text{ [s]}.$$

As a result we obtain:

$$t_0 = 9 \cdot 10^{-6} \text{[s]} \Rightarrow R_{eff} = \frac{3800}{9 \cdot 10^{-6}} = 0.42 \text{ [Gbit/s]}.$$

(e) When talking about the ARQ protocol family, we know from class that the number of transmissions needed to deliver a frame can be represented by a geometric random variable n_t. We also know from class that in the case of a Stop-And-Wait ARQ protocol with frame errors, the time needed to deliver a correct frame can be expressed as:

$$t_{sw} = (n_t - 1) \cdot t_{out} + t_0$$

The effective R_{eff} experienced by the network layer at station A can be then expressed as:

$$R_{eff} = \frac{n_f - n_0}{E[t_{sw}]} \tag{3}$$

Now, looking at the expression in (2), we can derive

$$E[t_{sw}] = E[(n_t - 1) \cdot t_{out} + t_0] = E[(n_t - 1)] \cdot t_{out} + t_0 \Rightarrow$$

$$\Rightarrow E[t_{sw}] = \frac{P_f}{1 - P_f} \cdot t_{out} + t_0$$

P_f can be computed from the bit error probability $p = 10^{-4}$ as:

$$P_f = 1 - (1-p)^{n_f} = 0.3297$$

We know that $t_{out} = 4 \cdot t_f$, while the value of t_0 was derived in part (d). As a result we get:

$$E[t_{sw}] = \frac{P_f}{1-P_f} \cdot t_{out} + t_0 = 16.87 \cdot 10^{-6}[\text{s}] \Rightarrow$$

$$\Rightarrow \; R_{eff} = \frac{n_f - n_0}{E[t_{sw}]} = 0.22 \text{ [Gbit/s]}$$

3. Consider a LAN with M attached stations. The protocol stack consists of a physical layer that provides a data rate $R = 1$ [Gbit/s], a data link layer, and a network layer. We assume that the physical layer is robust, i.e., no frame errors. The LAN medium length is 1000 [m] and the propagation speed is $c = 2 \times 10^8$ [m/s]. The network layer SDUs have a size of 5000 byte, and the data link layer overhead (i.e., due to header and trailer) is 32 byte.

   (a) Assume that the MAC layer uses a TDMA protocol where each station is assigned a fixed slot of duration T_s. The slot time is designed in such a way that (i) the bits transmitted in a slot can carry a whole network SDU, (ii) no collisions can occur at any receiving station, and (iii) to maximize the data rate for the stations. Find the value of M such that the average data rate experienced by the network layer of each station (i.e., R_{avg}) is at least 15 [Mbit/s]. (3p)

   (b) Let R_{agg} be the aggregated, useful data rate on the LAN, i.e., the average number of network SDU bits from all M stations transmitted on the LAN per second. Suppose that N of the M stations have data to transmit. What is the ratio of transmitting stations, i.e., N/M, to have the LAN operating at a normalized throughput, $\rho = R_{agg}/R$, of at least 80%? Assume the value of M and the TDMA scheme in part (a). (3p)

   (c) Repeat part (a) and part (b) when $R = 2.5$ [Gbit/s]. Comment on the results. (3p)

   (d) Suppose we replace the TDMA scheme with a simple reservation scheme. The reservation messages are 32 byte long. They are transmitted in reservation minislots of duration T_{ms}. A reservation minislot is designed such that (i) the bits transmitted in a slot can carry a whole reservation message, (ii) no collisions can occur at any receiving station, and (iii) to maximize the data rate for the stations. A cycle starts with the M stations sending one reservation message each. Then, the N stations with data to transmit send a single network SDU each, which completes the cycle. Assuming that all the stations have always data to transmit, find the value of M such that the average data rate experienced by the network layer of each station (i.e., R_{avg}) is at least 15 [Mbit/s]. (3p)

Solution Problem 3

(a) The slot duration assigned to each station can be computed as:

$$T_s = t_{prop-max} + T_f$$

where $t_{prop-max}$ is the maximum propagation delay and T_f is the time required to transmit (by the data link layer) the network layer SDU (i.e., consisting of n_{SDU} bits). We know that

$$T_f = \frac{n_{SDU} + n_0}{R} = 40.3\mu\text{s}$$

where n_0 is the transmission overhead introduced by the data link layer. We also know that

$$t_{prop-max} = L/c = 5\mu\text{s}$$

then,

$$T_s = L/c + T_f = 45.3\mu\text{s}$$

The average data rate experienced by the network layer of each station R_{avg} can be computed as:

$$R_{avg} = \frac{n_{SDU}}{M \cdot T_s}$$

Given the value of $R_{avg} = 15\text{Mbit/s}$ it is possible to derive the value of M as

$$M = \frac{n_{SDU}}{R_{avg} \cdot T_s} = 58.9$$

We need to take the floor of the value, so the correct answer is $M = 58$

(b) The aggregated rate R_{agg} can be computed as:

$$R_{agg} = N \cdot R_{avg} = N \cdot \frac{n_{SDU}}{M \cdot T_s}$$

since

$$\rho = \frac{R_{agg}}{R} = \frac{N}{R} \cdot \frac{n_{SDU}}{M \cdot T_s}$$

we can derive the value of N as:

$$N = \frac{\rho \cdot R \cdot M \cdot T_s}{n_{SDU}} = 52.5$$

Using the ceiling of the value of N, the answer to the question is

$$\frac{N}{M} = \frac{53}{58} = 0.9$$

This means that 90% of all the stations need to always be transmitting data to guarantee the LAN has a normalized throughput of at least 80%.

(c) With a higher rate R at the physical layer the value of T_f changes while the value of $t_{prop-max}$ stays the same. When $R = 2.5$ [Gbit/s] the new value of T_f is

$$T_f = \frac{n_{SDU} + n_0}{R} = 16.1\mu\text{s}$$

As a result

$$T_s = t_{prop-max} + T_f = 5 \cdot 10^{-6} + 16.1 \cdot 10^{-6} = 21.1\mu\text{s}$$

Given the value of $R_{avg} = 15$Mbit/s it is possible to derive the value of M as

$$M = \frac{n_{SDU}}{R_{avg} \cdot T_s} = 126.4 \Rightarrow M = 126$$

With an higher rate we can fit more station transmitting with the required R_{avg} value. This is because each station will require less time to transmit a SDU frame. On the other hand, with a smaller value of T_f the impact of the propagation time becomes more noticeable. Unfortunately this plays against the overall LAN performance. If we now compute the value of the aggregated rate R_{agg} when all M stations are transmitting, we obtain

$$R_{agg} = M \cdot R_{avg} = M \cdot \frac{n_{SDU}}{M \cdot T_s} = 1.89 \text{ [Gbit/s]}$$

which corresponds to a value of the normalized throughput for the LAN of

$$\rho = R_{agg}/R = 1.89/2.5 = 0.76$$

This means that with current value of R it is not possible to achieved the desired target normalized throughput value of 80%.

(d) In the case of a reservation scheme the the average data rate experienced by the network layer of each station (R_{avg}) can be computed as

$$R_{avg} = \frac{n_{SDU}}{M \cdot T_{ms} + N \cdot T_s}$$

If all the station have always data to transmit N=M and

$$R_{avg} = \frac{n_{SDU}}{M \cdot T_{ms} + M \cdot T_s} \Rightarrow$$

$$M = \frac{n_{SDU}}{R_{avg}(T_{ms} + T_s)}$$

The value of T_{ms} can be computed as

$$T_{ms} = t_{prop-max} + \frac{n_{res}}{R} = 5 \cdot 10^{-6} + 0.26 \cdot 10^{-6} = 5.26\mu\text{s}$$

while the value of T_s is the same as the one computed in part (a)

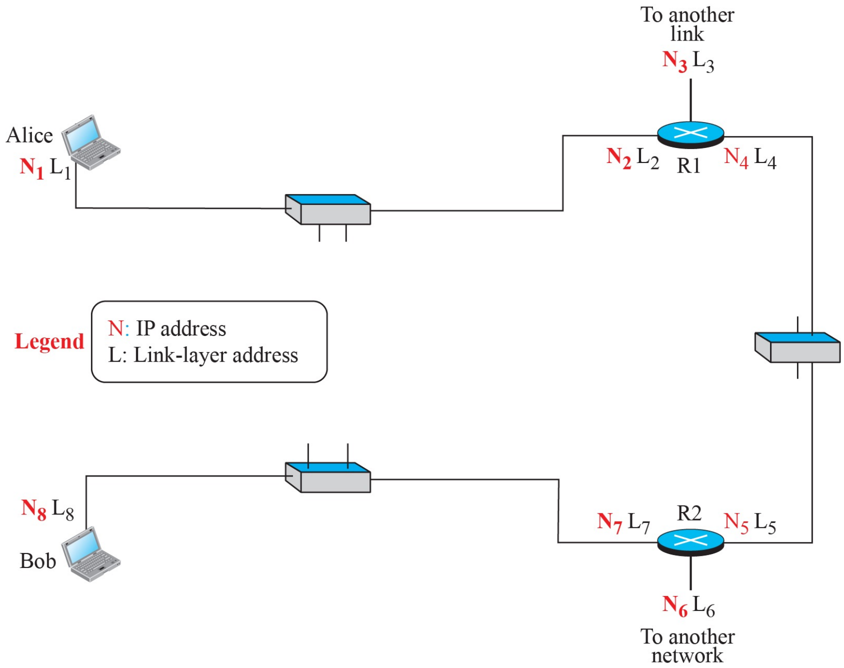
$$T_s = t_{prop-max} + T_f = 5 \cdot 10^{-6} + 40.3 \cdot 10^{-6} = 45.3\mu\text{s}$$

We can then derive the value of M as

$$M = \frac{n_{SDU}}{R_{avg}(T_{ms} + T_s)} = 52.8$$

This means the reservation system under exam I can fit at most 52 station on the LAN while guaranteeing that each station transmits on average 15 [Mbit/s]. This is 6 stations less than the number computed in part (a). The reason for this smaller number is due to the overhead of the reservation scheme, i.e., the M reservation minislot that I have to transmit. This is in agreement with the reasoning done in class whereas when we know that all the station in LAN will be transmitting all the time a TDMA system is more efficient that a reservation scheme.

4. Consider the LAN depicted in the figure below. Alice would like to send Bob UDP segments over a packet-switched network running a generic Physical (PHY) layer and Data Link (DL) layer protocol. The network layer protocol is IP. Each L_i (with $i = 1...8$) in the figure represents a link layer address and each N_i (with $i = 1...8$) represents a network layer address. R1 and R2 are routers, while the unnamed boxes represent hubs.



(a) List (in the protocol stack's correct order) the protocols involved in the transmission process at Alice's host, R1, R2, and Bob's host. (2p)

(b) Let PH and PT denote the header and the trailer (if it exists) of protocol P, respectively. For example, the DL header is denoted as DLH, and the DL trailer is denoted as DLT. Moreover, let the protocol data unit (PDU) of protocol P be denoted as P-PDU. Indicate what is the *network layer* PDU generated at Alice's host and what is the *data link layer* PDU received at Bob's host. (3p)

(c) Consider now the following three frames: (1) from Alice's host to R1, (2) from R1 to R2, and (3) from R2 to Bob's host.

i. List the link layer and network layer addresses contained in each frame. (2p)

ii. Using the notation explained in part (b), indicate in which field the link layer and network layer addresses are saved in each frame, respectively. (2p)

(d) Describe the content that needs to be stored at the forwarding tables at Alice's host, R1, R2 to allow the correct delivery of a UDP segment from Alice to Bob. (3p) Assume that the structure of a forwarding table is the following:

Destination IP	**Next hop IP**	**Interface**
N_i	N_j	L_n
...	...	...
...	...	...

Solution Problem 4

(a) The protocols involved at each node are the following (from lower to upper layer):

- Alice's host: PHY, DL, IP, UDP
- R1: PHY, DL, IP
- R2: PHY, DL, IP
- Bob's host: PHY, DL, IP, UDP

(b) Encaspulation at Alice's host: the network layer PDU is

|IPH|UDP-PDU|

Decapsulation at Bob's host: the data link layer PDU is

|DLH|IPH|UDP-PDU|DLT|

(c) i. The link and network layer addresses contained in each one of frames are the following:

- Alice's host to R1: L_1, L_2, N_1, N_8
- R1 to R2: L_4, L_5, N_1, N_8
- R2 to Bob's host: L_7, L_8, N_1, N_8

ii. The link layer addresses are stored in the DLH field of each frame, while the network layer addresses are store in the IPH field of each frame.

(d) The minimum amount of information that needs to be available in the forwarding tables is the following.

- Alice's host

Destination IP	**Next hop IP**	**Interface**
N_8	N_2	N_1
...	...	...
...	...	...

- R1:

Destination IP	**Next hop IP**	**Interface**
N_8	N_5	N_4
...	...	...
...	...	...

- R2:

Destination IP	**Next hop IP**	**Interface**
N_8	N_8	N_7
...	...	...
...	...	...

Note: due to a typo in the description of forwarding table content (i.e., L_n instead of N_n) we will accept an answer that reports the link layer address instead of the network layer address for a correct interface.