Exam in
SSY305 Communication Systems

Department of Electrical Engineering

Exam date: March 18, 2021, 14:00–18:00
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Material All material is allowed on this exam. Students are required to solve the exam problems individually. Cooperation, in any form or with anyone, is strictly forbidden.

Grading A correct, clear and well-motivated solution gives at most 12 points per problem.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. No fractional points are awarded.

Answers in any other language than Swedish or English are ignored.

Submission Exam problem solutions should be solved on paper as in a normal exam.

- Make sure that each paper is clearly marked with your name, exam problem number and page number.
- Scan or photograph your solutions.
- Name your image files `Problem_YY_Page_XX`. Example: `Problem_01_Page_02.jpg`.
- If you want, you can combine images for the same problem into a single document (e.g., Word or PDF) named `Problem_YY`.
- Submit your solutions by uploading the image files or documents via Canvas before **March 18, 18:30**.

Solutions Are made available at the earliest on March 19 at 19:00 on the course web page.

Results Exam results are posted on Canvas no later than on March 29, 2021. The grading reviews will have to be done remotely according to a process that will be explained in the course webpage.

Grades The final grade of the course will be decided by the project (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). Project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade.

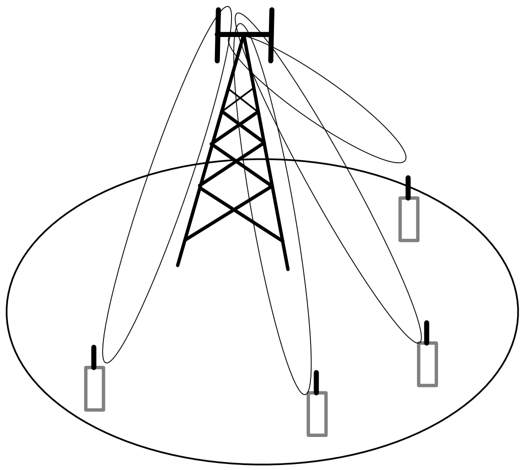
Total Score	0–39	40–68	69–79	≥ 80
Grade	Fail	3	4	5

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.
Good luck!

Table over the Q-function

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
0.00	5.0000E-01	0.76	2.2360E-01	1.52	6.4260E-02	2.28	1.1300E-02	3.04	1.1830E-03	3.80	7.2350E-05	4.56	2.5580E-06	5.32	5.1880E-08
0.01	4.9600E-01	0.77	2.2060E-01	1.53	6.3010E-02	2.29	1.1010E-02	3.05	1.1440E-03	3.81	6.9480E-05	4.57	2.4390E-06	5.33	4.9110E-08
0.02	4.9200E-01	0.78	2.1770E-01	1.54	6.1780E-02	2.30	1.0720E-02	3.06	1.1070E-03	3.82	6.6730E-05	4.58	2.3250E-06	5.34	4.6470E-08
0.03	4.8800E-01	0.79	2.1480E-01	1.55	6.0570E-02	2.31	1.0440E-02	3.07	1.0700E-03	3.83	6.4070E-05	4.59	2.2160E-06	5.35	4.3980E-08
0.04	4.8400E-01	0.80	2.1190E-01	1.56	5.9380E-02	2.32	1.0170E-02	3.08	1.0350E-03	3.84	6.1520E-05	4.60	2.1120E-06	5.36	4.1610E-08
0.05	4.8010E-01	0.81	2.0900E-01	1.57	5.8210E-02	2.33	9.9030E-03	3.09	1.0010E-03	3.85	5.9060E-05	4.61	2.0130E-06	5.37	3.9370E-08
0.06	4.7610E-01	0.82	2.0610E-01	1.58	5.7050E-02	2.34	9.6420E-03	3.10	9.6760E-04	3.86	5.6690E-05	4.62	1.9190E-06	5.38	3.7240E-08
0.07	4.7210E-01	0.83	2.0330E-01	1.59	5.5920E-02	2.35	9.3870E-03	3.11	9.3540E-04	3.87	5.4420E-05	4.63	1.8280E-06	5.39	3.5230E-08
0.08	4.6810E-01	0.84	2.0050E-01	1.60	5.4800E-02	2.36	9.1370E-03	3.12	9.0430E-04	3.88	5.2230E-05	4.64	1.7420E-06	5.40	3.3320E-08
0.09	4.6410E-01	0.85	1.9770E-01	1.61	5.3700E-02	2.37	8.8940E-03	3.13	8.7400E-04	3.89	5.0120E-05	4.65	1.6600E-06	5.41	3.1510E-08
0.10	4.6020E-01	0.86	1.9490E-01	1.62	5.2620E-02	2.38	8.6560E-03	3.14	8.4470E-04	3.90	4.8100E-05	4.66	1.5810E-06	5.42	2.9800E-08
0.11	4.5620E-01	0.87	1.9220E-01	1.63	5.1550E-02	2.39	8.4240E-03	3.15	8.1640E-04	3.91	4.6150E-05	4.67	1.5060E-06	5.43	2.8180E-08
0.12	4.5220E-01	0.88	1.8940E-01	1.64	5.0500E-02	2.40	8.1980E-03	3.16	7.8880E-04	3.92	4.4270E-05	4.68	1.4340E-06	5.44	2.6640E-08
0.13	4.4830E-01	0.89	1.8670E-01	1.65	4.9470E-02	2.41	7.9760E-03	3.17	7.6220E-04	3.93	4.2470E-05	4.69	1.3660E-06	5.45	2.5180E-08
0.14	4.4430E-01	0.90	1.8410E-01	1.66	4.8460E-02	2.42	7.7600E-03	3.18	7.3640E-04	3.94	4.0740E-05	4.70	1.3010E-06	5.46	2.3810E-08
0.15	4.4040E-01	0.91	1.8140E-01	1.67	4.7460E-02	2.43	7.5490E-03	3.19	7.1140E-04	3.95	3.9080E-05	4.71	1.2390E-06	5.47	2.2500E-08
0.16	4.3640E-01	0.92	1.7880E-01	1.68	4.6480E-02	2.44	7.3440E-03	3.20	6.8710E-04	3.96	3.7470E-05	4.72	1.1790E-06	5.48	2.1270E-08
0.17	4.3250E-01	0.93	1.7620E-01	1.69	4.5510E-02	2.45	7.1430E-03	3.21	6.6370E-04	3.97	3.5940E-05	4.73	1.1230E-06	5.49	2.0100E-08
0.18	4.2860E-01	0.94	1.7360E-01	1.70	4.4570E-02	2.46	6.9470E-03	3.22	6.4100E-04	3.98	3.4460E-05	4.74	1.0690E-06	5.50	1.8990E-08
0.19	4.2470E-01	0.95	1.7110E-01	1.71	4.3630E-02	2.47	6.7560E-03	3.23	6.1900E-04	3.99	3.3040E-05	4.75	1.0170E-06	5.51	1.7940E-08
0.20	4.2070E-01	0.96	1.6850E-01	1.72	4.2720E-02	2.48	6.5690E-03	3.24	5.9760E-04	4.00	3.1670E-05	4.76	9.6800E-07	5.52	1.6950E-08
0.21	4.1680E-01	0.97	1.6600E-01	1.73	4.1820E-02	2.49	6.3870E-03	3.25	5.7700E-04	4.01	3.0360E-05	4.77	9.2110E-07	5.53	1.6010E-08
0.22	4.1290E-01	0.98	1.6350E-01	1.74	4.0930E-02	2.50	6.2100E-03	3.26	5.5710E-04	4.02	2.9100E-05	4.78	8.7650E-07	5.54	1.5120E-08
0.23	4.0900E-01	0.99	1.6110E-01	1.75	4.0060E-02	2.51	6.0370E-03	3.27	5.3770E-04	4.03	2.7890E-05	4.79	8.3390E-07	5.55	1.4280E-08
0.24	4.0520E-01	1.00	1.5870E-01	1.76	3.9200E-02	2.52	5.8680E-03	3.28	5.1900E-04	4.04	2.6730E-05	4.80	7.9330E-07	5.56	1.3490E-08
0.25	4.0130E-01	1.01	1.5620E-01	1.77	3.8360E-02	2.53	5.7030E-03	3.29	5.0090E-04	4.05	2.5610E-05	4.81	7.5470E-07	5.57	1.2740E-08
0.26	3.9740E-01	1.02	1.5390E-01	1.78	3.7540E-02	2.54	5.5430E-03	3.30	4.8340E-04	4.06	2.4540E-05	4.82	7.1780E-07	5.58	1.2030E-08
0.27	3.9360E-01	1.03	1.5150E-01	1.79	3.6730E-02	2.55	5.3860E-03	3.31	4.6650E-04	4.07	2.3510E-05	4.83	6.8270E-07	5.59	1.1350E-08
0.28	3.8970E-01	1.04	1.4920E-01	1.80	3.5930E-02	2.56	5.2340E-03	3.32	4.5010E-04	4.08	2.2520E-05	4.84	6.4920E-07	5.60	1.0720E-08
0.29	3.8590E-01	1.05	1.4690E-01	1.81	3.5150E-02	2.57	5.0850E-03	3.33	4.3420E-04	4.09	2.1570E-05	4.85	6.1730E-07	5.61	1.0120E-08
0.30	3.8210E-01	1.06	1.4460E-01	1.82	3.4380E-02	2.58	4.9400E-03	3.34	4.1890E-04	4.10	2.0660E-05	4.86	5.8690E-07	5.62	9.5480E-09
0.31	3.7830E-01	1.07	1.4230E-01	1.83	3.3620E-02	2.59	4.7990E-03	3.35	4.0410E-04	4.11	1.9780E-05	4.87	5.5800E-07	5.63	9.0100E-09
0.32	3.7450E-01	1.08	1.4010E-01	1.84	3.2880E-02	2.60	4.6610E-03	3.36	3.8970E-04	4.12	1.8940E-05	4.88	5.3040E-07	5.64	8.5030E-09
0.33	3.7070E-01	1.09	1.3790E-01	1.85	3.2160E-02	2.61	4.5270E-03	3.37	3.7580E-04	4.13	1.8140E-05	4.89	5.0420E-07	5.65	8.0220E-09
0.34	3.6690E-01	1.10	1.3570E-01	1.86	3.1440E-02	2.62	4.3960E-03	3.38	3.6240E-04	4.14	1.7370E-05	4.90	4.7920E-07	5.66	7.5690E-09
0.35	3.6320E-01	1.11	1.3350E-01	1.87	3.0740E-02	2.63	4.2690E-03	3.39	3.4950E-04	4.15	1.6620E-05	4.91	4.5540E-07	5.67	7.1400E-09
0.36	3.5940E-01	1.12	1.3140E-01	1.88	3.0050E-02	2.64	4.1450E-03	3.40	3.3690E-04	4.16	1.5910E-05	4.92	4.3270E-07	5.68	6.7350E-09
0.37	3.5570E-01	1.13	1.2920E-01	1.89	2.9380E-02	2.65	4.0250E-03	3.41	3.2480E-04	4.17	1.5230E-05	4.93	4.1110E-07	5.69	6.3520E-09
0.38	3.5200E-01	1.14	1.2710E-01	1.90	2.8720E-02	2.66	3.9070E-03	3.42	3.1310E-04	4.18	1.4580E-05	4.94	3.9060E-07	5.70	5.9900E-09
0.39	3.4830E-01	1.15	1.2510E-01	1.91	2.8070E-02	2.67	3.7930E-03	3.43	3.0180E-04	4.19	1.3950E-05	4.95	3.7110E-07	5.71	5.6490E-09
0.40	3.4460E-01	1.16	1.2300E-01	1.92	2.7430E-02	2.68	3.6810E-03	3.44	2.9090E-04	4.20	1.3350E-05	4.96	3.5250E-07	5.72	5.3260E-09
0.41	3.4090E-01	1.17	1.2100E-01	1.93	2.6800E-02	2.69	3.5730E-03	3.45	2.8030E-04	4.21	1.2770E-05	4.97	3.3480E-07	5.73	5.0220E-09
0.42	3.3720E-01	1.18	1.1900E-01	1.94	2.6190E-02	2.70	3.4670E-03	3.46	2.7010E-04	4.22	1.2220E-05	4.98	3.1790E-07	5.74	4.7340E-09
0.43	3.3360E-01	1.19	1.1700E-01	1.95	2.5590E-02	2.71	3.3640E-03	3.47	2.6020E-04	4.23	1.1680E-05	4.99	3.0190E-07	5.75	4.4620E-09
0.44	3.3000E-01	1.20	1.1510E-01	1.96	2.5000E-02	2.72	3.2640E-03	3.48	2.5070E-04	4.24	1.1180E-05	5.00	2.8670E-07	5.76	4.2060E-09
0.45	3.2640E-01	1.21	1.1310E-01	1.97	2.4420E-02	2.73	3.1670E-03	3.49	2.4150E-04	4.25	1.0690E-05	5.01	2.7220E-07	5.77	3.9640E-09
0.46	3.2280E-01	1.22	1.1120E-01	1.98	2.3850E-02	2.74	3.0720E-03	3.50	2.3260E-04	4.26	1.0220E-05	5.02	2.5840E-07	5.78	3.7350E-09
0.47	3.1920E-01	1.23	1.0930E-01	1.99	2.3300E-02	2.75	2.9800E-03	3.51	2.2410E-04	4.27	9.7740E-06	5.03	2.4520E-07	5.79	3.5190E-09
0.48	3.1560E-01	1.24	1.0750E-01	2.00	2.2750E-02	2.76	2.8900E-03	3.52	2.1580E-04	4.28	9.3450E-06	5.04	2.3280E-07	5.80	3.3160E-09
0.49	3.1210E-01	1.25	1.0560E-01	2.01	2.2220E-02	2.77	2.8030E-03	3.53	2.0780E-04	4.29	8.9340E-06	5.05	2.2090E-07	5.81	3.1240E-09
0.50	3.0850E-01	1.26	1.0380E-01	2.02	2.1690E-02	2.78	2.7180E-03	3.54	2.0010E-04	4.30	8.5400E-06	5.06	2.0960E-07	5.82	2.9420E-09
0.51	3.0500E-01	1.27	1.0200E-01	2.03	2.1180E-02	2.79	2.6350E-03	3.55	1.9260E-04	4.31	8.1630E-06	5.07	1.9890E-07	5.83	2.7710E-09
0.52	3.0150E-01	1.28	1.0030E-01	2.04	2.0680E-02	2.80	2.5550E-03	3.56	1.8540E-04	4.32	7.8010E-06	5.08	1.8870E-07	5.84	2.6100E-09
0.53	2.9810E-01	1.29	9.8530E-02	2.05	2.0180E-02	2.81	2.4770E-03	3.57	1.7850E-04	4.33	7.4550E-06	5.09	1.7900E-07	5.85	2.4580E-09
0.54	2.9460E-01	1.30	9.6800E-02	2.06	1.9700E-02	2.82	2.4010E-03	3.58	1.7180E-04	4.34	7.1240E-06	5.10	1.6980E-07	5.86	2.3140E-09
0.55	2.9120E-01	1.31	9.5100E-02	2.07	1.9230E-02	2.83	2.3270E-03	3.59	1.6530E-04	4.35	6.8070E-06	5.11	1.6110E-07	5.87	2.1790E-09
0.56	2.8770E-01	1.32	9.3420E-02	2.08	1.8760E-02	2.84	2.2560E-03	3.60	1.5910E-04	4.36	6.5030E-06	5.12	1.5280E-07	5.88	2.0510E-09
0.57	2.8430E-01	1.33	9.1760E-02	2.09	1.8310E-02	2.85	2.1860E-03	3.61	1.5310E-04	4.37	6.2120E-06	5.13	1.4490E-07	5.89	1.9310E-09
0.58	2.8100E-01	1.34	9.0120E-02	2.10	1.7860E-02	2.86	2.1180E-03	3.62	1.4730E-04	4.38	5.9340E-06	5.14	1.3740E-07	5.90	1.8180E-09
0.59	2.7760E-01	1.35	8.8510E-02	2.11	1.7430E-02	2.87	2.0520E-03	3.63	1.4170E-04	4.39	5.6680E-06	5.15	1.3020E-07	5.91	1.7110E-09
0.60	2.7430E-01	1.36	8.6910E-02	2.12	1.7000E-02	2.88	1.9880E-03	3.64	1.3630E-04	4.40	5.4130E-06	5.16	1.2350E-07	5.92	1.6100E-09
0.61	2.7090E-01	1.37	8.5340E-02	2.13	1.6590E-02	2.89	1.9260E-03	3.65	1.3110E-04	4.41	5.1690E-06	5.17	1.1700E-07	5.93	1.5150E-09
0.62	2.6760E-01	1.38	8.3790E-02	2.14	1.6180E-02	2.90	1.8660E-03	3.66	1.2610E-04	4.42	4.9350E-06	5.18	1.1090E-07	5.94	1.4250E-09
0.63	2.6430E-01	1.39	8.2260E-02	2.15	1.5780E-02	2.91	1.8070E-03	3.67	1.2130E-04	4.43	4.7120E-06	5.19	1.0510E-07	5.95	1.3410E-09
0.64	2.6110E-01	1.40	8.0760E-02	2.16	1.5390E-02	2.92	1.7500E-03	3.68	1.1660E-04	4.44	4.4980E-06	5.20	9.9640E-08	5.96	1.2610E-09
0.65	2.5780E-01	1.41	7.9270E-02	2.17	1.5000E-02	2.93	1.6950E-03	3.69	1.1210E-04	4.45	4.2940E-06	5.21	9.4420E-08	5.97	1.1860E-09
0.66	2.5460E-01	1.42	7.7800E-02	2.18	1.4630E-02	2.94	1.6410E-03	3.70	1.0780E-04	4.46	4.0980E-06	5.22	8.9460E-08	5.98	1.1160E-09
0.67	2.5140E-01	1.43	7.6360E-02	2.19	1.4260E-02	2.95	1.5890E-03	3.71	1.0360E-04	4.47	3.9110E-06	5.23	8.4760E-08	5.99	1.0490E-09
0.68	2.4830E-01	1.44	7.4930E-02	2.20	1.3900E-02	2.96	1.5380E-03	3.72	9.9610E-05	4.48	3.7320E-06	5.24	8.0290E-08	6.00	9.8660E-10
0.69	2.4510E-01	1.45	7.3530E-02	2.21	1.3550E-02	2.97	1.4890E-03	3.73	9.5740E-05	4.49	3.5610E-06	5.25	7.6050E-08	6.01	9.2760E-10
0.70	2.4200E-01	1.46	7.2150E-02	2.22	1.3210E-02	2.98	1.4410E-03	3.74	9.2010E-05	4.50	3.3980E-06	5.26	7.2030E-08	6.02	8.7210E-10
0.71	2.3890E-01	1.47	7.0780E-02	2.23	1.2870E-02	2.99	1.3950E-03	3.75	8.8420E-05	4.51	3.2410E-06	5.27	6.8210E-08	6.03	8.1980E-10
0.72	2.3580E-01	1.48	6.9440E-02	2.24	1.2550E-02	3.00	1.3500E-03	3.76	8.4960E-05	4.52	3.0920E-06	5.28	6.4590E-08	6.04	7.7060E-10
0.73	2.3270E-01	1.49	6.8110E-02	2.25	1.2220E-02	3.01	1.3060E-03	3.77	8.1620E-05	4.53	2.9490E-06	5.29	6.1160E-08	6.05	7.2420E-10
0.74	2.2960E-01	1.50	6.6810E-02	2.26	1.1910E-02	3.02	1.2640E-03	3.78	7.8410E-05	4.54	2.8130E-06	5.30	5.7900E-08	6.06	6.8060E-10
0.75	2.2660E-01	1.51	6.5520E-02	2.27	1.1600E-02	3.03	1.2230E-03	3.79	7.5320E-05	4.55	2.6820E-06	5.31	5.4810E-08	6.07	6.3960E-10

1. Consider a simplified cellular system where a Base Station (BTS) transmits 100 byte frames to mobile users equipment (UE)s with a pulse amplitude modulation (PAM) over a bandwidth of $B = 200$ kHz. BTS can transmit the frames using either 2−PAM or 4−PAM with constellations $\mathcal{A}_{2PAM} = \{\pm c\}$, $\mathcal{A}_{4PAM} = \{\pm c, \pm 3c\}$. The symbol interval is fixed to $T_\mathrm{s} = 3\ \mu$s. We assume that the transmitted signal experience some attenuation that is proportional to the distance between BTS and UE and the average received power at a UE is P_r. UEs receivers add AWGN noise with power spectral density $N_0/2$, where $N_0 = 1.1 \times 10^{-20}$ W/Hz. To take a decision on whether 2-PAM or 4−PAM should be used to transmit the frame, the BTS send a training signal to UEs. Depending on the average received power P_r, a UE takes a decision whether to request a 2-PAM or 4−PAM from the BTS. The objective of the UE is to maximize the data rate satisfying at least a frame error rate of $P_f = 1\%$.



   (a) Propose a band-limited pulse with $B = 200$ kHz that satisfy ISI-free transmission at a symbol rate of $T_s = 3\ \mu$s. (1p)

   (b) What is the data rate experienced by a UE when BTS uses 2−PAM or 4−PAM ? (1p)

   (c) Express the frame error probability P_f as a function of symbol error probability P_e of M-PAM. (assume that symbol errors occur independently). (2p)

   (d) What is the minimum received power P_r at a UE, to achieve $P_f = 1\%$ when 2-PAM is used? (3p)

   (e) Repeat part (d) for 4-PAM. (3p)

   (f) Deduce for what intervals of P_r the UE should request 2-PAM or 4-PAM from the BTS such that (i) the frame error target is met and (ii) the data rate is maximized. (2p)

Hint:
- the symbol error probability for M-ary PAM is

$$P_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6}{M^2-1}\frac{E_s}{N_0}}\right)$$

where E_s is the symbol energy.
-The power can be related to the symbol energy as $P = E_s/T_s$.

Solution Problem 1 0.5p

(a) We can use a raised cosine pulse with a roll-off factor α that achieves $W = B = 200$ =kHz. We have that

$$W = \frac{1+\alpha}{2T_S} \implies \alpha = 2WT_S - 1 = 0.2 \quad \boxed{0.5p}$$

(b) The symbol rate $T_s = 3$ μs. We can compute the bit rate for both modulations following

$$R_b = \frac{\log_2(M)}{T_s}$$

Then, 0.5p 0.5p

$$R_b^{(M=2)} = 1/3 \text{ Mbit/s}, \quad R_b^{(M=4)} = 2/3 \text{ Mbit/s}$$

(c) Let $n_f = 100 \times 8 = 800$ bits be the length of frame in bits. We have $n_f/\log_2(M)$ symbols in each frame. The probability that all symbols composing a frame are transmitted and received correctly is

$$P_c = (1 - P_e)^{n_f/\log_2(M)}$$

The probability of frame error can then be expressed as

$$P_f = 1 - P_c = 1 - (1 - P_e)^{n_f/\log_2(M)}. \quad \boxed{2p}$$

(d) First, we can express P_e as a function of P_f 1p

$$P_e = 1 - (1 - P_f)^{\log_2(M)/n_f} = 1 - (1 - 0.01)^{1/800} = 1.2563 \times 10^{-5}$$

Then, since $P_\text{r} \times T_s = E_s$ we can write

$$P_e = Q\left(\sqrt{\frac{P_\text{r} T_s}{N_0/2}}\right)$$

1p

From the Q-function table we can get that $Q(4.22) = 1.222 \times 10^{-5}$. Then,

$$\frac{P_\text{r} T_s}{N_0/2} = \left[Q^{-1}(P_e)\right]^2 \implies$$

$$P_\text{r} = (4.22)^2 \times \frac{N_0/2}{T_s}$$

$$= 3.265 \times 10^{-14} \text{ w.} \quad \boxed{1p}$$

(e) Similarly, we can find the symbol error probability following

$$P_e = 1 - (1 - P_f)^{\log_2(M)/n_f} = 1 - (1 - 0.01)^{1/400} = 2.5126 \times 10^{-5} \quad \boxed{1p}$$

Then,

$$P_e' = \frac{3}{2} Q\left(\sqrt{\frac{1}{5}\frac{P_\text{r}' T_s}{N_0/2}}\right)$$

1p

From the Q-function table we get $Q(4.15) = 1.662 \times 10^{-5} \leq 2/3 P_e'$, then

$$\frac{1}{5}\frac{P_\text{r}' T_s}{N_0/2} = \left[Q^{-1}(2/3 P_e')\right]^2 \implies$$

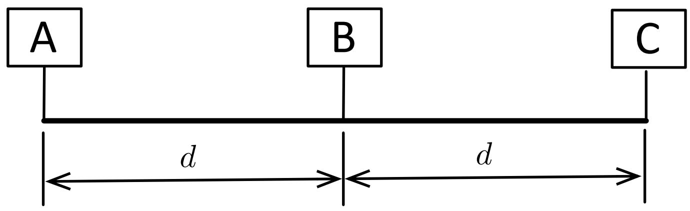
$$P_\text{r}' = (4.15)^2 \times 5 \times \frac{N_0/2}{T_s}$$

$$= 15.79 \times 10^{-14} \text{ w.} \quad \boxed{1p}$$

(f)

$\boxed{\textbf{2p}}$
$$\begin{cases} 4\ \text{PAM}: & 15.79 \times 10^{-14} \leq P_{\mathrm{r}} \\ 2\ \text{PAM}: & 3.265 \times 10^{-14} \leq P_{\mathrm{r}} < 15.79 \times 10^{-14} \end{cases}$$

2. Consider a LAN with three connected hosts A, B, and C. Suppose that $d = 400$ m and that the medium propagation speed is $c = 2c_0/3$, where $c_0 = 3 \times 10^8$ m/s is the speed of light in vacuum. The medium bit rate is $R = 10$ Gbit/s.



The link layer uses a Go-back-N ARQ protocol to provide a reliable, in-sequence, packet transmission service for the network layer. The ARQ protocol uses $m = 3$ bits sequence numbers . The link layer information frames are n_f bits long, including a 26 byte header and 32 CRC parity bits. An acknowledgement frame is 74 byte long. The medium introduces independent bit errors with probability p.

We assume that the error-detection is perfect (i.e., that no erroneous frames are accepted by the receiver at the link layer) and the receiver and transmitter processing times are negligible. The ARQ protocol ignores any information frames with detected errors. The transmission and reception of acknowledgement frames is assumed to be error free, i.e., bit errors affect information frames only.

Hosts use Aloha to share the channel. For simplicity, we will assume that only host B has data to send. Hence, there will be no collisions on the medium.

(a) What is the maximum send window size that we can use? Assuming a send window of the maximum size, compute the frame length n_f and the timeout values such that the bit rate experienced by the network layer is 90% of R when $p = 0$. (2p)

(b) Assume $p = 10^{-6}$. What is the experienced bit rate for the network layer for the frame length, timeout, and the send window values calculated in part (a)? (4p)

(c) Due to a fiber malfunctioning, the bit error probability p of the medium deteriorates drastically to $p = 10^{-5}$. What is now the bit rate experienced by the network layer? (2p)

(d) Assume we need to find a temporary solution to improve the bit rate experienced by the network layer until the malfunctioning fiber is found and repaired. Using the frame length and timeout values calculated in part (a), compute the value of the send window dimension that maximizes the bit rate experienced by the network layer when $p = 10^{-5}$. Motivate your answer. What can you conclude from this latest result? (4p)

Solution Problem 2

(a) Since the sequence numbers have 3 bits, then the maximum window size of Go-Back-N is $N \leq 2^3 - 1 = 7$. We have $n_o = 26 \times 8 + 32 = 240$ bits is the length of overhead bits, $n_a = 74 \times 8 = 592$ bits is the length of an ACK in bits. The turn-around time is given by $t_{out} = t_0 = t_f + t_a + 2\tau$, where $t_f = n_f/R$, $t_a = n_a/R$, are the frame and ACK transmission times (respectively), and $\tau = d/c = 2\mu$s is the propagation delay. The time-out should satisfy $t_{out} \geq t_0$, and to maximize efficiency we choose $t_{out} = t_0$. Assuming that the frame size is chosen such that we can send $N = 7$ frames during t_{out}, the effective rate of Go-Back-N in absence of frame errors is given by:

0.5p

1p

$$R_{eff} = \frac{N(n_f - n_o)}{t_{out}} = \frac{NR(n_f - n_o)}{n_f + n_a + 2\tau \times R} = 0.9R \implies$$
$$N(n_f - n_o) = 0.9(n_f + n_a + 2\tau R) \implies$$
$$n_f = \frac{0.9(n_a + 2\tau R) + Nn_0}{(N - 0.9)} = 6264.4\text{bits}$$

0.5p

We round this to the largest closest integer (we try to avoid idle time)

$$n_f = \lceil 6264.4 \rceil = 6265 \text{ bits.}$$

The efficiency is exactly equal to $\eta = 90.01\%$

Note that $N \times t_f < t_{out} = t_0$, this imply that the channel is idle for a time ($t_{idle} = Nt_f - t_{out} \approx 0.3\mu$s)

(b) First, given $n_f = 6265$ bits, $p = 10^{-6}$, the frame error rate is given by

Computing the frame error probability correctly for the frame length you chose gives 2p

$$P_f = 1 - (1-p)^{n_f} = 0.0062$$

2p

Second, we need to express the effective rate of Go-Back-N in presence of errors. Since the idle time for this choice of frame length and window size is negligibly small compared to t_0 ($t_{idle} \approx 1/16t_0$), we can assume that the channel keep transmitting frames continuously and thus the formula discussed in class approximate well the true effective rate.

1p

If you chose the correct frame length, motivation and explanation need to be included in order to use the formula derived in class

$$R_{eff,app} = R\left(1 - \frac{n_0}{n_f}\right)\frac{1 - P_f}{P_f(N-1) + 1} \tag{1}$$

(Note that this formula holds when the channel experiences no idle time, that is $N = \lceil t_0/t_f \rceil$)

Otherwise, if the idle time is not trascurable (i.e., $Nt_f < t_0 = t_{out}$), the exact efficiency can be derived as follows. Let Frame 0 be a reference frame. Let n_t be the number of transmissions needed to delivered correctly a frame.

$$\mathbb{E}\{n_t\} = \sum_{m=1}^{\infty}(1 - P_f)P_f^{m-1} = \frac{1}{1 - P_f}$$

Assuming that Frame 0 is received correctly after $n_t - 1$ failed transmission attemps, we can compute the expected time to deliver Frame 0 as:

$$\begin{aligned}
\mathbb{E}\{t_{GBN}\} &= t_0/N + \left(\frac{1}{1-P_f} - 1\right)t_0 \\
&= t_0/N\left[1 + \frac{NP_f}{1 - P_f}\right] \\
&= t_0/N\left[\frac{1 + P_f(N-1)}{1 - P_f}\right]
\end{aligned}$$

Note when Frame 0 has an error, we Go Back N frames, and retransmit all the outstanding frames. When Frame 0 is delivered correctly and since the channel was used to transmit $N-1$ other frames (total of N frames within $t_{out} = t_0$) while waiting for an ACK, we account that the time spent on frame 0 transmission is t_0/N. Thus, the exact effective rate, when the idle time is not trascurable, can be formulated as:

$$R_{eff,exact} = \frac{n_f - n_0}{\mathbb{E}\{t_{GBN}\}} = n_f\left(1 - \frac{n_o}{n_f}\right)\frac{1-P_f}{P_f(N-1)+1}\frac{N}{t_0} \tag{2}$$

Note that when $Nt_f = t_0 \implies R_{eff,exact} = R_{eff,app}$. Additionally when $N = 1$, the equation yields the efficiency of Stop and Wait Protocol:

$$R_{eff,SW} = \frac{n_f - n_0}{t_0}(1 - P_f) \tag{3}$$

Now we can readily compute the effective rate using both formulae.

This is assigned only if the correct frame length was used

1p

$$R_{eff,app} = 9.2117\text{Gbit/s}$$

$$R_{eff,exact} = 8.6215\text{Gbit/s}$$

(c) We compute the frame error probability corresponding to $p = 10^{-5}$,

1p

$$P_f = 1 - (1-p)^{n_f} = 0.0607$$

The effective rate is give by

1p

$$R_{eff,app} = 6.620\text{Gbit/s}$$

$$R_{eff,exact} = 6.196\text{Gbit/s}$$

(d) To answer this question we need to use the exact formula.

Deriving the exact formula 2p

Derivative 1p

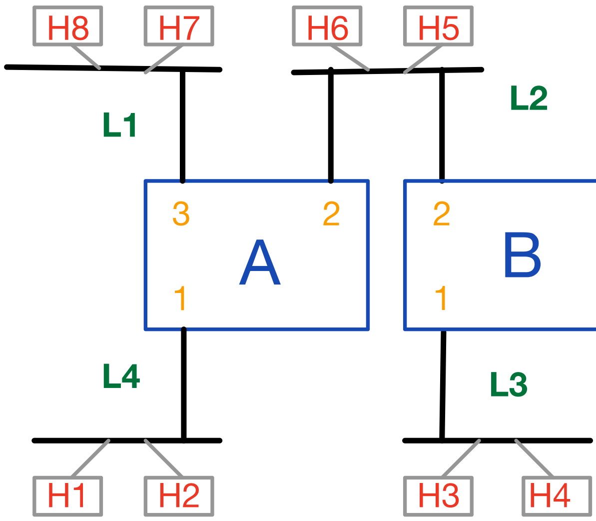
$$\begin{aligned}\frac{\mathrm{d}R_{eff,exact}}{\mathrm{d}N} &= C\frac{N}{1+P_f(N-1)}, \qquad C = n_f\left(1-\frac{n_o}{n_f}\right)(1-P_f)/t_0 > 0\\ &= C\frac{(1+P_f(N-1)-NP_f)}{(1+P_f(N-1))^2}\\ &= C\frac{1-P_f}{(1+P_f(N-1))^2} > 0\end{aligned}$$

Conclusion 1p

This implies that $R_{eff,exact}$ is increasing with N and thus $N = 7$ results in best efficiency. This example highlights how crucial it is, once the send window dimension of a Go Back N ARQ protocol is decided (i.e., under the assumption that n_f and t_0 are fixed), to make sure that the amount of idle time is always kept to a minimum. A Stop and Wait protocol might help improve the effective rate in the presence of a "bad" medium, but before moving to an SW ARQ to improve the value of the effective rate n_f should be recalculated, e.g., the same intuition we used in part (a).

OBS: Analyzing this using the approximation formula and a frame length that does not satisfy the assumptions yields maximum 1p/4p if the analysis is conducted correctly.

3. Consider a network where fours LANs (i.e., L1, L2, L3, and L4) are connected via two LAN transparent bridges. Each LAN comprises of two hosts connected and sharing the same medium, as depicted in the figure. All LANs support 100 Mbit/s transmission. The propagation speed in the medium is $c = 2c_0/3$, where $c_0 = 3 \times 10^8$ m/s. The bridges operate in a store-and-forward mode. More specifically, an incoming frame is completely received before it is forwarded to the corresponding output port. We assume that the processing time in the bridges and hosts is negligible. The numbers inside each bridge box close to where the transmission cable is attached represent the LAN bridge port number.



(a) The forwarding tables of the LAN bridges are defined as follows:

Host address	**Port ID**
a	x
b	y
$\vdots$	$\vdots$

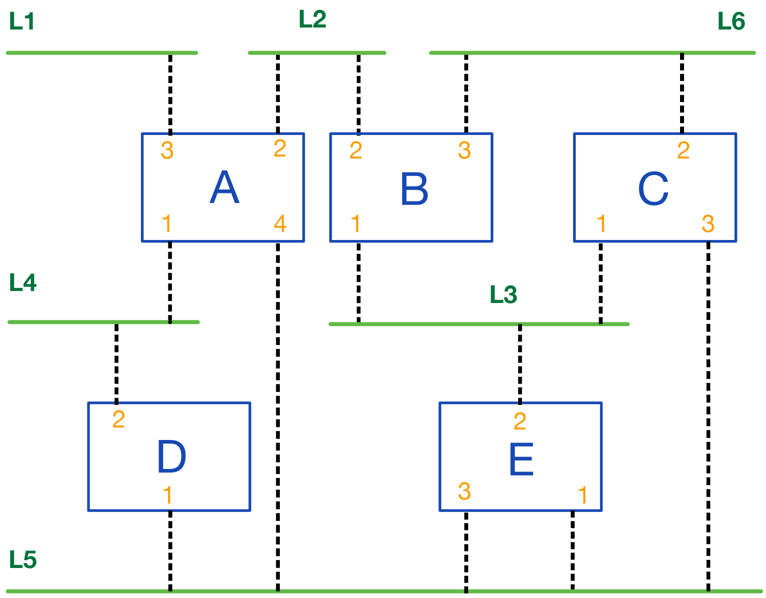
In other words, whenever a LAN bridge receives a frames for a host with address a it sends it to port x, etc. Suppose the LAN bridges are powered up at time $t = 0$, which implies that the forwarding tables are empty. Explain in details how the forwarding tables in bridge A and B change for each of the following transmissions.

Time t [s]	**Source Host**	**Destination Host**
1	H1	H7
2	H3	H5
3	H4	H3
4	H2	H4
5	H8	H1

Assume that backwards learning is used. (4p)

(b) Assume now a different scenario where data traffic is generated between H2 and H3 only. The distance between a host and the bridge port to which it is connected to is 50 m. Assume that the medium access control method chosen is CSMA-CD. Compute the minimum frame length n_f in bytes that should be used for transmission at the MAC layer of H2 and H3. Assume now that LAN bridges A and B are replaced by two network hubs (i.e., repeaters). Compute again the minimum frame length n_f in byes that should be used for transmission at the MAC layer of H2 and H3. What can you conclude from this latest results? (4p)

(c) Assume that after some time, the network grows in size, and 3 additional bridges (C, D, and E) are added to connect 2 new LANs (L5 and L6) as in the figure below.



Using the Spanning Tree algorithm, identify:

- the root bridge
- the root ports in the network
- for each LAN, the designated bridge and its associated designated port

While computing the spanning tree we assume the following. Ties should be resolved using the bridge ID or the port ID. More specifically, bridges or ports with the lowest ID value are always chosen first. The cost of a path to the root bridge is equal to the sum of the costs of the LANs traversed. More specifically assume the following cost values:

- Cost of traversing L1 is C1 = 2
- Cost of traversing L2 is C2 = 5
- Cost of traversing L3 is C3 = 1
- Cost of traversing L4 is C4 = 3
- Cost of traversing L5 is C5 = 1
- Cost of traversing L6 is C6 = 4

Draw the spanning tree as a result of Spanning Tree algorithm just computed. (2p)

(d) How would you modify the value of C5 defined in part (c) such that in the resulting spanning tree L5 is connected only to the root bridge and not to any other bridge in the network? Motivates your answer (2p)

Solution Problem 3

If only routing tables are included without explanation and mistakes occur at t=3,t=5, only 1p is awarded

(a) At $t = 1$, H1 sends a frame to H7. Frame is received by Bridge A on port 1. The forwarding table of Bridge A (FWD-A) is empty, so H1 is saved together with the ID of the port the frame came from (i.e., 1). FWD-A does not contain any information about about H7 either, so the frame is sent out to port 2 and 3 of the bridge. Bridge B receives the frame at port 2, the forwarding table of Bridge B (FWD-B) is empty, so it saves the information about H1 being reachable on port 2. The frame is also sent out to port 1. As a result FWD-A and FWD-B are:

1p

FWD-A	
Host address	**Port ID**
$H1$	1

FWD-B	
Host address	**Port ID**
$H1$	2

For t=1,2,4 (Flooding)

At $t = 2$, H3 sends a frame to H5. Frame is received by Bridge B on port 1. No entry in FWD-B about H3. H3 info saved together with port 1 ID. FWD-B has no information about H5, so the frame is sent out to port 2. Bridge A receives the frame at port 2. No entry in FWD-A about H3. H3 info saved together with port 2 ID. The frame is sent out to port 1 and 3. As a result FWD-A and FWD-B are:

FWD-A	
Host address	**Port ID**
$H1$	1
$H3$	2

FWD-B	
Host address	**Port ID**
$H1$	2
$H3$	1

At $t = 3$, H4 sends a frame to H3. Frame is received by Bridge B on port 1. No entry in FWD-B about H4. H4 info saved together with port 1 ID. FWD-B has information about H3. Knowing that H3 is reachable via port 1, the frame is not sent out to port 2. As a result FWD-A and FWD-B are:

1.5p

FWD-A	
Host address	**Port ID**
$H1$	1
$H3$	2

FWD-B	
Host address	**Port ID**
$H1$	2
$H3$	1
$H4$	1

For t=3 Backward learning

At $t = 4$, H2 sends a frame to H4. Frame is received by Bridge A on port 1. No entry in FWD-A about H2. H2 info saved together with port 1 ID. FWD-A has no information about H4, so the frame is sent out to port 2 and 3. Bridge B receives the frame at port 2. No entry in FWD-B about H2. H2 info saved together with port 2 ID. FWD-B has information about H4, so the frame is sent out to port 1. As a result FWD-A and FWD-B are:

FWD-A	
Host address	**Port ID**
$H1$	1
$H3$	2
$H2$	1

FWD-B	
Host address	**Port ID**
$H1$	2
$H3$	1
$H4$	1
$H2$	2

At $t = 5$, H8 sends a frame to H1. Frame is received by Bridge A on port 3. No entry in FWD-A about H8. H8 info saved together with port 3 ID. FWD-A has information about H1, the frame is sent out to port 1 and nowhere else. As a result FWD-A and FWD-B are:

1.5p

For t=5
Backward
learning

FWD-A		FWD-B	
Host address	**Port ID**	**Host address**	**Port ID**
$H1$	1	$H1$	2
$H3$	2	$H3$	1
$H2$	1	$H4$	1
$H8$	3	$H2$	1

(b) Transparent bridges (unlike hubs) have the property to separate the collision domains of the LANs connected to their ports. For this reason L4 belongs to a collision domain that is separate from L3. The propagation distance between H2 and port 1 of Bridge A and between H3 and port 1 of Bridge B is 50 m. This means that:

$$t_{prop} = 50/c = 0.25\mu s$$

2p

In order for CSMA-CD to be effective we know that $t_f \geq 2 \cdot t_{prop}$. Knowing that the medium has a transmission rate equal to R, so we can derive the vaue of n_f in byte as:

$$n_f \geq \lceil (2 \cdot t_{prop} \cdot R)/8 \rceil \Rightarrow n_f \geq 7 \text{ bytes}$$

Note that the communication between port 2 of Bridge A and port 2 of Bridge B belongs to yet another and separate collision domain. In that case the propagation delay is different and the value of n_f used there will also be different (but is outside the scope of the question in the exam).

In case hubs are used the propagation distance between H2 and H3 is 200 m. This means that:

$$t_{prop} = 200/c = 1\mu s$$

2p

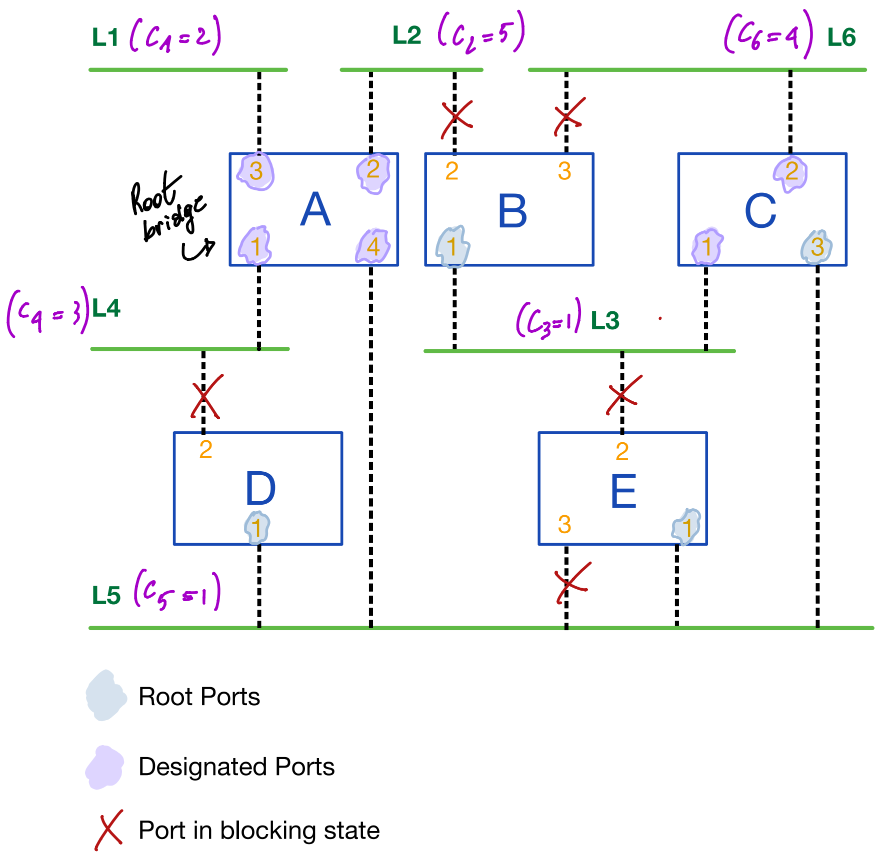
Using the same procedure as above we can derive that

$$n_f \geq \lceil (2 \cdot t_{prop} \cdot R)/8 \rceil \Rightarrow n_f \geq 25 \text{ bytes}$$

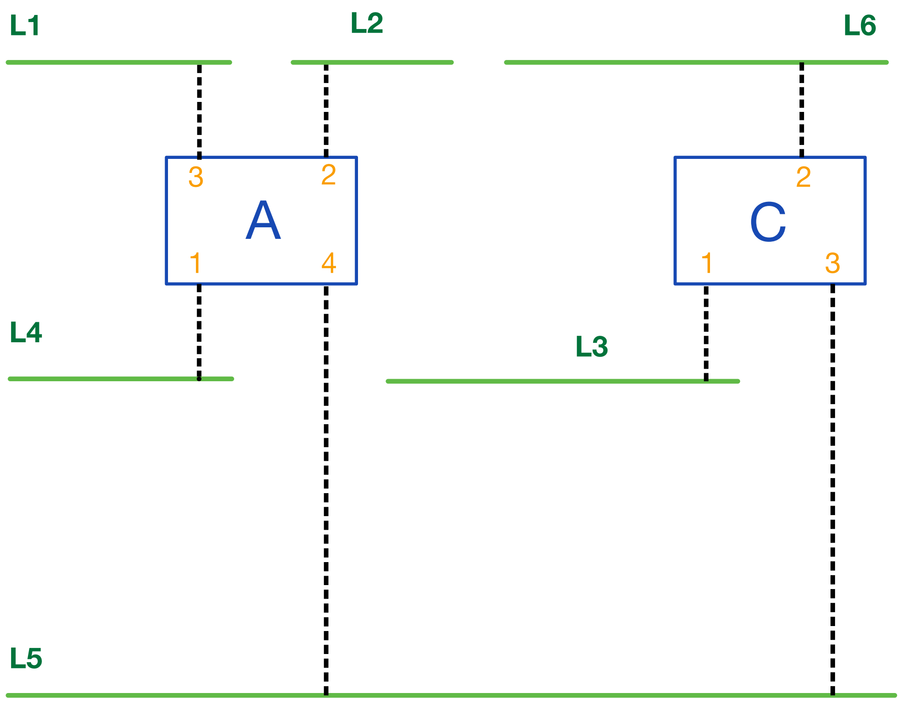
Results are consistent with the notion that with longer propagation delays we need to have a bigger value for the minimum frame length.

(c) Applying the Spanning Tree algorithm we obtain the result in the figure below

2p



While the spanning tree can be depicted as:

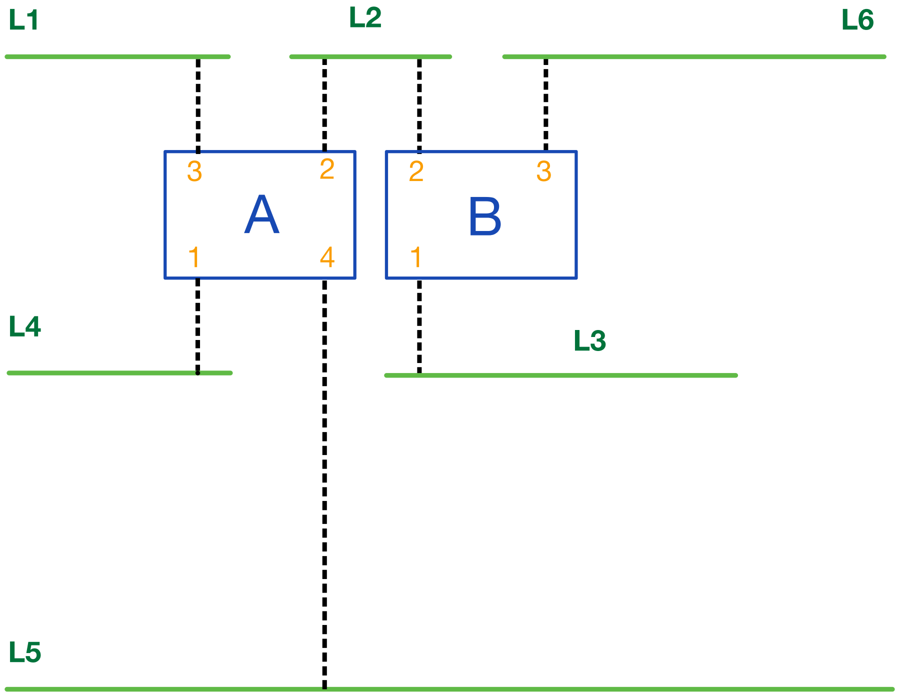


(d) If we would like to have a spanning tree solution where L5 is connected to the root bridge only, we need to enforce the following constraint

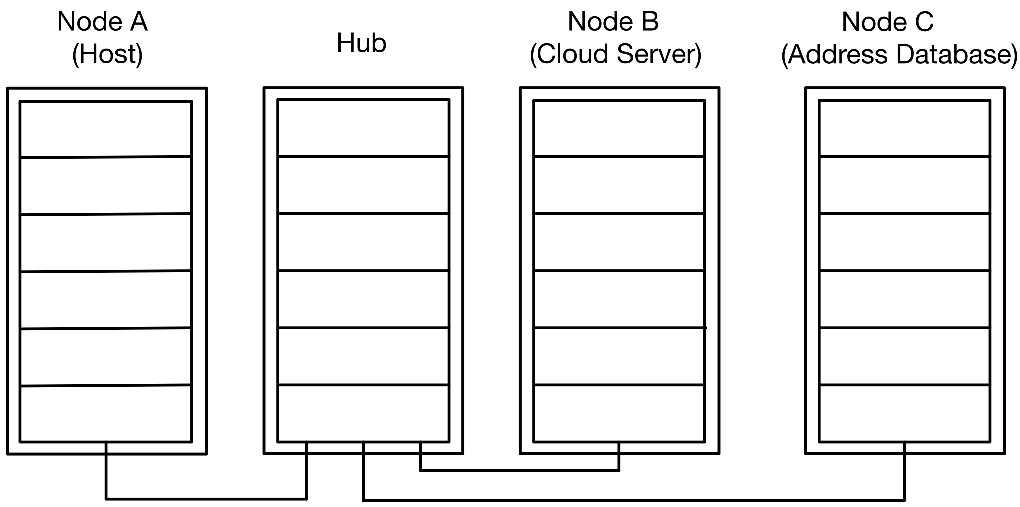
2p $$C_5 \geq C2 + C3 \Rightarrow C_5 \geq 6$$

Reason: L1, L4, and L2 are connected directly to the root bridge, they are not impacted by the cost of L5. On the other hand, we need to ensure that the L2 and

L3 become more appealing options for L6 to connect to the root bridge, compare to L5. The result is the following:



4. Consider the network in the figure. One Host runs at Node A. The host needs to retrieve a file stored in a Cloud Server running at Node B. Node C runs an Address Database with a mapping of internet names into IP addresses. The three nodes are connected via a Hub (i.e., repeater).



   (a) Let the physical (i.e., MAC) addresses in the network be denoted as 30:23:03:00:00:yy, where yy is a member of the set $\{00, 01, ..., FF\}$ (all two-digit hex numbers). Assign physical addresses to the appropriate places in the figure. Use physical addresses in order, i.e., start with assigning 30:23:03:00:00:00, then 30:23:03:00:00:01 and so on.

   Let the IP addresses in the network be denoted as 192.168.1.xxx, where xxx is a member of the set $\{0, 1, ..., 255\}$. Assign IP addresses to the appropriate places in the figure. Use IP addresses in order, i.e., start with assigning 192.168.1.0, then 192.168.1.1 and so on. (1p)

Assume the Host was just powered up and that the only information known by the Host after booting is its own MAC address, the internet name of the could server (i.e., "www.mystuff.com"), and the IP address of Node C.

To retrieve the file stored at Node B, the Host needs to query the Adddress Database at Node C to resolve the internet name of the Cloud Server. On the other hand, the Host needs first to discover the MAC address of Node C to initiate the query operation.

   (b) Name the protocol that the Host should use to discover Node C's MAC address from its IP address. Assume that the other protocols involved in the MAC address discovery operation are (in alphabetical order) 802.3 MAC, 802.3 PHY, LLC. Copy the figure above. For each network element (Node A, Hub, Node B, and Node C) list (in the protocol stack's correct order) which protocols are actively involved in receiving and/or processing the frames used for the MAC address discovery operation. Note that this list should include the protocol used to discover Node C's MAC address and that some boxes in the protocol stacks might be empty. (2p)

   (c) Let PH denote the header and PT denote the trailer (if it exists) for protocol P. For example, the LLC header is denoted as LLCH, and 802.3 trailer is denoted as 802.3T. Moreover, let the service data unit of protocol P be denoted as P-SDU. Indicate the order of the headers, trailer, and service data unit of a frame from Node A to the the Hub used for the MAC discovery operation. Do the same for a frame from the Hub to Node B, and from the Hub to Node C. (2p)

(d) What destination and source MAC addresses are used in the frame from Node A to the Hub? (1p)

Now that Node C's MAC address is known, the Host can query the Address Database to resolve the internet name "www.mystuff.com".

(e) Name the protocol that should be used to query the Address Database at Node C to resolve the internet name "www.mystuff.com". Assume that the other protocols needed for this operation are (in alphabetical order) 802.3 MAC, 802.3 PHY, IP, LLC, UDP. Copy the figure above. For each network element (Node A, Hub, Node B, and Node C) list (in the protocol stack's correct order) which protocols are actively involved in receiving and/or processing the frames used for resolving the "www.mystuff.com" internet name. Note that your list should include also the protocol used to query the Address Database at Node C and that some boxes in the protocol stacks might be empty. (2p)

(f) Using the notation explained in part (c) indicate the order of the headers, trailer, and service data unit of a frame from Node A to the Hub for the "www.mystuff.com" internet name resolution operation. Do the same for the frame going from the Hub to Node B and from the Hub to Node C. (2p)

(g) What destination and source MAC addresses are used in the frame originating at Node A for the "www.mystuff.com" internet name resolution operation? (1p)

Now that all the necessary information from the Address Database has been received, the Host at Node A can retrieve the file saved in the Cloud Server.

(h) Name a protocol that can be used to retrieve the file from "www.mystuff.com". What source and destination IP addresses are used in the frame originating at Node A for the file retrieving operation? (1p)

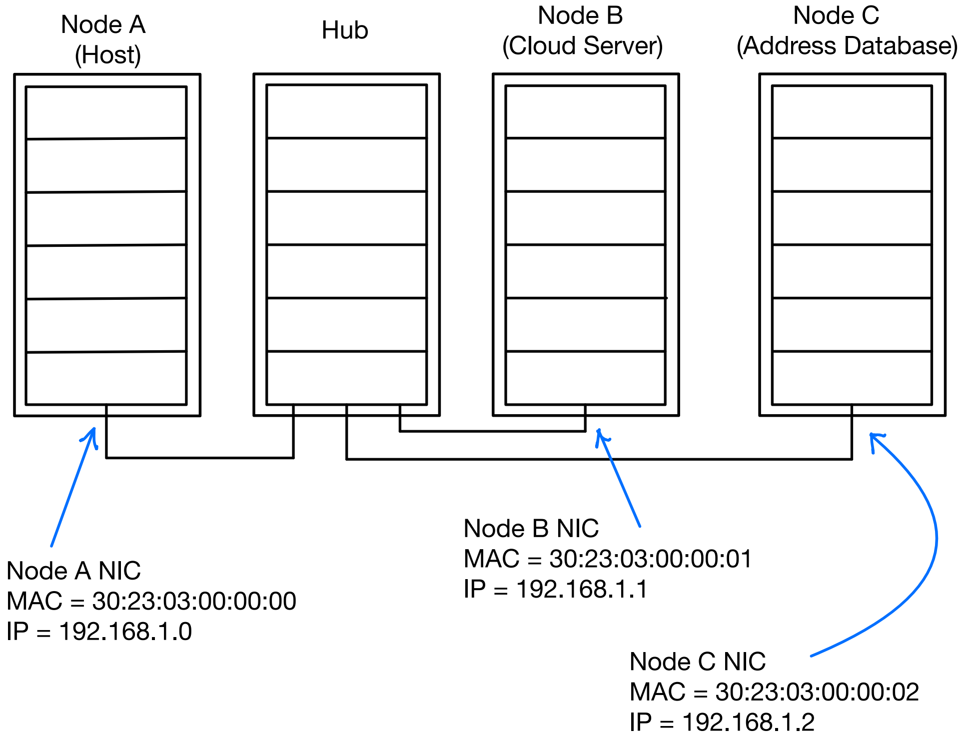
For correct indication of active protocols at node B for DNS querry processing

Solution Problem 4

(a) The hub is a PHY device able to only receive, regenerate, and forward the frame received at one of its ports. It does not have any MAC address. The MAC and IP addresses of Node A, B, and C are reported in the figure below

1p

Assigning addresses to the hub reduces the grade to 0.5/1p



(b) The protocol used to discovery the Node C's MAC address is ARP. 0.5p

(c) See next figure

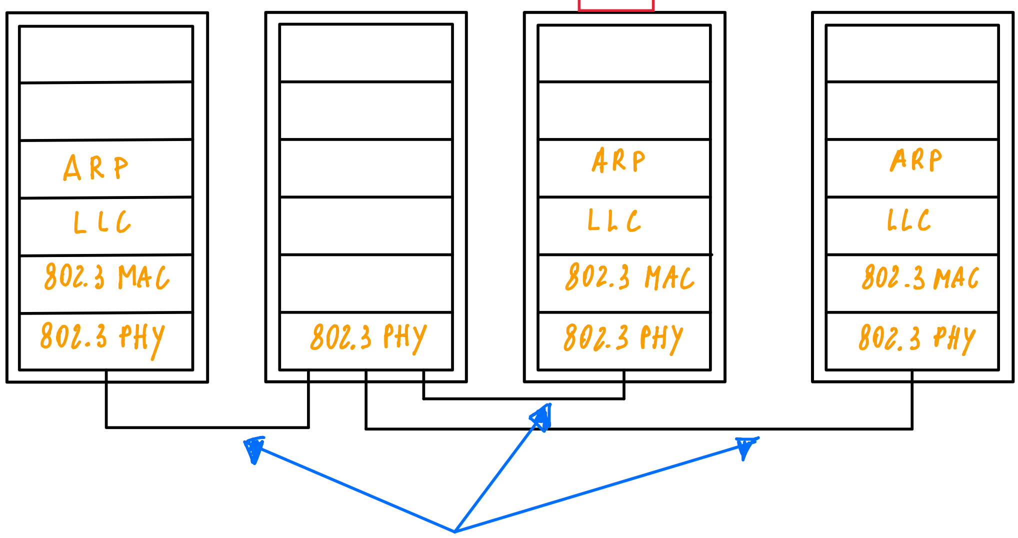
(d) See next figure

For correct indication that ARP protocol at node B is also active for ARP querry processing

0.5p

1p

For correct ordering of protocol stack



| 802.3H | LLCH | ARPH | ARP-SDU | 802.3T | 2p

MAC source address = 30:23:03:00:00:00
MAC destination address = FF:FF:FF:FF:FF:FF 1p

(e) The protocol used to query the Address Database at Node C to resolve the internet name www.mystuff.com is DNS.

For correct indication of active protocols at node B for DNS querry processing

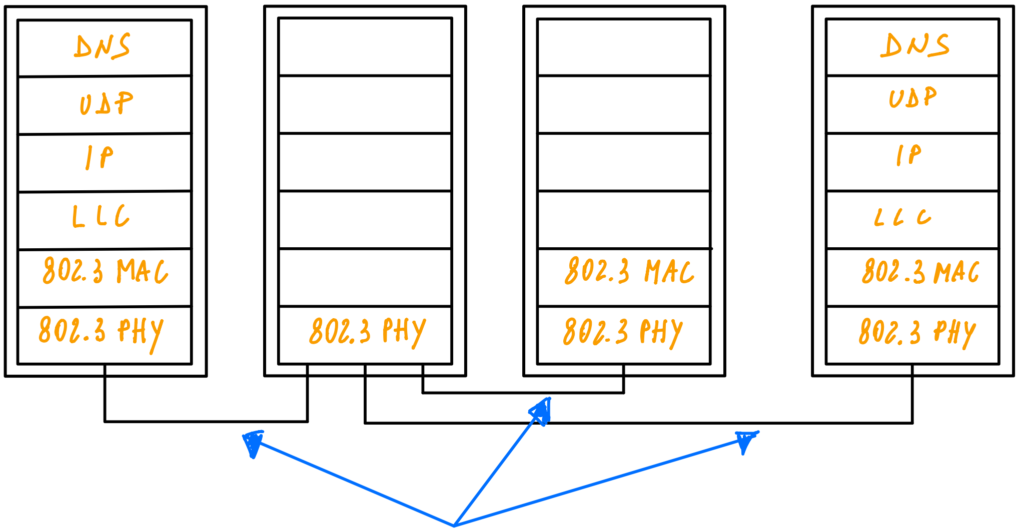
(f) See next figure

(g) See next figure

0.5p

1p

For correct ordering of protocol stack



| 802.3H | LLCH | IPH | UDPH | DNSH | DNS-SDU | 802.3T |

MAC source address = 30:23:03:00:00:00
MAC destination address = 30:23:03:00:00:02 1p

(h) A protocol that can be used to retrieve the file from "www.mystuff.com" is HTTP. 0.5p
The frame originating at Node A for the file retrieving operation has source address 192.168.1.0 and destination address 192.168.1.1
0.5p