# Exam in SSY305 Kommunikationssystem 

Department of Electrical Engineering<br>Exam date: June 12, 2019, 14:00-18:00<br>Document updated: June 11, 2019

## Teaching Staff

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Material Allowed material is

- Chalmers-approved calculator
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A paper-based dictionary, without added notes (electronic dictionaries are not allowed).

Grading A correct, clear and well-motivated solution gives a maximum of 12 points per problem.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. No fractional points are awarded.
Answers in any other language than Swedish or English are ignored.
Solutions Are made available at the earliest at 19:00 on the course web page.
Results Exam results are posted on Ping-Pong no later than on June 19, 2019. The grading review is on June 20 and June 26, 2019, 13:00-14:00 in room 6436 in the ED-building (Chouaib's office).

Grades The final grade on the course will be decided by the projects (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). Project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade according to the following table.

| Total Score | $0-39$ | $40-69$ | $70-79$ | $\geq 80$ |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Fail | 3 | 4 | 5 |

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.

## Table over the Q-function

|  | Q(x) |  | Q(x) |  | Q(x) |  | $Q(x)$ |  | Q(x) |  | Q(x) |  | Q(x) |  | $Q(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | $5.0000 \mathrm{E}-01$ | 0.76 | $2.2360 \mathrm{E}-01$ | 1.52 | $6.4260 \mathrm{E}-02$ | 2.28 | $1.1300 \mathrm{E}-02$ | 3.04 | $1.1830 \mathrm{E}-03$ | 3.80 | 7.2350E-05 | 4.56 | $2.5580 \mathrm{E}-06$ | 5.32 | 5.1880 E |
| 0.01 | $4.9600 \mathrm{E}-01$ | 0.77 | $2.2060 \mathrm{E}-0$ | 1.5 | $6.3010 \mathrm{E}-02$ | 2.29 | 1.1 | 3.05 | $1.1440 \mathrm{E}-03$ | 3.81 | $6.9480 \mathrm{E}-05$ | 4.57 | $2.4390 \mathrm{E}-06$ | 3 |  |
| 0.02 | 4.9200 | 0. | $2.1770 \mathrm{E}-01$ |  | $6.1780 \mathrm{E}-02$ | 2. | $1.0720 \mathrm{E}-02$ | 3.0 | $1.1070 \mathrm{E}-03$ | 3.8 | 5 | 8 | 6 | 5.34 | $4.6470 \mathrm{E}-08$ |
| 0.03 | $4.8800 \mathrm{E}-01$ | 0.79 | $2.1480 \mathrm{E}-01$ | 1.55 | 6.0 | 2.3 | 1.0 | 3.07 | $1.0700 \mathrm{E}-03$ |  | 5 | 4.59 | $2.2160 \mathrm{E}-06$ | 5.35 | $4.3980 \mathrm{E}-08$ |
| 0.04 | $4.8400 \mathrm{E}-01$ | 0.8 | 2.1 | 1.5 | 5.938 | 2.3 | 1.01 | 3.08 | 1.03 | 3.8 | 6.1 | 4.60 | $2.1120 \mathrm{E}-06$ | 5.36 | $4.1610 \mathrm{E}-08$ |
| 0.05 | 4.8010 | 0.81 | 2.0900 | 1.57 | 5.82 | 2.33 | 9.9030 | 3.09 | 1.00 | 3.85 | 5.9060 | 4.61 | 30 | 5.3 | $3.9370 \mathrm{E}-08$ |
| . 06 | 4.761 | 0.82 | 2.061 | 1.58 | 5.7 | 2.34 | 9.6 | 3.10 | 9.6760E-04 | 3.86 | $5.6690 \mathrm{E}-05$ | 4.6 | 6 | 8 | $3.7240 \mathrm{E}-08$ |
| 0.07 | 4.72 | 0.83 | 2.0330E-01 | 1.59 | 5. | 2. | 9. | 3.11 | $9.3540 \mathrm{E}-04$ | 3.87 | 5.4 | 4.63 | 1.8280E-06 | 5.39 | $3.5230 \mathrm{E}-08$ |
| 0.08 | 4.6 | 0.84 | 2.0 | 1.60 | 5. | 2.3 | 9.1 | 3.12 | $9.0430 \mathrm{E}-04$ | 3.8 | $5.2230 \mathrm{E}-05$ | 4.64 | 1.7420E-06 | 5.40 | 3.3 |
| 0.09 | $4.6410 \mathrm{E}-01$ | 0.85 | 770 | 1.61 | $5.3700 \mathrm{E}-02$ | . 3 | 8.8940 | 3.1 | 7400E-04 | 3.89 | $5.0120 \mathrm{E}-05$ | 4.6 | 6600E-06 | 5.4 | 3.1510E-08 |
| 0.10 | 4.6020 | 0.86 | 1.9490E-01 | 1.62 | 5. | 2.3 | 8.6 | 3.14 |  | 3.90 | 5 | 6 | 6 | 5.42 | $2.9800 \mathrm{E}-08$ |
| 0.11 | 4.5620 | 0.87 | $1.9220 \mathrm{E}-01$ | 1.63 | 5. | 2.39 | 8. | 3.15 | 8.1640E-04 | 3.9 | $4.6150 \mathrm{E}-05$ | 4.67 | 1.5060E-06 | 5.43 | $2.8180 \mathrm{E}-08$ |
| 0.12 | 4. | 0.88 | 1.8940E-0 | 1.64 | 5. | 2.40 | 8. | 3.16 | 7.888 | 3.9 | 4. | 4.68 | $1.4340 \mathrm{E}-06$ | 5.44 | 2.6 |
| 0.13 | $4.4830 \mathrm{E}-01$ | 0.8 | 1.8670E-01 | 1.65 | 4.9470 | 2.41 | 7.9760 | 3.17 | 6220 | 3.93 | $4.2470 \mathrm{E}-05$ | 4.69 | 3660E-06 | 5.45 | $2.5180 \mathrm{E}-08$ |
|  | 4.44 | 0 |  | 1. | 4.8460E-02 | 2.42 | $7.7600 \mathrm{E}-03$ | 3. | $7.3640 \mathrm{E}-04$ | 3.94 | $4.0740 \mathrm{E}-05$ | 0 | 1.3010E-06 |  | $2.3810 \mathrm{E}-08$ |
| 0.15 | 4.40 | 0 | $1.8140 \mathrm{E}-01$ | 1.67 | 4. | 2.43 | 7. | 3. | $7.1140 \mathrm{E}-04$ | 3.9 | 3.9 | 4.71 | 1.2390E-06 | 5.47 | $2.2500 \mathrm{E}-08$ |
| 0.16 | 4.364 | 0.92 | $1.7880 \mathrm{E}-01$ | 1.68 | 4. | 2.44 | 7. | 3. | 6. | 3.96 | 3.7 | 4.72 | $1.1790 \mathrm{E}-06$ | 5.48 | $2.1270 \mathrm{E}-08$ |
| 0.17 | 4.3250 | 0.93 | 1.7620E-01 | 1.69 | $4.5510 \mathrm{E}-02$ | 2.4 | 7.1430E-03 | 3.21 | 63 | 3.97 | $3.5940 \mathrm{E}-05$ | 4.73 | 1230E-06 | 9 | $2.0100 \mathrm{E}-08$ |
| 0.18 | 4.286 | 0 | 1.7360E-01 |  | $4.4570 \mathrm{E}-02$ | 2.46 |  | 3. |  | 3.9 | $3.4460 \mathrm{E}-05$ | 4.74 | 6 | 5.50 |  |
| 0.19 | 4.247 | 0.95 | 1.7110E-01 | 1.71 | 4.363 | 2.47 | 6. | 3.23 | $6.1900 \mathrm{E}-04$ | 3.9 | 3.3 | 4.75 | $1.0170 \mathrm{E}-06$ | 5.51 | 1.7 |
| 0.20 | 4.2070 | 0. | 1.6850 | 1. | 4.2720 | 2.48 | 6.5690 | 3.24 | 5.9760 | 4.00 | $3.1670 \mathrm{E}-05$ | 4.76 | 9.6800 | 5.52 | 1.6950E-08 |
| 0.21 | 4.168 | 0 | 1.6600 | 1. | 4.1820E-02 | 2.4 | $6.3870 \mathrm{E}-03$ | 3.25 |  | 4.01 | 3.0 | 7 | $9.2110 \mathrm{E}-07$ | 5.53 | $1.6010 \mathrm{E}-08$ |
| 0.22 | 4.1290 | 0.9 | 1.6350E-01 | 1.7 | 4.0930E-02 | 2.50 |  | 3.2 |  | 4.02 | 2.9 | 4.78 | $8.7650 \mathrm{E}-07$ |  | $1.5120 \mathrm{E}-08$ |
| 0.23 | 4.0900 | 0.99 | 1.6110 | 1.75 | 4.0060E-02 | 2.51 | 6.0 | 3. | 5.3 | 4.03 | 2.78 | 4.79 | 8.33 | 5.55 | 1.4 |
| 0.24 | 4.0520 | 1.00 | 1.5 | 1.76 | 3.9200 | 2.52 | 5.868 | 3.2 | 5.19 | 4.04 | $2.6730 \mathrm{E}-05$ | 4.80 | 9330E | 5.56 | .3490E-08 |
| 0.25 | 4.0130 | 1.0 | 1.5620 | 1. | . 83 | 2.5 | 5.7030 | 3. | 5.0090E-04 | 5 | 2.5 | 4.8 | $7.5470 \mathrm{E}-07$ | 5.57 | $1.2740 \mathrm{E}-08$ |
| 0.26 | 3.97 |  | 1.5390E-01 | 1.7 | $3.7540 \mathrm{E}-02$ |  |  | 3. | $4.8340 \mathrm{E}-04$ | 4.06 | 2. | 4.82 | $7.1780 \mathrm{E}-07$ | 5.58 | $1.2030 \mathrm{E}-08$ |
| 0.27 | 3.9360 | 1.03 | 1.5150 | 1.79 | 3. | 2.5 | 5.3 | 3.3 | 665 | 4.07 | 2.3510 |  | 6.8270 | 5.59 | $1.1350 \mathrm{E}-08$ |
| 0.28 | 3.897 | 1.04 | 20 | 1.80 | 3. | 2.56 | $5.2340 \mathrm{E}-03$ | 3.32 | 501 | 4.08 | $2.2520 \mathrm{E}-05$ | 4.84 | $6.4920 \mathrm{E}-07$ | . 6 | 08 |
| 0.29 | 3.859 | 1.05 | 1.4690 | 1.8 | 3.5150 | 2.5 | 5.0850 | 3.33 | $4.3420 \mathrm{E}-04$ | 4.09 | 2.1 | 4.8 | $6.1730 \mathrm{E}-07$ | 5.61 | $1.0120 \mathrm{E}-08$ |
| 0.30 | 3.8 |  | $1.4460 \mathrm{E}-01$ | 1.8 | $3.4380 \mathrm{E}-02$ | 2.5 | $4.9400 \mathrm{E}-03$ | 3.34 | $4.1890 \mathrm{E}-04$ | 4.10 | 2.0 |  | $5.8690 \mathrm{E}-07$ | 5.62 | 9.5 |
| 0.31 | 3.783 | 1.07 | $1.4230 \mathrm{E}-01$ | 1.83 | 3.3620 | 2.59 | $4.7990 \mathrm{E}-03$ | 3. | 4. | 4.11 | $1.9780 \mathrm{E}-05$ |  | 5.5800 | 5.63 | 9.0100E-09 |
| 0.32 | 3.745 | 1. | 1.4010 | 1.8 | 3.2880E-02 | 2.6 | $4.6610 \mathrm{E}-03$ | 3.36 | 3.8970E-04 | 4.12 | $1.8940 \mathrm{E}-05$ | 4.88 | $5.3040 \mathrm{E}-07$ | 5.64 | .5030E-09 |
| 0.33 | 3.70 | 1. | 1.3790 | 1.8 | 3.2160 | 2.6 | 4.52 | 3.3 | $3.7580 \mathrm{E}-04$ | 4.13 | $1.8140 \mathrm{E}-05$ | 4.89 | 5.0420E-07 | 5.65 | $8.0220 \mathrm{E}-09$ |
|  | 3.6 |  |  |  |  | 2.62 |  | 3.38 |  |  |  | 4.90 | 7 |  |  |
| 0.35 | $3.6320 \mathrm{E}-01$ |  | 35 | 1.87 | 3. | 2.63 | $4.2690 \mathrm{E}-03$ | 3.39 | 3.4950 | 4.15 | 1.6620 | 4. | 5540E-07 | 5.67 | .1400E-09 |
| 0.36 | 3.5940 | 1. | 1.31 | 1. | . 005 | 2.6 | 4.1450 | 3.40 | 369 | 4.16 | 1.59 | 4.9 | 327 | 5.68 | .7350E-09 |
| 0.37 | 3.5570 |  | 1.2920 |  | 2.938 |  | 4.0250 |  | $3.2480 \mathrm{E}-04$ | 4.17 | 1.52 | 4. | $4.1110 \mathrm{E}-07$ | 5.69 | $6.3520 \mathrm{E}-09$ |
|  | 3.52 |  |  |  | 2. | 2.66 |  |  | 3.1 | - | 1.4 |  | $3.9060 \mathrm{E}-07$ | 5.7 | 5.9900E-09 |
| 0.39 | 3.4830 | 1.1 | $1.2510 \mathrm{E}-01$ | 1.91 | 2.8070 | 2.67 | $3.7930 \mathrm{E}-03$ | 3.43 | 3.0180E-04 | - | $1.3950 \mathrm{E}-05$ | 4.95 | $3.7110 \mathrm{E}-07$ | 5.71 | 09 |
| 0.40 | 3.4460 | 1.1 | 1.230 | 1.92 | $2.7430 \mathrm{E}-02$ | 2.6 | 3.6 |  | 2.90 | - | 1.3350 | 4. | 3.5250 | 5.72 | 5.32 |
|  | 3.4090 |  | 1.2100E-01 | 1.93 | $2.6800 \mathrm{E}-02$ | 2.6 |  |  | 2.8 |  | 1.2 |  | , |  | $5.0220 \mathrm{E}-09$ |
|  | 3.3 | 1.18 | $1.1900 \mathrm{E}-01$ |  | 2. | 2.70 | 3. | 3.46 | $2.7010 \mathrm{E}-04$ | 4.22 | 20E-05 | 4.98 | $3.1790 \mathrm{E}-07$ | 5.74 | 09 |
|  | 3.3360 | 1.19 | 1.1700 | 1.9 | 2.5590 E | 2.7 | 3.3640 | 3.47 | 2.6 | 4.23 | $1.1680 \mathrm{E}-05$ | - | 3.0190 | 5.75 | 09 |
|  | 3.3000 | 1. | 1.1510 | 1.9 | 2.500 | 2.72 | 3.2 | 3. | 2.50 | 4.24 | $1.1180 \mathrm{E}-05$ | 5.00 | 2.8670 | 5.76 | . 2 |
|  | 3.2 |  |  |  |  | 2.7 |  |  | 2. |  | $1.0690 \mathrm{E}-05$ |  | , | 5.77 | $3.9640 \mathrm{E}-09$ |
|  | 3.2 |  | 1.1120 | 1.98 | 2.3850 E |  | $3.0720 \mathrm{E}-03$ | 3.50 | 2. |  | 5 | 5.02 | $2.5840 \mathrm{E}-07$ | 5.78 | 09 |
|  | 3.1920 | 1.23 | 93 | 1.99 | . 33 | 2.7 | 2.98 | 3.51 | 2.241 | 4.27 | $9.7740 \mathrm{E}-06$ | 5.03 | 2.4520 | 5.79 | 3.5 |
|  | 3.1560 |  | 1.0750 E | 2.0 | 2.2750 E | 2.7 | 2.8 | 3.52 | 2.15 | 4.28 | $9.3450 \mathrm{E}-06$ | 5. | $2.3280 \mathrm{E}-07$ | 5.80 | 3. |
|  | 3.1210 | 1.25 | $1.0560 \mathrm{E}-01$ | 2. | 2. | 2. | 2.8 |  | 2.0 |  | 8. | 5. | $2.2090 \mathrm{E}-07$ | 5.81 | $3.1240 \mathrm{E}-09$ |
|  | 3.0850 | 1.26 | 1.0380E-01 | 2.02 | 2.1690 | 2.7 | $2.7180 \mathrm{E}-03$ |  | . 001 | 4.30 | $8.5400 \mathrm{E}-06$ | 5.06 | - | 5.82 | -09 |
|  | 3.0500 | 1. | 1.0200 | 2.03 | . 1 | 2.7 | 2.6350 | 3. | 1.9260E-04 | 4.31 | 8.16 | 5.07 | 9890 | 5.83 | 2.7 |
|  | 3.01 | 1.2 | 1.0030E-01 | 2.0 | 2. | 2.8 | 2. | 3.56 | $1.8540 \mathrm{E}-04$ | 4.32 | $7.8010 \mathrm{E}-06$ | 8 | 1.8870E-07 |  | $2.6100 \mathrm{E}-09$ |
| . | 2.9810 | 1.29 | 9.8530 | . 0 | 2.0180 | 2.8 | $2.4770 \mathrm{E}-03$ | 3.5 | 50 | 4.33 | 4550 | 5.0 | 7900E-07 | 5.8 | $2.4580 \mathrm{E}-09$ |
|  | 2.9460 | 1.30 | $9.6800 \mathrm{E}-02$ | 2.0 | 1.9700 | 2.8 | . 40 | 3.5 | 1.7180E-04 | 4.3 | 7.1240 | 5. | 6980 | 5.86 | $2.3140 \mathrm{E}-09$ |
|  | 2.91 | 1.3 | 9.5100 | 2.07 | 1.9 | 2.83 | 2.32 | 3.5 | $1.6530 \mathrm{E}-04$ | 4. | 6.807 | 5. | 6110 | 5.87 | 2.1 |
|  | 2.8 | 1.32 | 9.3 | 2. | 1. | 2.8 | 2. | 3.6 | 1.5 |  | 6.5 | 5. | 1.5280E-07 | 5.88 | $2.0510 \mathrm{E}-09$ |
| 0.57 | 2.8430E-01 | 1. | 9.1760 | 2.09 | 1.831 | 2.85 | . 1860 | 3.6 | 1.5310 E | 4.37 | $6.2120 \mathrm{E}-06$ | 5. | $4490 \mathrm{E}-07$ | 5.8 | 1.9310E-09 |
| 0.58 | 2.8100 | 1. | 9.0120E-02 | 2.10 | 1.78 | 2.86 | 2.1180 | 3.6 | 1.47 | 4.3 | 5.9340 | 5. | 1.3740 | 5.90 | 1.8180E-09 |
|  | 2.7 | 1.35 | 8.8510E-02 | 2.1 | 1.7430 | 2.8 | 2.0520 | 3. | $1.4170 \mathrm{E}-04$ | 4.39 | 5.6 | 5. | 3020 | 5.91 | 1.7 |
| 0.60 | $2.7430 \mathrm{E}-01$ | 1. | 8.6910 | 2.1 | 1.7000 | 2.8 | 1.988 | 3.6 | 1.3630 | 4. | 5.4130 | 5.16 | $1.2350 \mathrm{E}-07$ | 5.9 | 1.6100E-09 |
|  | $2.7090 \mathrm{E}-01$ | 1.3 | 8.5340 | 2.13 | 1.6590 | 2.89 | 260 | 3.65 | 1.3110 | 4.4 | 5.1690 | 5.1 | $1.1700 \mathrm{E}-07$ | 5.93 | 1.5150E-09 |
| 0.62 | 2.676 | 1.38 | 37 | 2.1 | 1.61 | 2.90 | 1.8660E-03 | 3.6 | 1.26 | 4.4 | 4.9350 | 5.1 | 1090 | 5.94 | $1.4250 \mathrm{E}-09$ |
| 0.63 | 2.6 | 1.39 | 8. | 2.15 | 1.5 | 2.91 | 1. | 3.6 | 1.2 | 4.43 | $4.7120 \mathrm{E}-06$ | 5. |  | 5.95 | $1.3410 \mathrm{E}-09$ |
| 0.64 | $2.6110 \mathrm{E}-01$ | 1.40 | 8.0760 | 2.16 | 1.5390E | 2.9 | 1.7500 | 3.68 | $1.1660 \mathrm{E}-0$ | 4.4 | $4.4980 \mathrm{E}-06$ | 5.20 | $9.9640 \mathrm{E}-08$ | 5.96 | $1.2610 \mathrm{E}-09$ |
| 0.65 | $2.5780 \mathrm{E}-01$ | 1.4 | 9270 | 2.1 | 1.5000 | 2.9 | .6950 | 3.6 | 1.1210 | 4.4 | .2940 | 5.2 | $9.4420 \mathrm{E}-08$ | 5.9 | $1.1860 \mathrm{E}-09$ |
| 0.66 | 2.5460 |  | 800 | . 18 | 1.4630 | 2.94 | 641 | 3.70 | 1.0780 | 4.46 | 4.0980 | 5.2 | $8.9460 \mathrm{E}-08$ | 5.9 | $1.1160 \mathrm{E}-09$ |
| 0.67 | 2.5140 E | 1.43 | 360 | 2.19 | 1. | 2.95 | 1.589 | 3. | 1.0360 | 4. | 3.91 | 5.23 | 8.4760 | 5. | $1.0490 \mathrm{E}-09$ |
| 68 | $2.4830 \mathrm{E}-01$ | 1.44 | 4930 | 2.20 | 1.3900 E | 2.9 | 1.5380 | 3.7 | $9.9610 \mathrm{E}-0$ | 4.48 | $3.7320 \mathrm{E}-06$ | 5.24 | $8.0290 \mathrm{E}-08$ | 6.00 | $9.8660 \mathrm{E}-10$ |
| . 69 | $2.4510 \mathrm{E}-01$ | 1. | 3530E | 2.2 | 1.3550E | 2.9 | 1.4890 | 3.7 | 9.5740 | 4.4 | 3.5610 | 5.2 | 7.6050 E | 6.0 | 9.2760 |
| 0.70 | $2.4200 \mathrm{E}-01$ | 1.46 | 2150 | 2.22 | .3210 | 2.9 | $1.4410 \mathrm{E}-03$ | 3.7 | 9.2010 | 4.50 | 3.3980 | 5.26 | $7.2030 \mathrm{E}-08$ | 6.0 | $8.7210 \mathrm{E}-10$ |
| . 71 | $2.3890 \mathrm{E}-01$ | 1. | 7.0780 | 2.23 | 1.2870E | 2.99 | 1.3950 E | 3.7 | 8.8420E-05 | 4.51 | $3.2410 \mathrm{E}-06$ | 5.27 | $6.8210 \mathrm{E}-08$ | 6.03 | $8.1980 \mathrm{E}-10$ |
| 72 | $2.3580 \mathrm{E}-01$ | 1.48 | $6.9440 \mathrm{E}-0$ | 2.24 | 1.2550E-02 | 3.00 | $1.3500 \mathrm{E}-03$ | 3.76 | 8.4960E-05 | 4.52 | 3.0920E-06 | 5.28 | $6.4590 \mathrm{E}-08$ | 6.04 | 7.7060E-10 |
| 73 | $2.3270 \mathrm{E}-01$ | 1.49 | 6.8110 | 2.25 | $1.2220 \mathrm{E}-02$ | 3.01 | 1.3060 | 3.77 | $8.1620 \mathrm{E}-05$ | 4.53 | $2.9490 \mathrm{E}-06$ | 5.29 | $6.1160 \mathrm{E}-08$ | 6.05 | 7.2420 |
| 0.74 | $2.2960 \mathrm{E}-01$ | 1.50 | 6.6810E-02 | 2.26 | $1.1910 \mathrm{E}-02$ | . 02 | 1.2640 E | 3.78 | $7.8410 \mathrm{E}-05$ | 4.54 | $2.8130 \mathrm{E}-06$ | 5.30 | $5.7900 \mathrm{E}-08$ | 6.06 | $6.8060 \mathrm{E}-10$ |
| 0.75 | $2.2660 \mathrm{E}-01$ | 1.51 | 6.5520 E | 2.27 | $1.1600 \mathrm{E}-02$ | 3.03 | $1.2230 \mathrm{E}-03$ | 3.79 | $7.5320 \mathrm{E}-05$ | 4.55 | $2.6820 \mathrm{E}-06$ | 5.31 | $5.4810 \mathrm{E}-08$ | 6.07 | $6.3960 \mathrm{E}-1$ |

1. Consider a network of $N=100$ stations that are attached to a common physical medium. The medium access control method is a simple reservation protocol where communication is done in cycles. A cycle consists of $N$ minislots (reservation slots) followed by $M$ data slots. Here, $M$ is the number of stations that have data to send.
The network layer (NET) is located on top of the data link layer. The data link layer consists of the data link control (DLC) sublayer just above the medium access control (MAC) sublayer, which in turn is just above the physical (PHY) layer, see the figure below.


Suppose the data link (DL) layer header and trailer requires in total 25 byte. The DL header is added by the DLC before sending the data to the MAC sublayer. The MAC layer does not add any header bits to the data frames received from the DLC. The reservation messages (created by the MAC sublayer) are 25 byte long. The physical layer provides the data rate $R_{\text {PHY }}=100 \mathrm{Mbit} / \mathrm{s}$ to the MAC sublayer. For simplicity, we assume that the physical layer is error-free. Let $M=10$.

Let $R_{\mathrm{DL}}$ be the effective data of the DL layer service, i.e., the data rate experienced by the network layer.
(a) Compute $R_{\text {DL }}$ when the DL service data unit (SDU) is 400 byte long. Assume that the data slot duration is chosen to maximize the effective data rate. (3p)
(b) Suppose the data slot duration is as in Part (a). What is $R_{\text {DL }}$ when the DL SDU is 800 byte? Assume that the DLC split the DL SDU into a number of smaller fragments before pushing the data down to the MAC. (3p)
(c) Suppose the data slot duration is as in Part (a). What is $R_{\text {DL }}$ when the DL SDU is 200 byte? Assume that the DLC pads the DL SDU with zero bytes before pushing the data down to the MAC. (3p)
(d) Give a general expression for $R_{\text {DL }}$ when the DL SDU size is $n_{\text {DSDU }}$ byte long. Assume that the DLC does fragmentation and zero-padding as needed and that the data slot duration is as in Part (a). (3p)
2. Consider transmission of 1500 byte long packets over an AWGN channel with noise power spectral density $N_{0} / 2$ using $M$-ary PAM with the pulse shape $h(t)$ (depicted below) and where $N_{0}=1.1 \times 10^{-20} \mathrm{~W} / \mathrm{Hz}$. The bits that make up the packet are assumed to be independent and equally likely. The signal constellation is $\mathcal{A}=\{ \pm 1, \pm 3, \ldots, \pm(M-1)\}$, the data rate is $100 \mathrm{Mbit} / \mathrm{s}$, and the transmission should be ISI-free.

(a) What is the smallest $A$ that is required to achieve the bit error probability $P_{b}=10^{-3}$ for $M=2$ ? ( 4 p )
(b) What is the smallest $A$ that is required to achieve the packet error probability $P_{e}=10^{-1}$ for $M=2 ?(4 \mathrm{p})$
(c) Repeat Part (b) for $M=4$. (4p)

Hint: the symbol error probability for $M$-ary PAM is

$$
P_{e}=\frac{2(M-1)}{M} Q\left(\sqrt{\frac{6}{M^{2}-1} \frac{E_{s}}{N_{0}}}\right)
$$

where $E_{s}$ is the symbol energy.
3. (a) What is the purpose of the spanning tree protocol? (2p)
(b) Why is collision detection not used in WiFi? (2p)
(c) What is the purpose of the ARP protocol in the TCP/IP model? (2p)
(d) Define the security goal "authentication." (1p)
(e) Suppose we use a single-parity check code for error detection. If the number of information bits is 3 , list all detectable and undetectable error patterns. (2p)
(f) What is the purpose of using square-root Nyquist pulses for PAM transmission? (1p)
(g) Give examples of two practical systems that use static medium access control. (2p)
4. Consider a network consisting of four nodes, the two end-nodes (hosts) A and B and an intermediate routers R1 and R2. The distances between the nodes and the link data rates are defined in the figure below. The propagation speed on all links is $c=2 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

| Host | 200 m | Router R1 | 20000 m | Router R2 | 200 m | Host |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $100 \mathrm{Mbit} / \mathrm{s}$ |  | $1000 \mathrm{Mbit} / \mathrm{s}$ |  | $100 \mathrm{Mbit} / \mathrm{s}$ | B |

The hosts communicate using a Stop-And-Wait protocol, where the data packets are 1250 byte long, including a 50-byte header, and the ACK packets are 50 byte long. The routers are store-and-forward routers. That is, an incoming packet is completely received before it is forwarded to the output port. Processing times in hosts and routers are assumed to be negligible.
(a) Determine the value of the time-out such that (i) no unnecessary retransmissions occur and (ii) the effective data rate of the ARQ protocol is maximized. (4p)
(b) What is the effective data rate assuming error-free transmission? (2p)
(c) Suppose R1 drops packets with probability 0.1 and R2 drops packets with probability 0.2. Assume that the packet drops are independent of each other. Note that both data packets and ACK packets are subject to packet drops. What is the effective date of the ARQ protocol? (4p)
(d) Suppose your boss is not happy with the effective data rate in Part (c). She asks you to suggest another ARQ protocol to improve the effective data rate. What protocol would you suggest? Your boss wants a good motivation for your suggestion, not just an answer. (2p)

Hint: Suppose the success probability of trial is $p$, then the average number of independent trials needed until a successful trial is

$$
\sum_{k=1}^{\infty} k p(1-p)^{k-1}=\frac{1}{p}
$$

# Exam in SSY305 Kommunikationssystem 

Department of Electrical Engineering<br>Exam date: June 12, 2019, 14:00-18:00<br>Document updated: June 18, 2019

## Teaching Staff

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Material Allowed material is

- Chalmers-approved calculator
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A paper-based dictionary, without added notes (electronic dictionaries are not allowed).

Grading A correct, clear and well-motivated solution gives a maximum of 12 points per problem.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. No fractional points are awarded.
Answers in any other language than Swedish or English are ignored.
Solutions Are made available at the earliest at 19:00 on the course web page.
Results Exam results are posted on Ping-Pong no later than on June 19, 2019. The grading review is on June 20 and June 26, 2019, 13:00-14:00 in room 6436 in the ED-building (Chouaib's office).

Grades The final grade on the course will be decided by the projects (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). Project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade according to the following table.

| Total Score | $0-39$ | $40-69$ | $70-79$ | $\geq 80$ |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Fail | 3 | 4 | 5 |

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.

1. Consider a network of $N=100$ stations that are attached to a common physical medium. The medium access control method is a simple reservation protocol where communication is done in cycles. A cycle consists of $N$ minislots (reservation slots) followed by $M$ data slots. Here, $M$ is the number of stations that have data to send.
The network layer (NET) is located on top of the data link layer. The data link layer consists of the data link control (DLC) sublayer just above the medium access control (MAC) sublayer, which in turn is just above the physical (PHY) layer, see the figure below.


Suppose the data link (DL) layer header and trailer requires in total 25 byte. The DL header is added by the DLC before sending the data to the MAC sublayer. The MAC layer does not add any header bits to the data frames received from the DLC. The reservation messages (created by the MAC sublayer) are 25 byte long. The physical layer provides the data rate $R_{\text {PHY }}=100 \mathrm{Mbit} / \mathrm{s}$ to the MAC sublayer. For simplicity, we assume that the physical layer is error-free. Let $M=10$.

Let $R_{\mathrm{DL}}$ be the effective data of the DL layer service, i.e., the data rate experienced by the network layer.
(a) Compute $R_{\text {DL }}$ when the DL service data unit (SDU) is 400 byte long. Assume that the data slot duration is chosen to maximize the effective data rate. (3p)
(b) Suppose the data slot duration is as in Part (a). What is $R_{\text {DL }}$ when the DL SDU is 800 byte? Assume that the DLC split the DL SDU into a number of smaller fragments before pushing the data down to the MAC. (3p)
(c) Suppose the data slot duration is as in Part (a). What is $R_{\text {DL }}$ when the DL SDU is 200 byte? Assume that the DLC pads the DL SDU with zero bytes before pushing the data down to the MAC. (3p)
(d) Give a general expression for $R_{\text {DL }}$ when the DL SDU size is $n_{\text {DSDU }}$ byte long. Assume that the DLC does fragmentation and zero-padding as needed and that the data slot duration is as in Part (a). (3p)

## Solution

(a) Let $T_{\mathrm{r}}$ be the slot duration of one reservation frame, and $T_{\mathrm{D}}$ be the slot duration of one data frame. Assuming that the propagation and processing times are negligible, the effective data rate is maximized if the slot duration is equal to the transmission time of a frame, hence

$$
\begin{aligned}
T_{\mathrm{r}} & =\frac{n_{\mathrm{r}}}{R_{\mathrm{PHY}}} \\
T_{\mathrm{D}} & =\frac{n_{\mathrm{o}}+n_{\mathrm{DSDU}}}{R_{\mathrm{PHY}}}
\end{aligned}
$$

where $n_{\mathrm{o}}$ is the length of the header and trailer in bits, and $n_{\mathrm{r}}$ and $n_{\text {DSDU }}$ are the length of the reservation message and DL SDU in bits, respectively. Substituting with the numerical values, we get

$$
\begin{aligned}
T_{\mathrm{r}} & =\frac{25 \times 8}{10^{8}}=2 \mu \mathrm{~s} \\
T_{\mathrm{D}} & =\frac{425 \times 8}{10^{8}}=34 \mu \mathrm{~s}
\end{aligned}
$$

To compute the effective data rate experienced by the NET layer of a particular station, we take into account that the station can transmit one data frame each cycle of $N$ reservation slots and $M$ data slots, hence

$$
R_{\mathrm{DL}}=\frac{400 \times 8}{N T_{\mathrm{r}}+M T_{\mathrm{D}}}=5.926 \mathrm{Mbit} / \mathrm{s} .
$$

Note that we assume that the station has always data to transmit. Therefore the experienced data rate is the same whether the station sends data in the first data slot or the last one.
(b) In this case the station needs to send two data frames. This can be done in two transmission cycles, hence

$$
R_{\mathrm{DL}}^{(\mathrm{b})}=\frac{800 \times 8}{2\left(N T_{\mathrm{r}}+M T_{\mathrm{D}}\right)}=5.926 \mathrm{Mbit} / \mathrm{s} .
$$

(c) In this case only one transmission cycle is needed. However, since the data slot time is optimized for data frames that are 400 bytes long, a lower data rate is experienced

$$
R_{\mathrm{DL}}^{(\mathrm{c})}=\frac{200 \times 8}{N T_{\mathrm{r}}+M T_{\mathrm{D}}}=2.963 \mathrm{Mbit} / \mathrm{s}
$$

(d) In general, we need $\left\lceil\frac{n_{\text {DSDU }}}{(400 \times 8)}\right\rceil$ cycles to transmit all the data, hence

$$
R_{\mathrm{DL}}=\frac{n_{\mathrm{DSDU}}}{\left\lceil\frac{n_{\mathrm{DSDU}}}{(400 \times 8)}\right\rceil\left(N T_{\mathrm{r}}+M T_{\mathrm{D}}\right)}
$$

Note that $R_{\mathrm{DL}}^{\max }=5.926 \mathrm{Mbit} / \mathrm{s}$.
2. Consider transmission of 1500 byte long packets over an AWGN channel with noise power spectral density $N_{0} / 2$ using $M$-ary PAM with the pulse shape $h(t)$ (depicted below) and where $N_{0}=1.1 \times 10^{-20} \mathrm{~W} / \mathrm{Hz}$. The bits that make up the packet are assumed to be independent and equally likely. The signal constellation is $\mathcal{A}=\{ \pm 1, \pm 3, \ldots, \pm(M-1)\}$, the data rate is $100 \mathrm{Mbit} / \mathrm{s}$, and the transmission should be ISI-free.

(a) What is the smallest $A$ that is required to achieve the bit error probability $P_{b}=10^{-3}$ for $M=2$ ? ( 4 p )
(b) What is the smallest $A$ that is required to achieve the packet error probability $P_{e} P_{p}=10^{-1}$ for $M=2 ?(4 \mathrm{p})$
(c) Repeat Part (b) for $M=4$. ( 4 p )

Hint: the symbol error probability for $M$-ary PAM is

$$
P_{e}=\frac{2(M-1)}{M} Q\left(\sqrt{\frac{6}{M^{2}-1} \frac{E_{s}}{N_{0}}}\right)
$$

where $E_{s}$ is the symbol energy.
Solution
(a) We have for $M=2$

$$
P_{b}=P_{e}=Q\left(\sqrt{\frac{2 E_{s}}{N_{0}}}\right)
$$

where the symbol energy $E_{s}$ is given by

$$
\begin{aligned}
& E_{s}=E_{a} E_{h} . \\
& E_{a}=\frac{1}{|\mathcal{A}|} \sum_{i=1}^{M}\left|a_{i}\right|^{2} . \\
& E_{h}=\int_{-\infty}^{+\infty}|h(t)|^{2} d t=A^{2} T .
\end{aligned}
$$

For $M=2, E_{a}=1$. The symbol duration $T$ can be deduced from the data rate. Since, we are using a rectangular pulse

$$
T=\frac{\log _{2}(M)}{R}=10^{-8} \mathrm{~s}
$$

From the Q-function table we can get

$$
Q(x) \leq 10^{-3} \longrightarrow x=3.10 .
$$

Then,

$$
\begin{aligned}
x^{2} & =\frac{2 E_{s}}{N_{0}}=\frac{2 A^{2} T}{N_{0}} \\
A & =\sqrt{\frac{x^{2} N_{0}}{2 T}} \\
A & =2.3 \mu \mathrm{v}
\end{aligned}
$$

(b) Since symbol errors are independent for PAM over a baseband AWGN channel, the packet error probability is $P_{p}=1-\left(1-P_{e}\right)^{n_{s}}$, where $P_{e}$ is the symbol error probability and $n_{s}$ is the number of symbols in the packet. Cleary, $n_{s}=8 n_{b} / \log _{2}(M)$, where $n_{b}$ is the number of bytes in the packet and $\log _{2}(M)$ is the number of bits per symbol. Putting everything together, we can compute the symbol error probability as

$$
\begin{align*}
P_{p} & =1-\left(1-P_{e}\right)^{n_{s}} \Rightarrow \\
P_{e} & =1-\left(1-P_{p}\right)^{\log _{2}(M) /\left(8 n_{b}\right)}  \tag{1}\\
& =8.78^{-6}
\end{align*}
$$

Then, we continue in similar steps as in (a). From the Q-function table we get

$$
\begin{aligned}
Q(x) & \leq 8.78 \times 10^{-6} \longrightarrow x=4.30 . \\
A & =\sqrt{\frac{x^{2} N_{0}}{2 T}} \\
& =3.19 \mu \mathrm{~V} .
\end{aligned}
$$

(c) We use (1) to compute the symbol error probability needed to achieve $P_{p}=10^{-1}$.

$$
P_{e}=1.7560 \times 10^{-5}
$$

In the case $M=4$

$$
P_{e}=\frac{3}{2} Q\left(\sqrt{\frac{2}{5} \frac{E_{s}}{N_{0}}}\right)
$$

Using the Q-function table, we get

$$
Q(x) \leq \frac{2}{3} 1.7560 \times 10^{-5} \longrightarrow x=4.23
$$

For $\mathcal{A}=\{-3,-1,1,3\}$, we have that $E_{a}=5$ and $E_{s}=5 A^{2} T$, where $T=\log _{2}(M) / R=$ $2.10^{-8}$. Then,

$$
\begin{aligned}
x^{2} & =\frac{2}{5} \frac{E_{s}}{N_{0}}=\frac{2}{5} \frac{5 A^{2} T}{N_{0}}=\frac{2 A^{2} T}{N_{0}} \\
A & =\sqrt{\frac{N_{0}}{2 T} x^{2}} \\
& =2.22 \mu \mathrm{v} .
\end{aligned}
$$

3. (a) What is the purpose of the spanning tree protocol? (2p)

The purpose of the protocol is to create a logical loop-free topology in LAN networks, even if the physical topology might contain loops]. This removes the risk of having a broadcast storm in the network.
(b) Why is collision detection not used in WiFi? (2p)

If a station is transmitting over the wireless medium, it is hard to detect if other stations are transmitting at the same time. That is because the difference between the transmitted power and the received power might be very high in Wi-Fi. Hence, sensing the medium while transmitting is not practical in Wi-Fi systems.
(c) What is the purpose of the ARP protocol in the TCP/IP model? (2p)

To find the MAC address corresponding to a destination IP address.
(d) Define the security goal "authentication." (1p)

The goal is to ensure that a receiver can verify that that a received message is from the claimed sender.
(e) Suppose we use a single-parity check code for error detection. If the number of information bits is 3 , list all detectable and undetectable error patterns. (2p)
Error patterns can be modeled with an error vector composed of 4 bits. Then any error pattern that contain an even number of 1's is undetectable. All the remaining error patterns are detectable. We list these, Undetectable error patterns: 0011, 0101, 0110, 1001, 1010, 1100, 1111. Detectable error patterns: 0001, 0010, 0100, 0111, 1000, 1011, 1101, 1110.
(f) What is the purpose of using square-root Nyquist pulses for PAM transmission? (1p) To avoid Inter Symbol Interference (ISI) between the transmitted symbols.
(g) Give examples of two practical systems that use static medium access control. (2p) FM radio broadcasting and TV broadcasting.
4. Consider a network consisting of four nodes, the two end-nodes (hosts) A and B and an intermediate routers R1 and R2. The distances between the nodes and the link data rates are defined in the figure below. The propagation speed on all links is $c=2 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

| Host | 200 m | Router R1 | 20000 m | Router R2 | 200 m | Host |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $100 \mathrm{Mbit} / \mathrm{s}$ |  | $1000 \mathrm{Mbit} / \mathrm{s}$ |  | $100 \mathrm{Mbit} / \mathrm{s}$ | B |

The hosts communicate using a Stop-And-Wait protocol, where the data packets are 1250 byte long, including a 50 -byte header, and the ACK packets are 50 byte long. The routers are store-and-forward routers. That is, an incoming packet is completely received before it is forwarded to the output port. Processing times in hosts and routers are assumed to be negligible.
(a) Determine the value of the time-out such that (i) no unnecessary retransmissions occur and (ii) the effective data rate of the ARQ protocol is maximized. (4p)
(b) What is the effective data rate assuming error-free transmission? (2p)
(c) Suppose R1 drops packets with probability 0.1 and R2 drops packets with probability 0.2 . Assume that the packet drops are independent of each other. Note that both data packets and ACK packets are subject to packet drops. What is the effective date of the ARQ protocol? (4p)
(d) Suppose your boss is not happy with the effective data rate in Part (c). She asks you to suggest another ARQ protocol to improve the effective data rate. What protocol would you suggest? Your boss wants a good motivation for your suggestion, not just an answer. (2p)

Hint: Suppose the success probability of trial is $p$, then the average number of independent trials needed until a successful trial is

$$
\sum_{k=1}^{\infty} k p(1-p)^{k-1}=\frac{1}{p}
$$

## Solution

(a) Let $\tau_{\text {prop, } 1}$ be the propagation delay between Host $\mathrm{A} / \mathrm{B}$ and $\mathrm{R} 1 / \mathrm{R} 2$, and let $\tau_{\text {prop, } 2}$ be the propagation delay between R1 and R2. Then,

$$
\begin{aligned}
& \tau_{\text {prop }, 1}=\tau_{\text {prop }, 3}=\frac{200}{c}=1 \mu \mathrm{~s} \\
& \tau_{\text {prop }, 2}=\frac{20000}{c}=100 \mu \mathrm{~s}
\end{aligned}
$$

Let $n_{\mathrm{d}}$ and $n_{\mathrm{a}}$ be the length of data packet and ACK packet in bits, respectively. The round-trip-time (RTT) for the stop and wait protocol can be computed as

$$
\begin{align*}
t_{0} & =\frac{n_{\mathrm{d}}}{10^{8}}+\tau_{\text {prop }, 1}+\frac{n_{\mathrm{d}}}{10^{9}}+\tau_{\text {prop }, 2}+\frac{n_{\mathrm{d}}}{10^{8}}+\tau_{\text {prop }, 3}  \tag{2}\\
& +\frac{n_{\mathrm{a}}}{10^{8}}+\tau_{\text {prop }, 1}+\frac{n_{\mathrm{a}}}{10^{9}}+\tau_{\text {prop }, 2}+\frac{n_{\mathrm{a}}}{10^{8}}+\tau_{\text {prop }, 3}=0.423 \mathrm{~ms} \tag{3}
\end{align*}
$$

To avoid unnecessary retransmissions and maximize the effective data rate of the protocol we choose the time out to be equal to the RTT, that is

$$
t_{\text {out }}=t_{0}=0.423 \mathrm{~ms}
$$

(b) Let $n_{\mathrm{o}}$ be the length of the header. The effective data rate in absence of errors can be computed as

$$
R_{\mathrm{sw}}^{0}=\frac{n_{\mathrm{d}}-n_{\mathrm{o}}}{t_{0}}=22.7 \mathrm{Mbit} / \mathrm{s} .
$$

(c) Let $p$ be the probability that a transmission trial is successful. This can be expressed as
$p=\operatorname{Pr}\{" P a c k e t$ frame is not dropped by R1" $\cap$ "Packet frame is not dropped by R2" $\cap$ "ACK frame is not dropped by R2" $\cap$ "ACK frame is not dropped by R1" $\}$

Under the assumption that packets drops in R1 and R2 are independent and the assumption that data packets drops and ACK packets drops are independent, $p$ is given by
$p=\operatorname{Pr}\{" P a c k e t$ frame is not dropped by R1" $\} \times \operatorname{Pr}\{"$ Packet frame is not dropped by R2" $\}$ $\times \operatorname{Pr}\{"$ ACK frame is not dropped by R2" $\} \times \operatorname{Pr}\{"$ ACK frame is not dropped by R1" $\}$

$$
\begin{aligned}
p & =(1-0.1)^{2}(1-0.2)^{2} \\
& =0.5184 .
\end{aligned}
$$

Then we can readily compute the effective data rate taking into account that the average number of retransmissions needed until a frame is successfully transmitted is $\frac{1}{p}$ (check the hint), as

$$
R_{\mathrm{sw}}=\frac{n_{\mathrm{d}}-n_{\mathrm{o}}}{\frac{1}{p} t_{\mathrm{out}}}=11.8 \mathrm{Mbit} / \mathrm{s}
$$

(d) We can use either Go-Back-N or Selective Repeat protocols to improve the performance. In error-free conditions, Go-Back-N (with a sufficiently large send window and full-duplex links) is able to transmit and deliver data continuously. That is, the effective rate is close to the data rate of the bottleneck links along the path, which in this case is $100 \mathrm{Mbit} / \mathrm{s}$. However, all packets in the send window needs to be retransmitted in case of an unsuccessful transmission attempt. Since the probability for success is low in this example, it is better to use Selective Repeat, which is less sensitive to unsuccessful transmissions compared to Go-Back-N.

