

Exam in SSY305 Kommunikationssystem

Department of Electrical Engineering

Exam date: March 21, 2019, 14:00–18:00

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Teaching Staff

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Material Allowed material is

- Chalmers-approved calculator
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A paper-based dictionary, without added notes (electronic dictionaries are not allowed).

Grading A correct, clear and well-motivated solution gives a maximum of 12 points per problem.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. No fractional points are awarded.

Answers in any other language than Swedish or English are ignored.

Solutions Are made available at the earliest at 19:00 on the course web page.

Results Exam results are posted on Ping-Pong no later than March 27, 2019. The grading review is on March 27 and April 4, 2019, 12:00–13:00 in room 3340 in the ED-building.

Grades The final grade on the course will be decided by the projects (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). Project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade according to the following table.

Total Score	0–39	40–69	70–79	≥ 80
Grade	Fail	3	4	5

**PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY
ORDERED IN DIFFICULTY.**

Good luck!

Table over the Q-function

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
0.00	5.0000E-01	0.76	2.2360E-01	1.52	6.4260E-02	2.28	1.1300E-02	3.04	1.1830E-03	3.80	7.2350E-05	4.56	2.5580E-06
0.01	4.9600E-01	0.77	2.2060E-01	1.53	6.3010E-02	2.29	1.1010E-02	3.05	1.1440E-03	3.81	6.9480E-05	4.57	2.4390E-06
0.02	4.9200E-01	0.78	2.1770E-01	1.54	6.1780E-02	2.30	1.0720E-02	3.06	1.1070E-03	3.82	6.6730E-05	4.58	2.3250E-06
0.03	4.8800E-01	0.79	2.1480E-01	1.55	6.0570E-02	2.31	1.0440E-02	3.07	1.0700E-03	3.83	6.4070E-05	4.59	2.2160E-06
0.04	4.8400E-01	0.80	2.1190E-01	1.56	5.9380E-02	2.32	1.0170E-02	3.08	1.0350E-03	3.84	6.1520E-05	4.60	2.1120E-06
0.05	4.8010E-01	0.81	2.0900E-01	1.57	5.8210E-02	2.33	9.9030E-03	3.09	1.0010E-03	3.85	5.9060E-05	4.61	2.0130E-06
0.06	4.7610E-01	0.82	2.0610E-01	1.58	5.7050E-02	2.34	9.6420E-03	3.10	9.6760E-04	3.86	5.6690E-05	4.62	1.9190E-06
0.07	4.7210E-01	0.83	2.0330E-01	1.59	5.5920E-02	2.35	9.3870E-03	3.11	9.3540E-04	3.87	5.4420E-05	4.63	1.8280E-06
0.08	4.6810E-01	0.84	2.0050E-01	1.60	5.4800E-02	2.36	9.1370E-03	3.12	9.0430E-04	3.88	5.2230E-05	4.64	1.7420E-06
0.09	4.6410E-01	0.85	1.9770E-01	1.61	5.3700E-02	2.37	8.8940E-03	3.13	8.7400E-04	3.89	5.0120E-05	4.65	1.6600E-06
0.10	4.6020E-01	0.86	1.9490E-01	1.62	5.2620E-02	2.38	8.6560E-03	3.14	8.4470E-04	3.90	4.8100E-05	4.66	1.5810E-06
0.11	4.5620E-01	0.87	1.9220E-01	1.63	5.1550E-02	2.39	8.4240E-03	3.15	8.1640E-04	3.91	4.6150E-05	4.67	1.5060E-06
0.12	4.5220E-01	0.88	1.8940E-01	1.64	5.0500E-02	2.40	8.1980E-03	3.16	7.8880E-04	3.92	4.4270E-05	4.68	1.4340E-06
0.13	4.4830E-01	0.89	1.8670E-01	1.65	4.9470E-02	2.41	7.9760E-03	3.17	7.6220E-04	3.93	4.2470E-05	4.69	1.3660E-06
0.14	4.4430E-01	0.90	1.8410E-01	1.66	4.8460E-02	2.42	7.7600E-03	3.18	7.3600E-04	3.94	4.0740E-05	4.70	1.3010E-06
0.15	4.4040E-01	0.91	1.8140E-01	1.67	4.7460E-02	2.43	7.5490E-03	3.19	7.1140E-04	3.95	3.9080E-05	4.71	1.2390E-06
0.16	4.3640E-01	0.92	1.7880E-01	1.68	4.6480E-02	2.44	7.3440E-03	3.20	6.8710E-04	3.96	3.7470E-05	4.72	1.1790E-06
0.17	4.3250E-01	0.93	1.7620E-01	1.69	4.5510E-02	2.45	7.1430E-03	3.21	6.6370E-04	3.97	3.5940E-05	4.73	1.1230E-06
0.18	4.2860E-01	0.94	1.7360E-01	1.70	4.4570E-02	2.46	6.9470E-03	3.22	6.4100E-04	3.98	3.4460E-05	4.74	1.0690E-06
0.19	4.2470E-01	0.95	1.7110E-01	1.71	4.3630E-02	2.47	6.7560E-03	3.23	6.1900E-04	3.99	3.3040E-05	4.75	1.0170E-06
0.20	4.2070E-01	0.96	1.6850E-01	1.72	4.2720E-02	2.48	6.5690E-03	3.24	5.9760E-04	4.00	3.1670E-05	4.76	9.6800E-07
0.21	4.1680E-01	0.97	1.6600E-01	1.73	4.1820E-02	2.49	6.3870E-03	3.25	5.7700E-04	4.01	3.0360E-05	4.77	9.2110E-07
0.22	4.1290E-01	0.98	1.6350E-01	1.74	4.0930E-02	2.50	6.2100E-03	3.26	5.5710E-04	4.02	2.9100E-05	4.78	8.7650E-07
0.23	4.0900E-01	0.99	1.6110E-01	1.75	4.0060E-02	2.51	6.0370E-03	3.27	5.3770E-04	4.03	2.7890E-05	4.79	8.3390E-07
0.24	4.0520E-01	1.00	1.5870E-01	1.76	3.9200E-02	2.52	5.8680E-03	3.28	5.1900E-04	4.04	2.6730E-05	4.80	7.9330E-07
0.25	4.0130E-01	1.01	1.5620E-01	1.77	3.8360E-02	2.53	5.7030E-03	3.29	5.0090E-04	4.05	2.5610E-05	4.81	7.5470E-07
0.26	3.9740E-01	1.02	1.5390E-01	1.78	3.7540E-02	2.54	5.5430E-03	3.30	4.8340E-04	4.06	2.4540E-05	4.82	7.1780E-07
0.27	3.9360E-01	1.03	1.5150E-01	1.79	3.6730E-02	2.55	5.3860E-03	3.31	4.6650E-04	4.07	2.3510E-05	4.83	6.8270E-07
0.28	3.8970E-01	1.04	1.4920E-01	1.80	3.5930E-02	2.56	5.2340E-03	3.32	4.5010E-04	4.08	2.2520E-05	4.84	6.4920E-07
0.29	3.8590E-01	1.05	1.4690E-01	1.81	3.5150E-02	2.57	5.0850E-03	3.33	4.3420E-04	4.09	2.1570E-05	4.85	6.1730E-07
0.30	3.8210E-01	1.06	1.4460E-01	1.82	3.4380E-02	2.58	4.9400E-03	3.34	4.1890E-04	4.10	2.0660E-05	4.86	5.8690E-07
0.31	3.7830E-01	1.07	1.4230E-01	1.83	3.3620E-02	2.59	4.7990E-03	3.35	4.0410E-04	4.11	1.9780E-05	4.87	5.5800E-07
0.32	3.7450E-01	1.08	1.4010E-01	1.84	3.2880E-02	2.60	4.6610E-03	3.36	3.8970E-04	4.12	1.8940E-05	4.88	5.3040E-07
0.33	3.7070E-01	1.09	1.3790E-01	1.85	3.2160E-02	2.61	4.5270E-03	3.37	3.7580E-04	4.13	1.8140E-05	4.89	5.0420E-07
0.34	3.6690E-01	1.10	1.3570E-01	1.86	3.1440E-02	2.62	4.3960E-03	3.38	3.6240E-04	4.14	1.7370E-05	4.90	4.7920E-07
0.35	3.6320E-01	1.11	1.3350E-01	1.87	3.0740E-02	2.63	4.2690E-03	3.39	3.4950E-04	4.15	1.6620E-05	4.91	4.5540E-07
0.36	3.5940E-01	1.12	1.3140E-01	1.88	3.0050E-02	2.64	4.1450E-03	3.40	3.3690E-04	4.16	1.5910E-05	4.92	4.3270E-07
0.37	3.5570E-01	1.13	1.2920E-01	1.89	2.9380E-02	2.65	4.0250E-03	3.41	3.2480E-04	4.17	1.5230E-05	4.93	4.1110E-07
0.38	3.5200E-01	1.14	1.2710E-01	1.90	2.8720E-02	2.66	3.9070E-03	3.42	3.1310E-04	4.18	1.4580E-05	4.94	3.9060E-07
0.39	3.4830E-01	1.15	1.2510E-01	1.91	2.8070E-02	2.67	3.7930E-03	3.43	3.0180E-04	4.19	1.3950E-05	4.95	3.7110E-07
0.40	3.4460E-01	1.16	1.2300E-01	1.92	2.7430E-02	2.68	3.6810E-03	3.44	2.9090E-04	4.20	1.3350E-05	4.96	3.5250E-07
0.41	3.4090E-01	1.17	1.2100E-01	1.93	2.6800E-02	2.69	3.5730E-03	3.45	2.8030E-04	4.21	1.2770E-05	4.97	3.3480E-07
0.42	3.3720E-01	1.18	1.1900E-01	1.94	2.6190E-02	2.70	3.4670E-03	3.46	2.7010E-04	4.22	1.2220E-05	4.98	3.1790E-07
0.43	3.3360E-01	1.19	1.1700E-01	1.95	2.5590E-02	2.71	3.3640E-03	3.47	2.6020E-04	4.23	1.1680E-05	4.99	3.0190E-07
0.44	3.3000E-01	1.20	1.1510E-01	1.96	2.5000E-02	2.72	3.2640E-03	3.48	2.5070E-04	4.24	1.1180E-05	5.00	2.8670E-07
0.45	3.2640E-01	1.21	1.1310E-01	1.97	2.4420E-02	2.73	3.1670E-03	3.49	2.4150E-04	4.25	1.0690E-05	5.01	2.7220E-07
0.46	3.2280E-01	1.22	1.1120E-01	1.98	2.3850E-02	2.74	3.0720E-03	3.50	2.3260E-04	4.26	1.0220E-05	5.02	2.5840E-07
0.47	3.1920E-01	1.23	1.0930E-01	1.99	2.3300E-02	2.75	2.9800E-03	3.51	2.2410E-04	4.27	9.7740E-06	5.03	2.4520E-07
0.48	3.1560E-01	1.24	1.0750E-01	2.00	2.2750E-02	2.76	2.8900E-03	3.52	2.1580E-04	4.28	9.3450E-06	5.04	2.3280E-07
0.49	3.1210E-01	1.25	1.0560E-01	2.01	2.2220E-02	2.77	2.8030E-03	3.53	2.0780E-04	4.29	8.9340E-06	5.05	2.2090E-07
0.50	3.0850E-01	1.26	1.0380E-01	2.02	2.1690E-02	2.78	2.7180E-03	3.54	2.0010E-04	4.30	8.5400E-06	5.06	2.0960E-07
0.51	3.0500E-01	1.27	1.0200E-01	2.03	2.1180E-02	2.79	2.6350E-03	3.55	1.9260E-04	4.31	8.1630E-06	5.07	1.9890E-07
0.52	3.0150E-01	1.28	1.0030E-01	2.04	2.0680E-02	2.80	2.5550E-03	3.56	1.8540E-04	4.32	7.8010E-06	5.08	1.8870E-07
0.53	2.9810E-01	1.29	9.8530E-02	2.05	2.0180E-02	2.81	2.4770E-03	3.57	1.7850E-04	4.33	7.4550E-06	5.09	1.7900E-07
0.54	2.9460E-01	1.30	9.6800E-02	2.06	1.9700E-02	2.82	2.4010E-03	3.58	1.7180E-04	4.34	7.1240E-06	5.10	1.6980E-07
0.55	2.9120E-01	1.31	9.5100E-02	2.07	1.9230E-02	2.83	2.3270E-03	3.59	1.6530E-04	4.35	6.8070E-06	5.11	1.6110E-07
0.56	2.8770E-01	1.32	9.3420E-02	2.08	1.8760E-02	2.84	2.2560E-03	3.60	1.5910E-04	4.36	6.5030E-06	5.12	1.5280E-07
0.57	2.8430E-01	1.33	9.1760E-02	2.09	1.8310E-02	2.85	2.1860E-03	3.61	1.5310E-04	4.37	6.2120E-06	5.13	1.4490E-07
0.58	2.8100E-01	1.34	9.0120E-02	2.10	1.7860E-02	2.86	2.1180E-03	3.62	1.4730E-04	4.38	5.9340E-06	5.14	1.3740E-07
0.59	2.7760E-01	1.35	8.8510E-02	2.11	1.7430E-02	2.87	2.0520E-03	3.63	1.4170E-04	4.39	5.6680E-06	5.15	1.3020E-07
0.60	2.7430E-01	1.36	8.6910E-02	2.12	1.7000E-02	2.88	1.9880E-03	3.64	1.3630E-04	4.40	5.4130E-06	5.16	1.2350E-07
0.61	2.7090E-01	1.37	8.5340E-02	2.13	1.6590E-02	2.89	1.9260E-03	3.65	1.3110E-04	4.41	5.1690E-06	5.17	1.1700E-07
0.62	2.6760E-01	1.38	8.3790E-02	2.14	1.6180E-02	2.90	1.8660E-03	3.66	1.2610E-04	4.42	4.9350E-06	5.18	1.1090E-07
0.63	2.6430E-01	1.39	8.2260E-02	2.15	1.5780E-02	2.91	1.8070E-03	3.67	1.2130E-04	4.43	4.7120E-06	5.19	1.0510E-07
0.64	2.6110E-01	1.40	8.0760E-02	2.16	1.5390E-02	2.92	1.7500E-03	3.68	1.1660E-04	4.44	4.4980E-06	5.20	9.9640E-08
0.65	2.5780E-01	1.41	7.9270E-02	2.17	1.5000E-02	2.93	1.6950E-03	3.69	1.1210E-04	4.45	4.2940E-06	5.21	9.4420E-08
0.66	2.5460E-01	1.42	7.7800E-02	2.18	1.4630E-02	2.94	1.6410E-03	3.70	1.0780E-04	4.46	4.0980E-06	5.22	8.9460E-08
0.67	2.5140E-01	1.43	7.6360E-02	2.19	1.4260E-02	2.95	1.5890E-03	3.71	1.0360E-04	4.47	3.9110E-06	5.23	8.4760E-08
0.68	2.4830E-01	1.44	7.4930E-02	2.20	1.3900E-02	2.96	1.5380E-03	3.72	9.9610E-05	4.48	3.7320E-06	5.24	8.0290E-08
0.69	2.4510E-01	1.45	7.3530E-02	2.21	1.3550E-02	2.97	1.4890E-03	3.73	9.5740E-05	4.49	3.5610E-06	5.25	7.6050E-08
0.70	2.4200E-01	1.46	7.2150E-02	2.22	1.3210E-02	2.98	1.4410E-03	3.74	9.2010E-05				

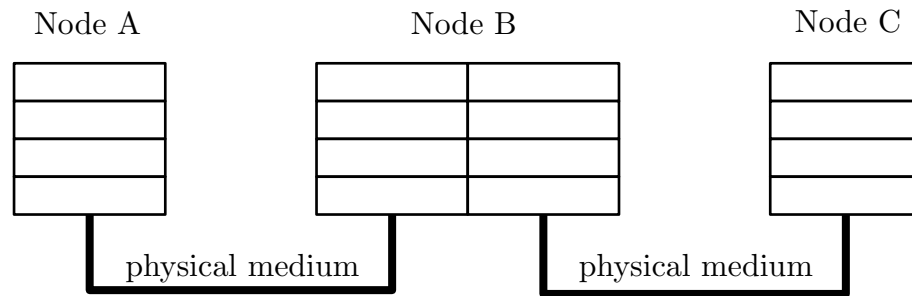
1. Consider transmission of 1500 byte long packets over an AWGN channel with noise power spectral density $N_0/2$ using M -ary PAM, where $N_0 = 1.1 \times 10^{-20}$ W/Hz. The bits that make up the packet are assumed to be independent and equally likely. The received signal power is 10^{-12} W and the packet error probability should not exceed 0.1.
 - (a) Give an expression for the packet error probability in terms of the symbol error probability P_e . (2p)
 - (b) What is the maximum data rate if $M = 2$? (3p)
 - (c) Repeat Part (b) for $M = 4$. (3p)
 - (d) Suppose we use root-raised cosine pulse shapes with roll-off factor $\alpha = 0.1$ and a matched filter receiver. If the transmission should be free of ISI, what is the bandwidth of the transmitted signal in Part (b) and (c)? (2p)

Hint: the symbol error probability for M -ary PAM is

$$P_e = \frac{2(M-1)}{M} Q \left(\sqrt{\frac{6}{M^2-1} \frac{E_s}{N_0}} \right)$$

2.
 - (a) Explain what is piggy-backing is and what the advantage if it is. (2p)
 - (b) Explain what the hidden-terminal problem is and what measures can taken to counteract it. (3p)
 - (c) Consider an Ethernet LAN consisting of a number of hosts with full-duplex connections to an Ethernet switch. Explain why the Ethernet CSMA/CD medium access method is not needed for this setup. (2p)
 - (d) Define the security goal “integrity.” (1p)
 - (e) What is the purpose of the demodulator block in the Shannon communication model? (2p)
 - (f) Describe the service provided by the TCP protocol in the TCP/IP model. (2p)

3. Consider a network consisting of three nodes, the two end-nodes (hosts) A and C, and an intermediate router B.



The protocol stack in each node consists of four layers. In alphabetical order, the layers are the data link (D), network (N), physical (P), and transport (T) layers. For simplicity, we assume that protocol overhead of layer x is in the form of a header field denoted xH , where the layer $x \in \{D, N, P, T\}$. The service data unit of layer x is denoted $xSDU$. A generic data unit consisting of the fields (i.e., headers or SDUs) $F1$, $F2$, and $F3$ is denoted $|F1|F2|F3|$.

The transport layer protocol implements a Stop-and-Wait ARQ mechanism to provide a reliable service. The ACK frame of the protocol is simply the transport layer header (TH) with an empty payload. That is, the transport layer information packet is $|TH|TSDU|$ and the transport layer ACK packet is $|TH|$.

The physical link is full-duplex with data rate 1 Mbit/s.

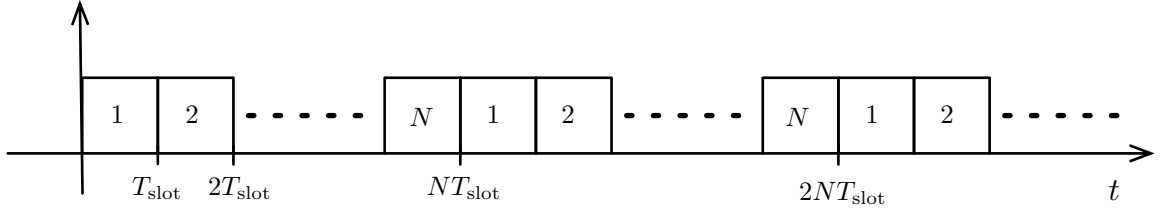
For simplicity, we assume that all data transfer is error-free.

- Copy the figure above and identify the protocol layers in the nodes. (2p)
- Draw a line with arrows to indicate the path a TSDU takes from Node A to Node C. Add another line to show the path of the corresponding transport layer ACK packet. Specify all data units (using the notation $|F1|F2| \dots |Fn|$) that passes over the physical link between Node A and B. (3p)
- Suppose we add a Stop-and-Wait protocol to the link layer as well. The link layer ACK message is just a link layer header with no payload. Note that both the transport layer and the data link layer are executing ARQ protocols. Repeat Part (b), i.e., draw a line with arrows to indicate the path a TSDU takes from Node A to Node C. Add lines with arrows to indicate the paths of all ACK packets that result. Specify all data units that passes over the physical link between Node A and B. (3p)
- Suppose the TSDU is 1500 byte and that all headers are 25 byte long. Assume that the propagation delay of each link is $10 \mu s$ and that processing delays are negligible. Give reasonable time-out values to be used by the transport layer and data link layer ARQ protocols. Motivate. (4p)

4. Consider a network with $N > 1$ users that share access to a wireless channel with total bandwidth W Hz using TDMA or FDMA. Suppose the users want to transmit n_f -bit long packets. The physical layer is able to support a data rate of B bit/s, where B is the channel bandwidth for TDMA or FDMA, respectively.

For the FDMA scheme, the total bandwidth is divided into N frequency channels of bandwidth W/N , and the n th user is allocated to the n th frequency channel.

For the TDMA scheme, the time axis is divided into N time slots of duration T_{slot} , and the n th user is allocated time slots $n, n+N, n+2N$, etc. In particular, User 1 is allocated the slots $[0, T_{\text{slot}})$, $[NT_{\text{slot}}, (N+1)T_{\text{slot}})$, $[2NT_{\text{slot}}, (2N+1)T_{\text{slot}})$, $\dots$, see figure below. The slot time is chosen such it takes exactly K time slots to transmit n_f bits.



The packet delivery time τ is defined as $\tau = t_d - t_a$, where t_a is the time when the packet arrives at the transmitted MAC layer and t_d is the time when it is delivered from the receiver MAC layer. We assume that the processing delays at the MAC and PHY layers are negligible and that the transmission is error-free.

For the purpose of this problem, we will consider the packet delivery time for the first packet from user number 1. That is, User 1 has no packets currently in transmission. The arrival time for the first user's first packet is assumed to be uniformly distributed in the interval $(-NT_{\text{slot}}, 0]$, that is $t_a \sim \mathcal{U}(-NT_{\text{slot}}, 0]$. For the considered TDMA scheme, we cannot transmit during a part of a slot. That is, we cannot start transmission in the middle of a slot.

- Give an expression for the packet delivery time for FDMA. (2p)
- Give expressions for the minimum, average, and maximum packet delivery time for User 1 in the TDMA scheme when $K = 1$. (4p)
- Give expressions for the minimum, average, and maximum packet delivery time for User 1 in the TDMA scheme when $K > 1$. (4p)
- Compare the the minimum, average, and maximum packet delivery time for the TDMA scheme with the FDMA delivery time for $K = 1$ and when $K \rightarrow \infty$. What conclusions can be drawn? (2p)

Hint: if $X \sim \mathcal{U}(a, b]$, then $E[X] = (b - a)/2$.

Exam in SSY305 Kommunikationssystem

Department of Electrical Engineering

Exam date: March 21, 2019, 14:00–18:00

Document updated: March 26, 2019

Teaching Staff

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Material Allowed material is

- Chalmers-approved calculator
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A paper-based dictionary, without added notes (electronic dictionaries are not allowed).

Grading A correct, clear and well-motivated solution gives a maximum of 12 points per problem.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. No fractional points are awarded.

Answers in any other language than Swedish or English are ignored.

Solutions Are made available at the earliest at 19:00 on the course web page.

Results Exam results are posted on Ping-Pong no later than March 27, 2019. The grading review is on March 27 and April 4, 2019, 12:00–13:00 in room 3340 in the ED-building.

Grades The final grade on the course will be decided by the projects (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). Project and exam must be passed (see course-PM for rules). The sum of all scores will decide the grade according to the following table.

Total Score	0–39	40–69	70–79	≥ 80
Grade	Fail	3	4	5

**PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY
ORDERED IN DIFFICULTY.**

Good luck!

Table over the Q-function

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
0.00	5.0000E-01	0.76	2.2360E-01	1.52	6.4260E-02	2.28	1.1300E-02	3.04	1.1830E-03	3.80	7.2350E-05	4.56	2.5580E-06
0.01	4.9600E-01	0.77	2.2060E-01	1.53	6.3010E-02	2.29	1.1010E-02	3.05	1.1440E-03	3.81	6.9480E-05	4.57	2.4390E-06
0.02	4.9200E-01	0.78	2.1770E-01	1.54	6.1780E-02	2.30	1.0720E-02	3.06	1.1070E-03	3.82	6.6730E-05	4.58	2.3250E-06
0.03	4.8800E-01	0.79	2.1480E-01	1.55	6.0570E-02	2.31	1.0440E-02	3.07	1.0700E-03	3.83	6.4070E-05	4.59	2.2160E-06
0.04	4.8400E-01	0.80	2.1190E-01	1.56	5.9380E-02	2.32	1.0170E-02	3.08	1.0350E-03	3.84	6.1520E-05	4.60	2.1120E-06
0.05	4.8010E-01	0.81	2.0900E-01	1.57	5.8210E-02	2.33	9.9030E-03	3.09	1.0010E-03	3.85	5.9060E-05	4.61	2.0130E-06
0.06	4.7610E-01	0.82	2.0610E-01	1.58	5.7050E-02	2.34	9.6420E-03	3.10	9.6760E-04	3.86	5.6690E-05	4.62	1.9190E-06
0.07	4.7210E-01	0.83	2.0330E-01	1.59	5.5920E-02	2.35	9.3870E-03	3.11	9.3540E-04	3.87	5.4420E-05	4.63	1.8280E-06
0.08	4.6810E-01	0.84	2.0050E-01	1.60	5.4800E-02	2.36	9.1370E-03	3.12	9.0430E-04	3.88	5.2230E-05	4.64	1.7420E-06
0.09	4.6410E-01	0.85	1.9770E-01	1.61	5.3700E-02	2.37	8.8940E-03	3.13	8.7400E-04	3.89	5.0120E-05	4.65	1.6600E-06
0.10	4.6020E-01	0.86	1.9490E-01	1.62	5.2620E-02	2.38	8.6560E-03	3.14	8.4470E-04	3.90	4.8100E-05	4.66	1.5810E-06
0.11	4.5620E-01	0.87	1.9220E-01	1.63	5.1550E-02	2.39	8.4240E-03	3.15	8.1640E-04	3.91	4.6150E-05	4.67	1.5060E-06
0.12	4.5220E-01	0.88	1.8940E-01	1.64	5.0500E-02	2.40	8.1980E-03	3.16	7.8880E-04	3.92	4.4270E-05	4.68	1.4340E-06
0.13	4.4830E-01	0.89	1.8670E-01	1.65	4.9470E-02	2.41	7.9760E-03	3.17	7.6220E-04	3.93	4.2470E-05	4.69	1.3660E-06
0.14	4.4430E-01	0.90	1.8410E-01	1.66	4.8460E-02	2.42	7.7600E-03	3.18	7.3600E-04	3.94	4.0740E-05	4.70	1.3010E-06
0.15	4.4040E-01	0.91	1.8140E-01	1.67	4.7460E-02	2.43	7.5490E-03	3.19	7.1140E-04	3.95	3.9080E-05	4.71	1.2390E-06
0.16	4.3640E-01	0.92	1.7880E-01	1.68	4.6480E-02	2.44	7.3440E-03	3.20	6.8710E-04	3.96	3.7470E-05	4.72	1.1790E-06
0.17	4.3250E-01	0.93	1.7620E-01	1.69	4.5510E-02	2.45	7.1430E-03	3.21	6.6370E-04	3.97	3.5940E-05	4.73	1.1230E-06
0.18	4.2860E-01	0.94	1.7360E-01	1.70	4.4570E-02	2.46	6.9470E-03	3.22	6.4100E-04	3.98	3.4460E-05	4.74	1.0690E-06
0.19	4.2470E-01	0.95	1.7110E-01	1.71	4.3630E-02	2.47	6.7560E-03	3.23	6.1900E-04	3.99	3.3040E-05	4.75	1.0170E-06
0.20	4.2070E-01	0.96	1.6850E-01	1.72	4.2720E-02	2.48	6.5690E-03	3.24	5.9760E-04	4.00	3.1670E-05	4.76	9.6800E-07
0.21	4.1680E-01	0.97	1.6600E-01	1.73	4.1820E-02	2.49	6.3870E-03	3.25	5.7700E-04	4.01	3.0360E-05	4.77	9.2110E-07
0.22	4.1290E-01	0.98	1.6350E-01	1.74	4.0930E-02	2.50	6.2100E-03	3.26	5.5710E-04	4.02	2.9100E-05	4.78	8.7650E-07
0.23	4.0900E-01	0.99	1.6110E-01	1.75	4.0060E-02	2.51	6.0370E-03	3.27	5.3770E-04	4.03	2.7890E-05	4.79	8.3390E-07
0.24	4.0520E-01	1.00	1.5870E-01	1.76	3.9200E-02	2.52	5.8680E-03	3.28	5.1900E-04	4.04	2.6730E-05	4.80	7.9330E-07
0.25	4.0130E-01	1.01	1.5620E-01	1.77	3.8360E-02	2.53	5.7030E-03	3.29	5.0090E-04	4.05	2.5610E-05	4.81	7.5470E-07
0.26	3.9740E-01	1.02	1.5390E-01	1.78	3.7540E-02	2.54	5.5430E-03	3.30	4.8340E-04	4.06	2.4540E-05	4.82	7.1780E-07
0.27	3.9360E-01	1.03	1.5150E-01	1.79	3.6730E-02	2.55	5.3860E-03	3.31	4.6650E-04	4.07	2.3510E-05	4.83	6.8270E-07
0.28	3.8970E-01	1.04	1.4920E-01	1.80	3.5930E-02	2.56	5.2340E-03	3.32	4.5010E-04	4.08	2.2520E-05	4.84	6.4920E-07
0.29	3.8590E-01	1.05	1.4690E-01	1.81	3.5150E-02	2.57	5.0850E-03	3.33	4.3420E-04	4.09	2.1570E-05	4.85	6.1730E-07
0.30	3.8210E-01	1.06	1.4460E-01	1.82	3.4380E-02	2.58	4.9400E-03	3.34	4.1890E-04	4.10	2.0660E-05	4.86	5.8690E-07
0.31	3.7830E-01	1.07	1.4230E-01	1.83	3.3620E-02	2.59	4.7990E-03	3.35	4.0410E-04	4.11	1.9780E-05	4.87	5.5800E-07
0.32	3.7450E-01	1.08	1.4010E-01	1.84	3.2880E-02	2.60	4.6610E-03	3.36	3.8970E-04	4.12	1.8940E-05	4.88	5.3040E-07
0.33	3.7070E-01	1.09	1.3790E-01	1.85	3.2160E-02	2.61	4.5270E-03	3.37	3.7580E-04	4.13	1.8140E-05	4.89	5.0420E-07
0.34	3.6690E-01	1.10	1.3570E-01	1.86	3.1440E-02	2.62	4.3960E-03	3.38	3.6240E-04	4.14	1.7370E-05	4.90	4.7920E-07
0.35	3.6320E-01	1.11	1.3350E-01	1.87	3.0740E-02	2.63	4.2690E-03	3.39	3.4950E-04	4.15	1.6620E-05	4.91	4.5540E-07
0.36	3.5940E-01	1.12	1.3140E-01	1.88	3.0050E-02	2.64	4.1450E-03	3.40	3.3690E-04	4.16	1.5910E-05	4.92	4.3270E-07
0.37	3.5570E-01	1.13	1.2920E-01	1.89	2.9380E-02	2.65	4.0250E-03	3.41	3.2480E-04	4.17	1.5230E-05	4.93	4.1110E-07
0.38	3.5200E-01	1.14	1.2710E-01	1.90	2.8720E-02	2.66	3.9070E-03	3.42	3.1310E-04	4.18	1.4580E-05	4.94	3.9060E-07
0.39	3.4830E-01	1.15	1.2510E-01	1.91	2.8070E-02	2.67	3.7930E-03	3.43	3.0180E-04	4.19	1.3950E-05	4.95	3.7110E-07
0.40	3.4460E-01	1.16	1.2300E-01	1.92	2.7430E-02	2.68	3.6810E-03	3.44	2.9090E-04	4.20	1.3350E-05	4.96	3.5250E-07
0.41	3.4090E-01	1.17	1.2100E-01	1.93	2.6800E-02	2.69	3.5730E-03	3.45	2.8030E-04	4.21	1.2770E-05	4.97	3.3480E-07
0.42	3.3720E-01	1.18	1.1900E-01	1.94	2.6190E-02	2.70	3.4670E-03	3.46	2.7010E-04	4.22	1.2220E-05	4.98	3.1790E-07
0.43	3.3360E-01	1.19	1.1700E-01	1.95	2.5590E-02	2.71	3.3640E-03	3.47	2.6020E-04	4.23	1.1680E-05	4.99	3.0190E-07
0.44	3.3000E-01	1.20	1.1510E-01	1.96	2.5000E-02	2.72	3.2640E-03	3.48	2.5070E-04	4.24	1.1180E-05	5.00	2.8670E-07
0.45	3.2640E-01	1.21	1.1310E-01	1.97	2.4420E-02	2.73	3.1670E-03	3.49	2.4150E-04	4.25	1.0690E-05	5.01	2.7220E-07
0.46	3.2280E-01	1.22	1.1120E-01	1.98	2.3850E-02	2.74	3.0720E-03	3.50	2.3260E-04	4.26	1.0220E-05	5.02	2.5840E-07
0.47	3.1920E-01	1.23	1.0930E-01	1.99	2.3300E-02	2.75	2.9800E-03	3.51	2.2410E-04	4.27	9.7740E-06	5.03	2.4520E-07
0.48	3.1560E-01	1.24	1.0750E-01	2.00	2.2750E-02	2.76	2.8900E-03	3.52	2.1580E-04	4.28	9.3450E-06	5.04	2.3280E-07
0.49	3.1210E-01	1.25	1.0560E-01	2.01	2.2220E-02	2.77	2.8030E-03	3.53	2.0780E-04	4.29	8.9340E-06	5.05	2.2090E-07
0.50	3.0850E-01	1.26	1.0380E-01	2.02	2.1690E-02	2.78	2.7180E-03	3.54	2.0010E-04	4.30	8.5400E-06	5.06	2.0960E-07
0.51	3.0500E-01	1.27	1.0200E-01	2.03	2.1180E-02	2.79	2.6350E-03	3.55	1.9260E-04	4.31	8.1630E-06	5.07	1.9890E-07
0.52	3.0150E-01	1.28	1.0030E-01	2.04	2.0680E-02	2.80	2.5550E-03	3.56	1.8540E-04	4.32	7.8010E-06	5.08	1.8870E-07
0.53	2.9810E-01	1.29	9.8530E-02	2.05	2.0180E-02	2.81	2.4770E-03	3.57	1.7850E-04	4.33	7.4550E-06	5.09	1.7900E-07
0.54	2.9460E-01	1.30	9.6800E-02	2.06	1.9700E-02	2.82	2.4010E-03	3.58	1.7180E-04	4.34	7.1240E-06	5.10	1.6980E-07
0.55	2.9120E-01	1.31	9.5100E-02	2.07	1.9230E-02	2.83	2.3270E-03	3.59	1.6530E-04	4.35	6.8070E-06	5.11	1.6110E-07
0.56	2.8770E-01	1.32	9.3420E-02	2.08	1.8760E-02	2.84	2.2560E-03	3.60	1.5910E-04	4.36	6.5030E-06	5.12	1.5280E-07
0.57	2.8430E-01	1.33	9.1760E-02	2.09	1.8310E-02	2.85	2.1860E-03	3.61	1.5310E-04	4.37	6.2120E-06	5.13	1.4490E-07
0.58	2.8100E-01	1.34	9.0120E-02	2.10	1.7860E-02	2.86	2.1180E-03	3.62	1.4730E-04	4.38	5.9340E-06	5.14	1.3740E-07
0.59	2.7760E-01	1.35	8.8510E-02	2.11	1.7430E-02	2.87	2.0520E-03	3.63	1.4170E-04	4.39	5.6680E-06	5.15	1.3020E-07
0.60	2.7430E-01	1.36	8.6910E-02	2.12	1.7000E-02	2.88	1.9880E-03	3.64	1.3630E-04	4.40	5.4130E-06	5.16	1.2350E-07
0.61	2.7090E-01	1.37	8.5340E-02	2.13	1.6590E-02	2.89	1.9260E-03	3.65	1.3110E-04	4.41	5.1690E-06	5.17	1.1700E-07
0.62	2.6760E-01	1.38	8.3790E-02	2.14	1.6180E-02	2.90	1.8660E-03	3.66	1.2610E-04	4.42	4.9350E-06	5.18	1.1090E-07
0.63	2.6430E-01	1.39	8.2260E-02	2.15	1.5780E-02	2.91	1.8070E-03	3.67	1.2130E-04	4.43	4.7120E-06	5.19	1.0510E-07
0.64	2.6110E-01	1.40	8.0760E-02	2.16	1.5390E-02	2.92	1.7500E-03	3.68	1.1660E-04	4.44	4.4980E-06	5.20	9.9640E-08
0.65	2.5780E-01	1.41	7.9270E-02	2.17	1.5000E-02	2.93	1.6950E-03	3.69	1.1210E-04	4.45	4.2940E-06	5.21	9.4420E-08
0.66	2.5460E-01	1.42	7.7800E-02	2.18	1.4630E-02	2.94	1.6410E-03	3.70	1.0780E-04	4.46	4.0980E-06	5.22	8.9460E-08
0.67	2.5140E-01	1.43	7.6360E-02	2.19	1.4260E-02	2.95	1.5890E-03	3.71	1.0360E-04	4.47	3.9110E-06	5.23	8.4760E-08
0.68	2.4830E-01	1.44	7.4930E-02	2.20	1.3900E-02	2.96	1.5380E-03	3.72	9.9610E-05	4.48	3.7320E-06	5.24	8.0290E-08
0.69	2.4510E-01	1.45	7.3530E-02	2.21	1.3550E-02	2.97	1.4890E-03	3.73	9.5740E-05	4.49	3.5610E-06	5.25	7.6050E-08
0.70	2.4200E-01	1.46	7.2150E-02	2.22	1.3210E-02	2.98	1.4410E-03	3.74	9.2010E-05				

1. Consider transmission of 1500 byte long packets over an AWGN channel with noise power spectral density $N_0/2$ using M -ary PAM, where $N_0 = 1.1 \times 10^{-20}$ W/Hz. The bits that make up the packet are assumed to be independent and equally likely. The received signal power is 10^{-12} W and the packet error probability should not exceed 0.1.
 - (a) Give an expression for the packet error probability in terms of the symbol error probability P_e . (2p)
 - (b) What is the maximum data rate if $M = 2$? (4p)
 - (c) Repeat Part (b) for $M = 4$. (4p)
 - (d) Suppose we use root-raised cosine pulse shapes with roll-off factor $\alpha = 0.1$ and a matched filter receiver. If the transmission should be free of ISI, what is the bandwidth of the transmitted signal in Part (b) and (c)? (2p)

Hint: the symbol error probability for M -ary PAM is

$$P_e = \frac{2(M-1)}{M} Q \left(\sqrt{\frac{6}{M^2-1} \frac{E_s}{N_0}} \right)$$

Solution:

- (a) We recall that the symbol energy is $E_s = P/R_s$, where P is the received power and R_s is the symbol rate. Since symbol errors are independent for PAM over a baseband AWGN channel (assuming that there is no ISI), the packet error probability is $P_p = 1 - (1 - P_e)^{n_s}$, where P_e is the symbol error probability and n_s is the number of symbols in the packet. Clearly, $n_s = 8n_b/\log_2(M)$, where n_b is the number of bytes in the packet and $\log_2(M)$ is the number of bits per symbol. Putting everything together, we can compute the symbol error probability as

$$\begin{aligned} P_p &= 1 - (1 - P_e)^{n_s} \Rightarrow \\ P_e &= 1 - (1 - P_p)^{\log_2(M)/(8n_b)} \end{aligned} \quad (1)$$

- (b) We start by solving for E_s as

$$\begin{aligned} P_e &= \frac{2(M-1)}{M} Q \left(\sqrt{\frac{6}{M^2-1} \frac{E_s}{N_0}} \right) \Rightarrow \\ E_s &= N_0 \frac{M^2-1}{6} \left[Q^{-1} \left(\frac{M}{2(M-1)} P_e \right) \right]^2 \end{aligned} \quad (2)$$

Finally, the data rate R_b (in bit/s) can be written as $R_b = R_s \log_2(M)$, and since $R_s = P/E_s$, we have that

$$R_b = R_s \log_2(M) = \frac{P}{E_s} \log_2(M) \quad (3)$$

By substituting the known quantities $n_b = 1500$, $P_p = 0.1$, $N_0 = 1.1 \times 10^{-20}$ in Equations (1)–(3), we can compute the maximum data rates for $M = 2$ as $R_b = 9.82$ Mbit/s.

- (c) Repeating the solution for Part (a) with $M = 4$ yields the data rate $R_b = 4.05$ Mbit/s. The reason for the lower data rate with $M = 4$ compared to $M = 2$ is that a higher E_b/N_0 is required to reach the packet error target for $M = 4$ compared to $M = 2$ (recall that the power efficiency for M -ary PAM is decreasing with M). Hence, for a fixed received power, the $M = 4$ must operate at a lower data rate to reach the required E_b/N_0 compared to the $M = 2$ system.

- (d) The bandwidth for an M -ary PAM system with symbol rate R_s and RRC pulse shaping with roll-off factor α is $W = R_s(1 + \alpha)/2$. Hence, the bandwidth required is 5.40 MHz and 1.11 MHz for $M = 2$ and $M = 4$, respectively. As expected, the bandwidth requirement is less for the $M = 4$ system compared to the $M = 2$ system (recall that the spectral efficiency for M -ary PAM is increasing with M).

2. (a) Explain what is piggy-backing is and what the advantage if it is. (2p)

Solution: if we use an ARQ protocol, the transmitting node sends information frames to the receiving node and ACK frames go in the opposite direction. Now, suppose Node A and Node B transmit data frames in both directions, then ACK frames also go in both directions. We can then embed the ACK-frame payloads into data frames (i.e., let the ACK-frame payload piggy-back onto the data frame). This will overall save communication resources, since there is no need for the overhead (addresses, etc.) needed to transmit explicit ACK frames.

- (b) Explain what the hidden-terminal problem is and what measures can be taken to counteract it. (3p)

Solution: the hidden-terminal problem occurs in carrier-sense multiple access systems when an on-going transmission might not be detected by all nodes. This is a problem in practice for Wi-Fi networks. Suppose Node A and Node B are out of radio range of each other (i.e., they cannot detect each other's transmissions). However, the nodes are in radio range of the access point (AP). Suppose that Node A transmits a frame to the AP. During the transmission, Node B senses the channel to determine if it also can transmit a frame. Node B will not hear Node A's transmission and therefore thinks (wrongly) that the channel is idle and starts a transmission. Node A and Node B's transmissions will then collide at the AP, which could lead to that one or both of the frames are lost. Here we say that Node A is a *hidden terminal* to Node B. To counteract this problem, we can use a request-to-send/clear-to-send (RTS/CTS) handshake procedure. The handshake works like this: if Node A wants to transmit a data frame to the AP, it first transmits a RTS frame to the AP. The AP answers the request by sending a CTS frame to Node A. The latter transmission, i.e., the CTS from the AP to Node A is also picked up by Node B. Hence, Node B now knows that the channel will be busy, in spite of the fact that Node B did not hear the RTS packet. (The CTS packet contains information on how long the channel will be busy in order to complete transmission of the subsequent information frame from Node A.) The RTS/CTS procedure is not perfect since simultaneously transmitted RTS frames from Node A and Node B can collide. However, since the RTS/CTS frames are much shorter than typical data frames, the probability for RTS collision is smaller than the probability for data frame collisions (when RTS/CTS is not used). However, RTS/CTS implies a significant overhead and is not used very often in practice.

- (c) Consider an Ethernet LAN consisting of a number of hosts with full-duplex connections to an Ethernet switch. Explain why the Ethernet CSMA/CD medium access method is not needed for this setup. (2p)

Solution: since all the hosts are connected to the switch with full duplex point-to-point links, collisions can't occur. Hence there is no need to have a collision detection mechanism. In such setting, the only event that may result in a frame loss, is buffer overflowing at the switch.

- (d) Define the security goal "integrity". (1p)

Solution: the goal of integrity is to "guarantee that messages have not been altered in transmission" by a third party.

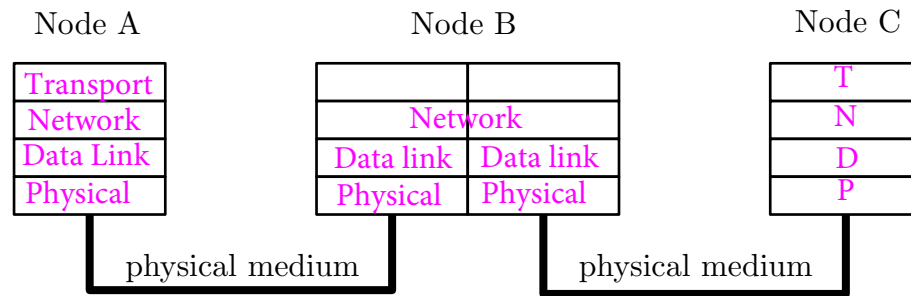
- (e) What is the purpose of the demodulator block in the Shannon communication model? (2p)

Solution: the purpose of the demodulator is to convert the received analog signal to decisions on the transmitted bits.

- (f) Describe the service provided by the TCP protocol in the TCP/IP model. (2p)

Solution: TCP provides a connection-oriented, full-duplex, reliable stream transmission service. This is realized using SNs, ACKs, error check codes, and re-transmissions.

3. Consider a network consisting of three nodes, the two end-nodes (hosts) A and C, and an intermediate router B.



The protocol stack in each node consists of four layers. In alphabetical order, the layers are the data link (D), network (N), physical (P), and transport (T) layers. For simplicity, we assume that protocol overhead of layer x is in the form of a header field denoted xH , where the layer $x \in \{D, N, P, T\}$. The service data unit of layer x is denoted $xSDU$. A generic data unit consisting of the fields (i.e., headers or SDUs) F1, F2, and F3 is denoted $|F1|F2|F3|$.

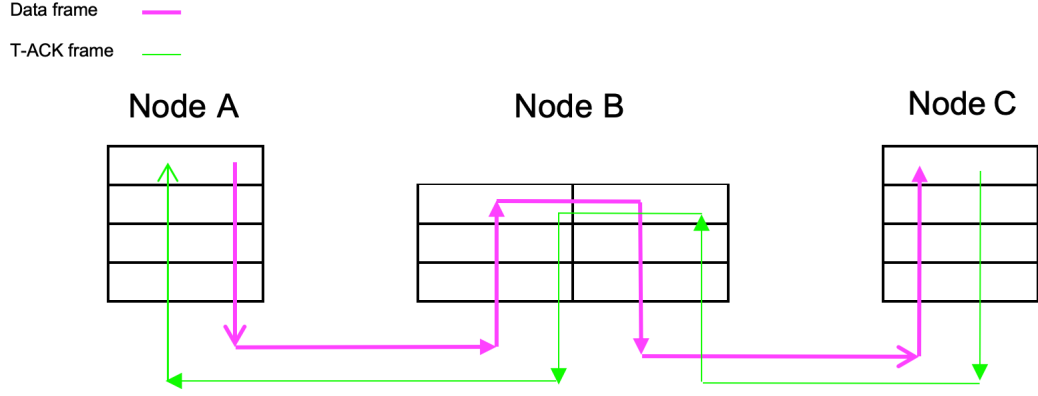
The transport layer protocol implements a Stop-and-Wait ARQ mechanism to provide a reliable service. The ACK frame of the protocol is simply the transport layer header (TH) with an empty payload. That is, the transport layer information packet is $|TH|TSDU|$ and the transport layer ACK packet is $|TH|$.

The physical link is full-duplex with data rate 1 Mbit/s.

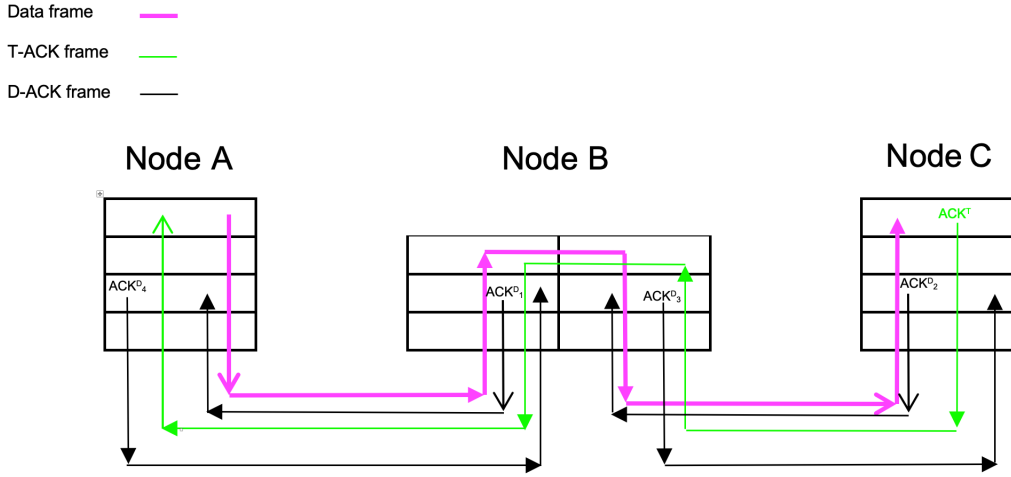
For simplicity, we assume that all data transfer is error-free.

- Copy the figure above and identify the protocol layers in the nodes. (2p)
- Draw a line with arrows to indicate the path a TSDU takes from Node A to Node C. Add another line to show the path of the corresponding transport layer ACK packet. Specify all data units (using the notation $|F1|F2|\dots|Fn|$) that passes over the physical link between Node A and B. (3p)
- Suppose we add a Stop-and-Wait protocol to the link layer as well. The link layer ACK message is just a link layer header with no payload. Note that both the transport layer and the data link layer are executing ARQ protocols. Repeat Part (b), i.e., draw a line with arrows to indicate the path a TSDU takes from Node A to Node C. Add lines with arrows to indicate the paths of all ACK packets that result. Specify all data units that passes over the physical link between Node A and B. (3p)
- Suppose the TSDU is 1500 byte and that all headers are 25 byte long. Assume that the propagation delay of each link is $10 \mu s$ and that processing delays are negligible. Give reasonable time-out values to be used by the transport layer and data link layer ARQ protocols. Motivate. (4p)

- Check the figure above. Note that Node B has only 3 layers. Also, it may have 2 different network interfaces (Data link and Physical layers)
- There are two data units types in this scenario:
 Transport layer data frame : $|DH|NH|TH|TSDU|$
 Transport layer ACK frame: $|DH|NH|TH|$



- (c) We have three data units types in this scenario.
 Transport layer data frame : |DH|NH|TH|TSDU|
 Transport layer ACK frame: |DH|NH|TH|
 Data link layer ACK frame: |DH|

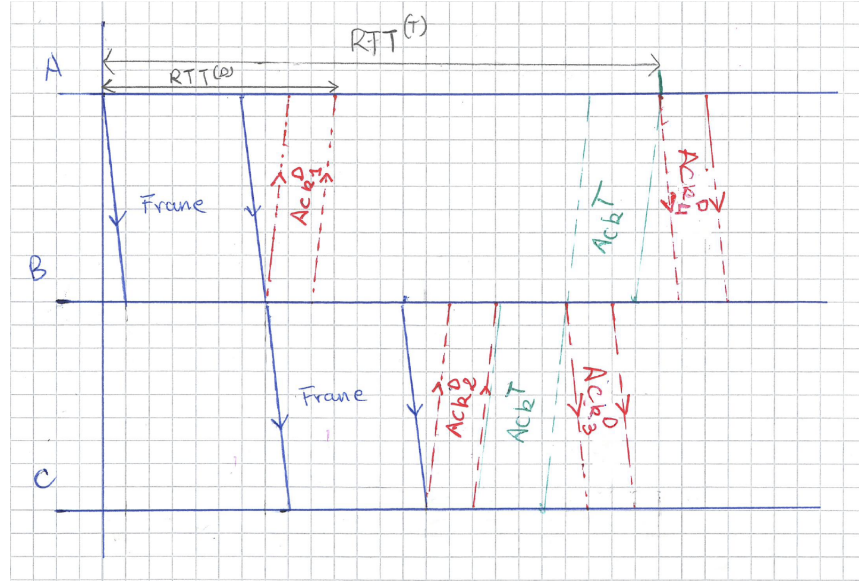


- (d) A reasonable time-out for the data link layer's Stop and Wait (S&W) ARQ need to be deduced based on the round-trip-time: $RTT^{(D)} = t_f + t_{a,D} + 2t_{\text{prop}}$ where t_f is information frame duration and $t_{a,D}$ data link ACK frame duration. Note that this round-trip time is the same in both hops (A to B and B to C). Let n_{SDU} be the length of the transport layer SDU in bits, and n_H be the length of any layer's header in bits. Then,

$$\begin{aligned}
 t_f &= \frac{n_f}{R} = \frac{n_{SDU} + 3n_H}{R} = \frac{1575 \times 8}{10^8} = 12.6 \text{ ms} \\
 t_{a,D} &= \frac{n_H}{R} = \frac{25 \times 8}{10^8} = 0.2 \text{ ms} \\
 RTT^{(D)} &= 12.82 \text{ ms}
 \end{aligned} \tag{4}$$

In absence of errors, a reasonable time-out for the data link ARQ is $t_{out}^{(D)} = RTT^{(D)} = 12.82 \text{ ms}$

The round-trip time of the transport layer $RTT^{(T)}$ can be computed from the figure below. Note that we assume that the Data link ACK is transmitted first before the transport layer ACK (from C to B).



Let $t_{a,T}$ be the duration of a transport layer ACK. We can readily write

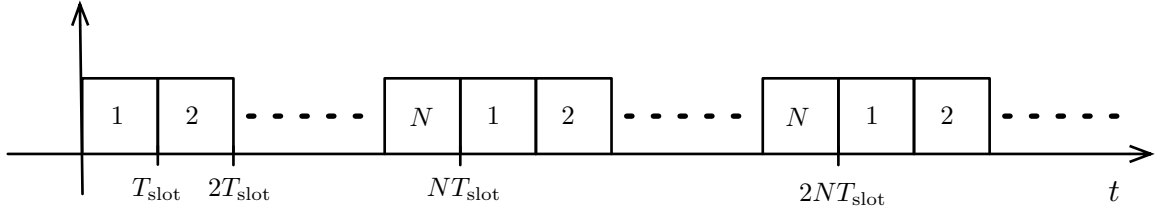
$$\begin{aligned}
 RTT^{(T)} &= 2t_f + 2t_{a,T} + t_{a,D} + 4t_{\text{prop}} \\
 t_{a,T} &= \frac{3 \times n_H}{R} = 0.6 \text{ ms} \\
 RTT^{(T)} &= 26.64 \text{ ms}
 \end{aligned} \tag{5}$$

In absence of errors, a reasonable choice of the transport layer ARQ's time-out is $t_{out}^{(T)} = RTT^{(T)} = 26.64 \text{ ms}$

4. Consider a network with $N > 1$ users that share access to a wireless channel with total bandwidth W Hz using TDMA or FDMA. Suppose the users want to transmit n_f -bit long packets. The physical layer is able to support a data rate of B bit/s, where B is the channel bandwidth for TDMA or FDMA, respectively.

For the FDMA scheme, the total bandwidth is divided into N frequency channels of bandwidth W/N , and the n th user is allocated to the n th frequency channel.

For the TDMA scheme, the time axis is divided into N time slots of duration T_{slot} , and the n th user is allocated time slots $n, n+N, n+2N$, etc. In particular, User 1 is allocated the slots $[0, T_{\text{slot}})$, $[NT_{\text{slot}}, (N+1)T_{\text{slot}})$, $[2NT_{\text{slot}}, (2N+1)T_{\text{slot}})$, $\dots$, see figure below. The slot time is chosen such it takes exactly K time slots to transmit n_f bits.



The packet delivery time τ is defined as $\tau = t_d - t_a$, where t_a is the time when the packet arrives at the transmitted MAC layer and t_d is the time when it is delivered from the receiver MAC layer. We assume that the processing delays at the MAC and PHY layers are negligible and that the transmission is error-free.

For the purpose of this problem, we will consider the packet delivery time for the first packet from user number 1. That is, User 1 has no packets currently in transmission. The arrival time for the first user's first packet is assumed to be uniformly distributed in the interval $(-NT_{\text{slot}}, 0]$, that is $t_a \sim \mathcal{U}(-NT_{\text{slot}}, 0]$. For the considered TDMA scheme, we cannot transmit during a part of a slot. That is, we cannot start transmission in the middle of a slot.

- Give an expression for the packet delivery time for FDMA. (2p)
- Give expressions for the minimum, average, and maximum packet delivery time for User 1 in the TDMA scheme when $K = 1$. (4p)
- Give expressions for the minimum, average, and maximum packet delivery time for User 1 in the TDMA scheme when $K > 1$. (4p)
- Compare the the minimum, average, and maximum packet delivery time for the TDMA scheme with the FDMA delivery time for $K = 1$ and when $K \rightarrow \infty$. What conclusions can be drawn? (2p)

Hint: if $X \sim \mathcal{U}(a, b]$, then $E[X] = (b - a)/2$.

Solution

- The channel bandwidth for FDMA is W/N , hence the physical layer can support a data rate of $R_{\text{FDMA}} = \frac{W}{N}$ bits/s. Since the channel is dedicated for the user, transmission can start right away after packet arrival. Then, the packet delivery time is equal to the packet transmission time, that is

$$\tau_{\text{FDMA}} = \frac{n_f}{R_{\text{FDMA}}} = \frac{Nn_f}{W}.$$

- (b) In the TDMA scheme each user uses the full bandwidth but only for T_{slot} sec. We first start by finding the length of T_{slot} . We know that each user is served by $R_{\text{TDMA}} = W$ bits/s, and in the general case, K slots are required to send the full packet, hence

$$\begin{aligned} KT_{\text{slot}} &= \frac{n_f}{W} \\ T_{\text{slot}} &= \frac{n_f}{KW} \end{aligned} \quad (6)$$

Now in the case $K = 1$, since $t_a > -NT_{\text{slot}}$, User 1 will wait until $t=0$ to start transmitting (because it has missed its slot in the previous frame). The transmission will finish at $t_d = T_{\text{slot}}$. Then the packet delivery time can be readily computed

$$\begin{aligned} \tau_{\text{TDMA}} &= t_d - t_a \\ &= T_{\text{slot}} - t_a \end{aligned} \quad (7)$$

Then, the minimum, maximum and average times are

$$\begin{aligned} \tau_{\text{TDMA}}^{(\min)} &= \min \tau_{\text{TDMA}} = T_{\text{slot}} - \max t_a = T_{\text{slot}} \\ \tau_{\text{TDMA}}^{(\max)} &= \max \tau_{\text{TDMA}} = T_{\text{slot}} - \min t_a = T_{\text{slot}} + NT_{\text{slot}} \\ \bar{\tau}_{\text{TDMA}} &= E[\tau_{\text{TDMA}}] = T_{\text{slot}} + E[t_a] = T_{\text{slot}} + \frac{N}{2}T_{\text{slot}} \end{aligned}$$

- (c) In case $K > 1$, User 1 – as previously – will not be able to start transmitting until $t = 0$. Then, KT_{slot} are necessary to finish the packet transmission. The first slot of User 1 is the interval $[0, T_{\text{slot}})$, the second is $[NT_{\text{slot}}, NT_{\text{slot}} + T_{\text{slot}})$ and the K^{th} slot is in $[(K-1)NT_{\text{slot}}, (K-1)NT_{\text{slot}} + T_{\text{slot}})$. User 1 finishes its transmission after transmitting the K^{th} slot, hence in this case $t_d = (K-1)NT_{\text{slot}} + T_{\text{slot}}$. To compute the packet delivery time τ'_{TDMA} we need to accommodate for the time User 1 waited for before $t = 0$, that is

$$\tau'_{\text{TDMA}} = (K-1)NT_{\text{slot}} + T_{\text{slot}} - t_a \quad (8)$$

Then, the minimum, average and maximum packet delivery times are

$$\begin{aligned} \tau'_{\text{TDMA}}^{(\min)} &= \min \tau'_{\text{TDMA}} = (K-1)NT_{\text{slot}} + T_{\text{slot}} - \max t_a = (K-1)NT_{\text{slot}} + T_{\text{slot}} \\ \tau'_{\text{TDMA}}^{(\max)} &= \max \tau'_{\text{TDMA}} = (K-1)NT_{\text{slot}} + T_{\text{slot}} - \min t_a = KNT_{\text{slot}} + T_{\text{slot}} \\ \bar{\tau}'_{\text{TDMA}} &= E[\tau'_{\text{TDMA}}] = (K-1)NT_{\text{slot}} + T_{\text{slot}} - E[t_a] = (K-1/2)NT_{\text{slot}} + T_{\text{slot}} \end{aligned}$$

- (d) We use (6) to express the packet delivery time in terms of W , n_f and N . Hence, $\lim_{K \rightarrow \infty} KT_{\text{slot}} = n_f/W$ and $\lim_{K \rightarrow \infty} T_{\text{slot}} = 0$. We summarize the results in the following table.

	$\tau_{\text{TDMA}} (K = 1)$	$\tau'_{\text{TDMA}} (K \rightarrow \infty)$	τ_{FDMA}
Minimum	$\frac{n_f}{W}$	$\rightarrow N\frac{n_f}{W}$	$N\frac{n_f}{W}$
Maximum	$(N+1)\frac{n_f}{W}$	$\rightarrow N\frac{n_f}{W}$	$N\frac{n_f}{W}$
Average	$(N/2+1)\frac{n_f}{W}$	$\rightarrow N\frac{n_f}{W}$	$N\frac{n_f}{W}$

We can note that for $K = 1$, the TDMA average packet delivery time is less than the FDMA's packet delivery time (for $N > 2$). However, as K increases the packet delivery time of the TDMA scheme converges to the packet delivery time of the FDMA scheme from below.

We can conclude that if the time slot is designed such that the packet can be transmitted within one T_{slot} , i.e., when $K = 1$, TDMA has on average better packet delivery time. However, if the packet requires many T_{slot} for transfer, i.e., when K becomes very large, then TDMA has no significant advantage compared to FDMA in terms of the packet delivery time.

This can be seen from another perspective. We can interpret that $K \rightarrow \infty$ means that users in the system have steady traffic, hence users have always some data to transmit. In such data traffic model, TDMA and FDMA have similar performance in terms of packet delivery time. (This was seen in Problem set 8, P5)