Name:

Exam in the course Antenna Engineering 2011-05-28

ANTENNA ENGINEERING (SSY100)
(E4) 2010/11 (Period IV)

Saturday 28 May 1400-1800 hours.

Teachers: Adjunct Prof Jan Carlsson, Prof Per-Simon Kildal, Associate Prof Jian Yang
Questions: Jan Carlsson, tel. 0703 665169

The exam consists of 2 parts. Part A is printed on colored paper and must be solved without using the textbook. When you have delivered the colored text and the solutions of Part A (latest 17:00), the textbook can be used for Part B of the exam.

You are allowed to use the following:

For Part A:	**Pocket calculator of your own choice**
For Part B only:	**Mathematical tables including Beta**
	Pocket calculator of your own choice
	Kildal's compendium "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(The textbook can contain own notes and marks on its original printed pages. No other notes are allowed.)

Tentamen består av 2 delar. Del A har tryckts på färgade papper och skall lösas utan att använda läroboken. När du har inlämnat dom färgade arken med uppgifterna för del A och dina svar på dessa uppgifter (senast 17:00), kan du ta fram läroboken för att lösa del B.

Tillåtna hjälpmedel:

För del A:	Valfri räknedosa
För del B:	Matematiska Tabeller inkluderad Beta
	Valfri räknedosa
	Kildals lärobok "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(Boken kan innehålla egna notater skrivna på dom inbundna sidorna. Extra ark med notater tillåts inte.)

Name:

Name:

PART A (must be delivered before textbook can be used)

1.0 On Labs (25p)

1.1. Some questions about Lab1. Be careful that you use the correct terminology from the course literature.

1.1.1. Which environment is emulated by reverberation rhamber and which environment is emulated by anechoic chamber? Explain.
1.1.2. Which quantities can we measure in reverberation chamber, and which ones can we measure in anechoic chamber?
1.1.3. Explain the different steps involved when measuring in a reverberation chamber.

1.2. In Lab 2, you have measured a standard gain horn and a microstrip slot array antenna; see Fig. 1.1. The measured radiation patterns are shown in Fig. 1.2 and Fig. 1.3.

1.2.1. Which of the two figures show radiation patterns for standard gain horn and which one for microstrip slot array antenna? Explain briefly.
1.2.2. Please sketch (redraw) the patterns and mark which curve (blue or red in the original graphs) is for E-plane and which one is for H-plane. Explain briefly.
1.2.3. In Fig. 1.2, the radiation pattern in one plane (red curve) has much broader beam width than the other, why?
1.2.4. In Fig.1.3, the radiation pattern in one plane (red curve) has lower first sidelobe than the other, why?
1.2.5. What is the length of the slot in the slot array antenna in mm? The operating frequency for the slot array is 9.5 GHz.
1.2.6. As you know, the slot array antenna has the maximum radiation direction at 30^o from the broadside. What is the maximum allowed spacing between the slots to avoid grating lobes? If you do not remember the complete formula, you can give the value for the full scan case. Explain.

1.3. In Lab 3, you could choose to realize one of several different patch antennas; see Fig.1.4. Explain which polarization(s) each of these patch antenna shapes radiate, i.e. horizontal linear, vertical linear or circular, provided the dimensions are properly chosen.

Name:


Standard gain horn

Microstrip slot array antenna

Figure 1.1 Antennas in Lab2.

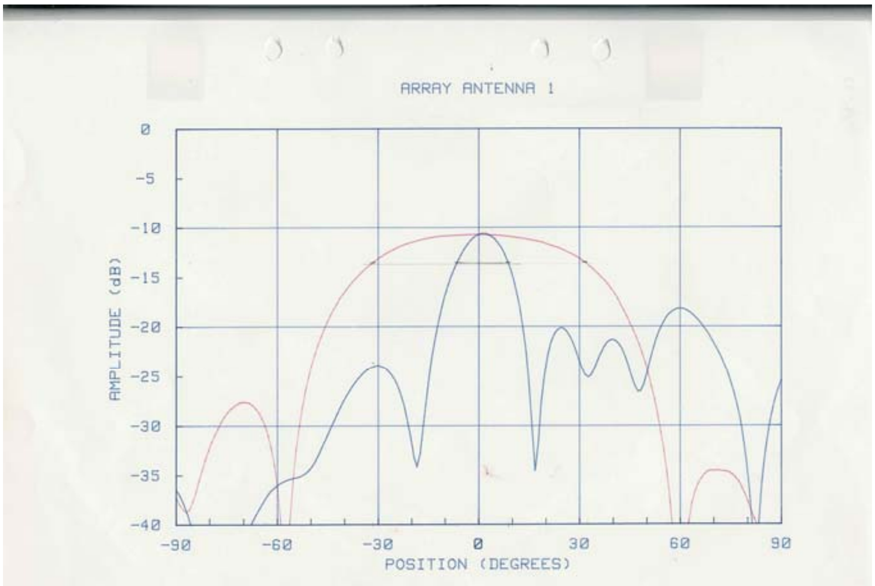

Figure 1.2 Measured radiation patterns on both E- and H-planes for antenna 1

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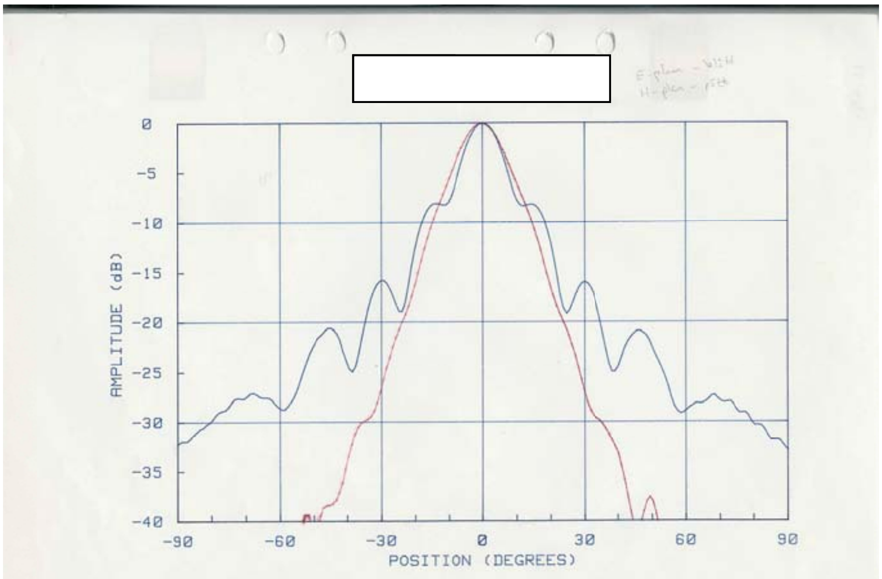

Fig. 1.3 Measured radiation patterns on both E- and H-planes for antenna 2

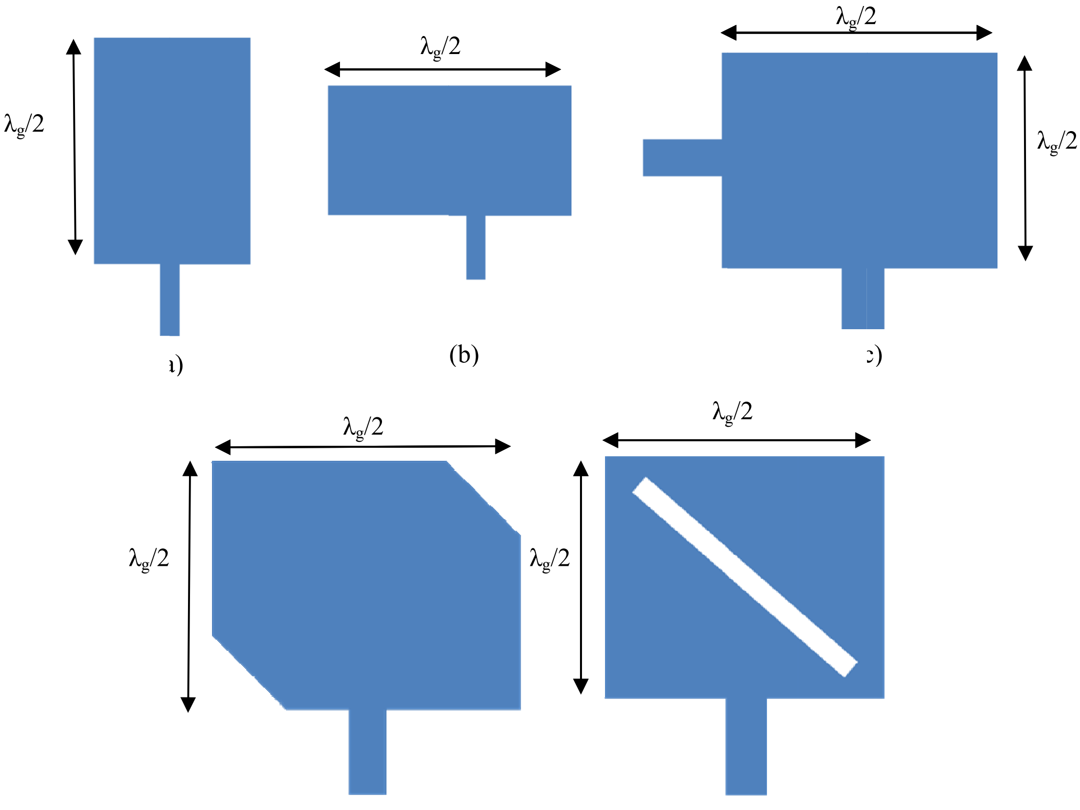

Figure 1.4 different patch antennas.

Name:

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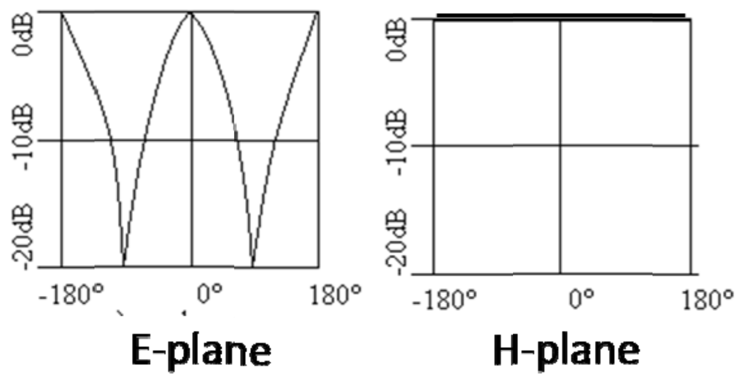
2.0 Far-field functions of different antennas (25p)

In Home assignment 2, you were asked to sketch the co- and cross- polar radiation patterns in E- and H- planes for several different basic antennas. However, in practice, the question is opposite: you have some specifications on antenna's radiation patterns from customers, and you should choose proper antennas to fulfill the specifications. For each of the following questions, the co-polar radiation patterns in E- and H-planes are given. You should

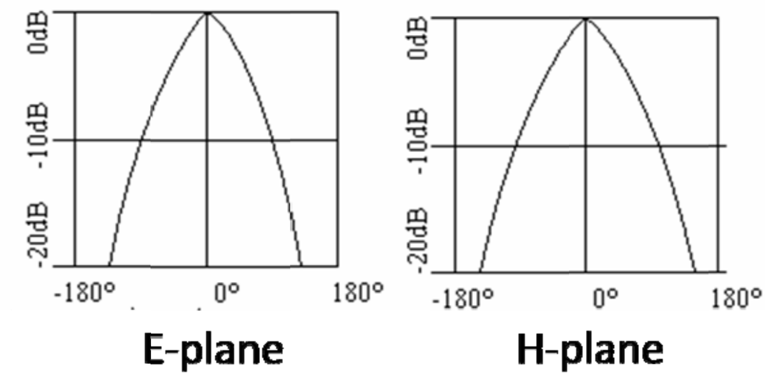
a) Choose a proper antenna that can provide the required radiation patterns. Write the name of the antenna or incremental source;
b) Sketch a coordinate system and draw your antenna in it, and indicate the polarization.
c) Explain which planes are E-plane and H-plane.

Note that there may exist more than one solution for the same radiation pattern requirement.

2.1 Radiation patterns:



2.2 Radiation patterns:



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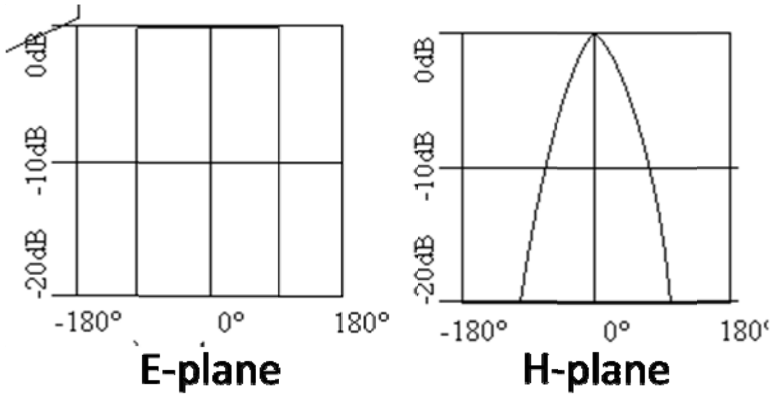
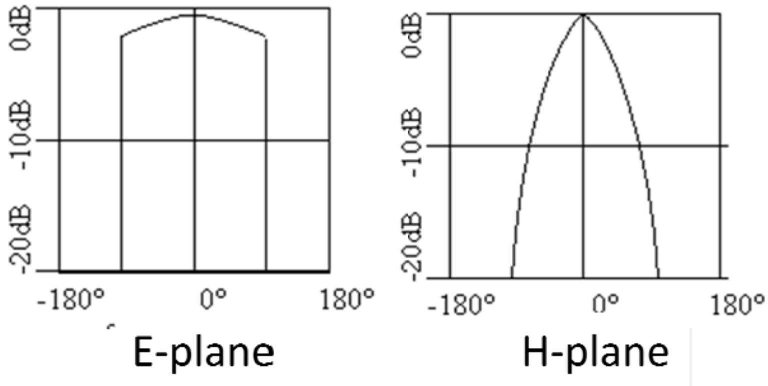
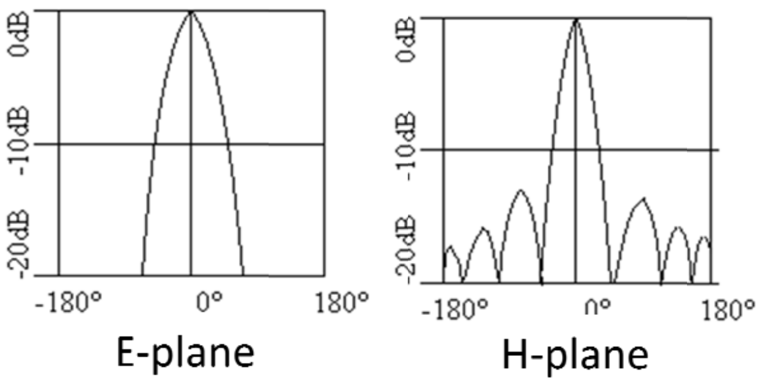
2.3 Radiation patterns:



2.4 Radiation patterns:



2.5 Radiation patterns:



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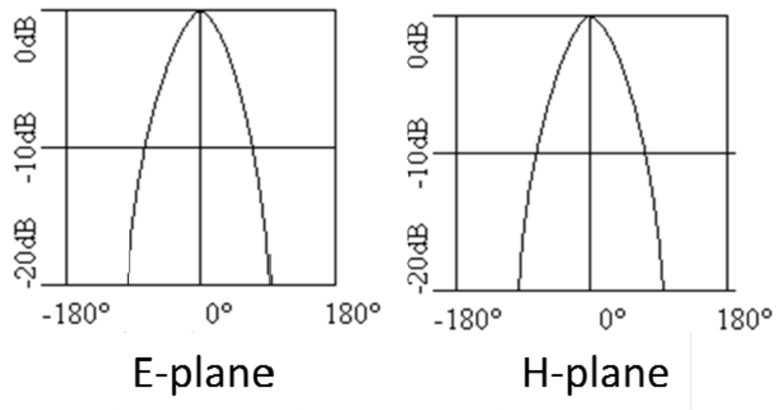
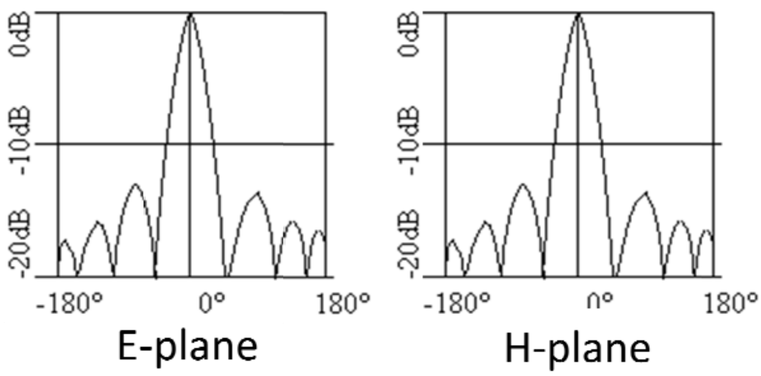
2.6 Radiation patterns:



2.7 Radiation patterns:



Name:

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PART B (You can use the textbook to solve this problem, but only after PART A has been delivered)

3.0 Diversity antenna (25p)

We have access to two dipole antennas that we would like to use as a diversity antenna in order to improve the performance of a radio unit that is used in an isotropic multipath environment. In the figure below we show the CDF curves for the two antennas measured separately (i.e. without the other antenna present) as well as the CDF when the two antennas are placed close to each other, as they would be when used on the radio, and the CDF when both antenna ports are selection combined. We also know that antenna 2 has an efficiency of 0 dB.

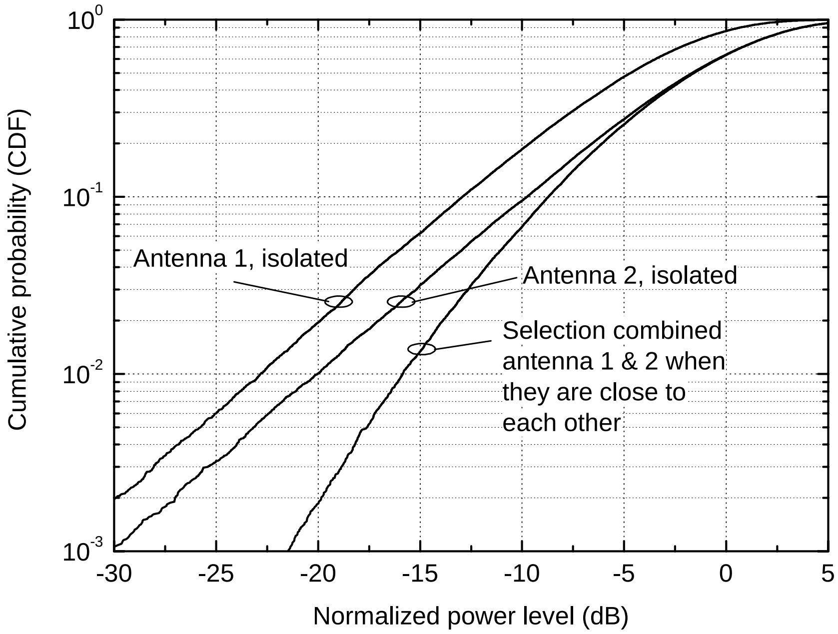

Figure 3.1. CDF:s for the two dipole antennas measured separately without the other antenna present and when placed close to each other when selection combination is used, respectively.

3.1 Why are the CDF curves for antennas 1 and 2 different and what does this difference represents?

3.2 In the figure, sketch how the CDF curves change for the two ports (branches) when the antennas are placed close to each other. Explain.

3.3 What is the gain of using the two antennas and selection combining if we compare with using only antenna 1? Assume a CDF level of 1%.

Name:

3.4 What is the gain of using the two antennas and selection combining if we compare with using an ideal reference antenna? Assume a CDF level of 1%.

3.5 Assume that we know that the efficiency for each branch decreases by 5 dB when the antennas are closely spaced with the same spacing as for the diversity combined curve. Determine then the absolute value of the complex correlation coefficient between the two branches?

3.6 Now, we have got an offer from an antenna vendor to buy a diversity antenna instead of using the two dipoles. The only information we have is that the antenna is lossless and that it has the equivalent circuit shown in the figure below. The values of the impedances in the figure are;
$Z_{11} = Z_{22} = 82.4 + j32.1$; $Z_{12} = Z_{21} = 76.1 - j0.7$ ohm .

What are the effective and apparent diversity gains of this antenna if selection combining and a CDF-level of 1% are assumed?

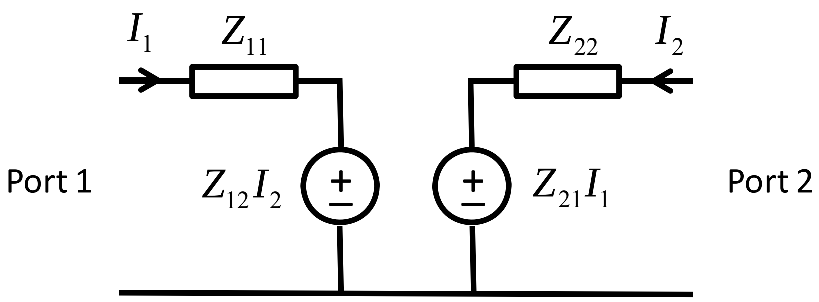

Figure 3.2. Equivalent circuit for diversity antenna.

3.7 If Port 1 is excited with a generator with 50 ohm internal impedance and Port 2 is terminated with 50 ohm, how much power is dissipated in the 50 ohm load at Port 2 and how much power is radiated?

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4.0 Dipole Array (25p)

Fig. 4.1 shows an array antenna consisting of four halfwave dipoles.

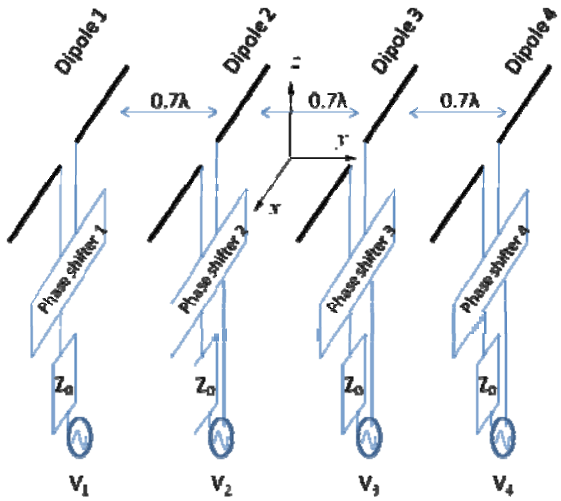

Figure 4.3 A 4-dipole linear array antenna

4.1 Draw the equivalent circuit of the array.

4.2 What are the values for the phase shifters when the maximum radiation direction (main beam direction) is along the z-axis? Here we assume uniform excitations (V_1=V_2=V_3=V_4).

4.3 Write the expression for the scan impedance of dipole 3 under the condition of 4.2. (Here you do not need to produce any numerical value. It is sufficient to write the expression and explain the procedure of your derivation. All the parameters you use should be known or easy to obtain from formulas or figures. Explain which formulas and figures in the compendium you are using.)

4.4 Write the expression of the embedded radiation field function of dipole 3 when dipole 3 is excited and the rest of the dipoles are terminated with loads (same as the source impedances) Z_0. NOTE that now V_1=V_2=V_4=0. (Here you do not need to produce any numerical value. It is sufficient to write the expression and explain the procedure of your derivation. All the parameters you use should be known or easy to obtain from formulas or figures. Explain which formulas and figures in the compendium you are using.)

4.5 Write the expression for the radiation field function of the whole array.

4.6 Now we want to steer the main beam to 30^{o} from the z-axis. What are now the values of the phase shifters? Are there any grating lobes now? If yes, in which direction?

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PART A (must be delivered before textbook can be used)

1.0 On Labs (25p)

1.1. Some questions about Lab1. Be careful that you use the correct terminology from the course literature.

1.1.1. Which environment is emulated by reverberation chamber and which environment is emulated by anechoic chamber? Explain. **(2p)**

Solution:
A reverberation chamber emulates the multipath environment (due to multiple reflections…). An anechoic chamber emulates the free-space environment (no refection…). (Any answers explaining the correct physics will get full points.)

1.1.2. Which quantities can we measure in reverberation chamber, and which ones can we measure in anechoic chamber? **(2p)**

Solution:
We can measure the diversity gain, efficiency, MIMO capacity, TIS, TRP… in a reverberation chamber. (Mentioning three quantities is enough.) We can radiation pattern (CO & XP), gain, … in an anechoic chamber. (Mentioning one quantities is enough.)

1.1.3. Explain the different steps involved when measuring in a reverberation chamber. **(3p)**

Solution:
For an efficiency measurement, one need to first perform a reference measurement with a reference antenna with unity efficiency to get the average power level; then do an efficiency measurement with the antenna under test (AUT) to get another power level. The difference of the two power level (in dB) is the efficiency of the AUT. (Any correct measurement procedure for any application, e.g. diversity, TRP…, will get full points.)

1.2. In Lab 2, you have measured a standard gain horn and a microstrip slot array antenna; see Fig. 1.1. The measured radiation patterns are shown in Fig. 1.2 and Fig. 1.3.

1.2.1. Which of the two figures show radiation patterns for standard gain horn and which one for microstrip slot array antenna? Explain briefly. **(2p)**

Solution:

Fig.1.2 is for microstrip slot array antenna, Fig1.3 is for standard gain horn. Because microstrip slot array antenna has very different beamwidth for E- and H-planes.

1.2.2. Please sketch (redraw) the patterns and mark which curve (blue or red in the original graphs) is for E-plane and which one is for H-plane. Explain briefly. **(2p)**

Solution:
Blue curve in Fig.1.2 is for E-plane of microstrip slot array antenna, in Fig1.3 is for E-plane for standard gain horn. Red curves are for H-planes.

1.2.3. In Fig. 1.2, the radiation pattern in one plane (red curve) has much broader beam width than the other, why? **(3p)**

Solution:
Size in H-plane is much smaller than that in E-plane.

1.2.4. In Fig.1.3, the radiation pattern in one plane (red curve) has lower first sidelobe than the other, why? **(3p)**

Solution:
Tapered field distribution in H-plane.

1.2.5. What is the length of the slot in the slot array antenna in mm? The operating frequency for the slot array is 9.5 GHz. **(2p)**

Solution:
Half wavelength in free space. $l == \frac{\lambda}{2} = 15.8mm$

1.2.6. As you know, the slot array antenna has the maximum radiation direction at 30^{o} from the broadside. What is the maximum allowed spacing between the slots to avoid grating lobes? If you do not remember the complete formula, you can give the value for the full scan case. Explain. **(2p)**

Solution:
$$d < \frac{\lambda}{1+\cos\alpha} = 21mm$$

1.3. In Lab 3, you could choose to realize one of several different patch antennas; see Fig.1.4. Explain which polarization(s) each of these patch antenna shapes radiate, i.e. horizontal linear, vertical linear or circular, provided the dimensions are properly chosen. **(4p)**

Solution:

(a) Linear polar: vertical;
(b) Linear polar: horizontal;
(c) Dual-linear polar;
(d) Circular polar;
(e) Circular Polar;

Name:

Standard gain horn

Microstrip slot array antenna

Figure 1.1 Antennas in Lab2.

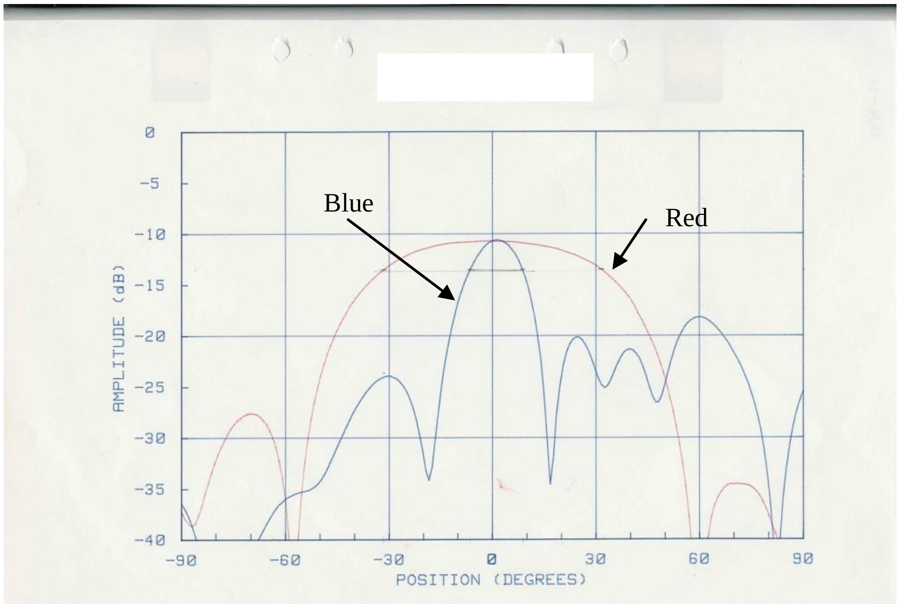

Figure 1.2 Measured radiation patterns on both E- and H-planes for antenna 1

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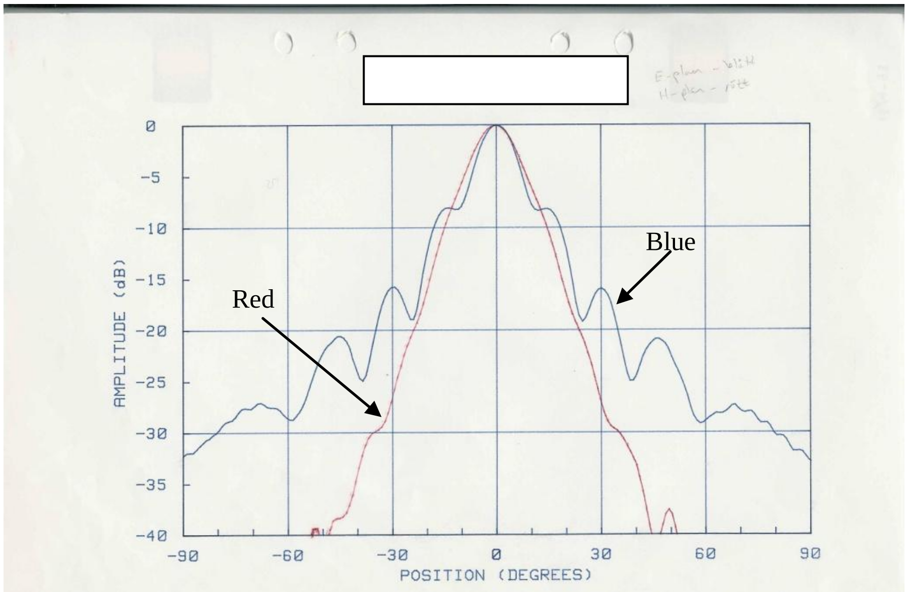


Fig. 1.3 Measured radiation patterns on both E- and H-planes for antenna 2

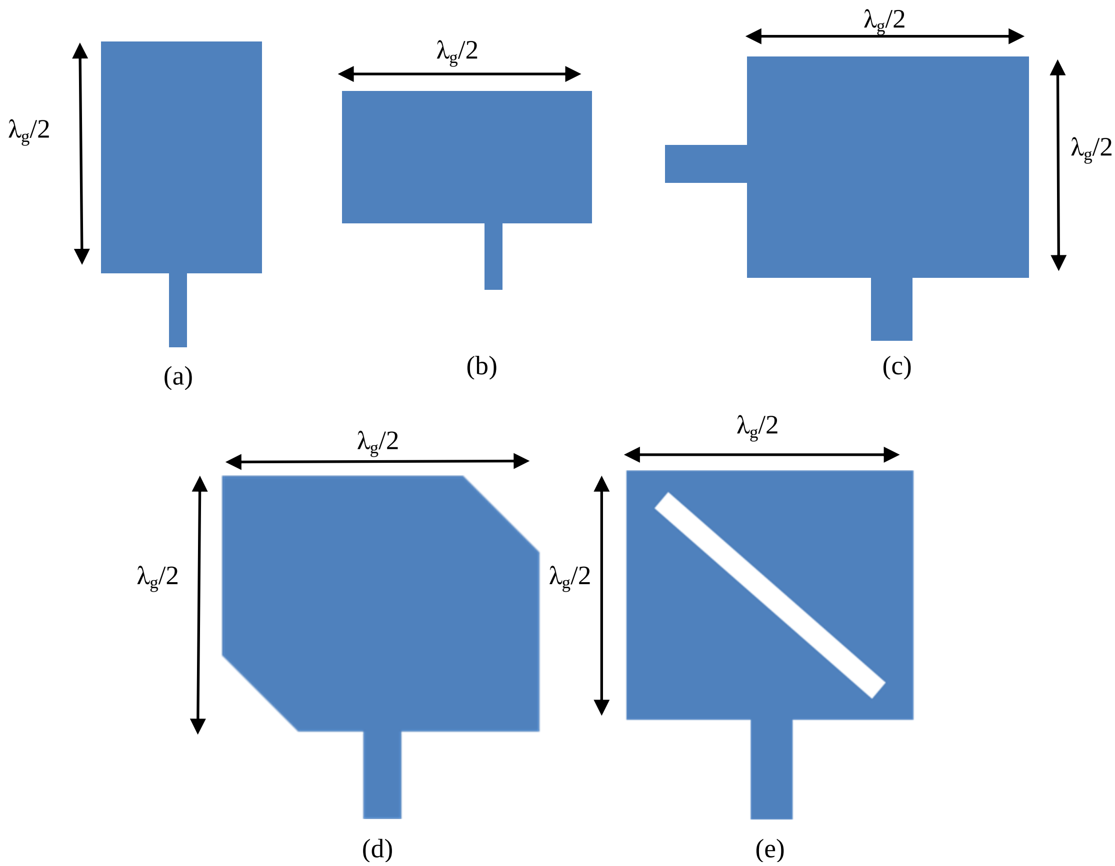


Figure 1.4 different patch antennas.

Name:

Name:

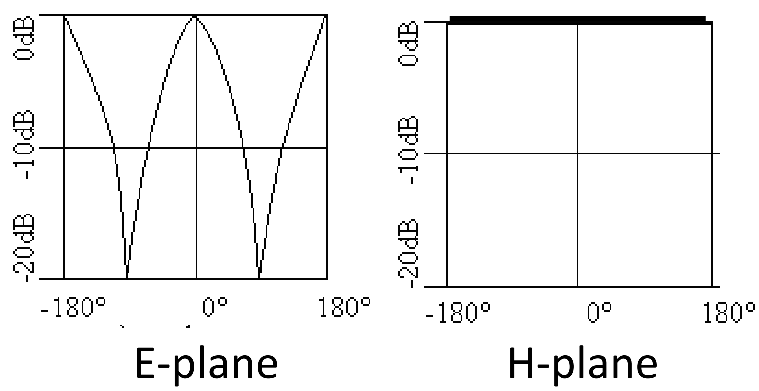
2.0 Far-field functions of different antennas (25p)

In Home assignment 2, you were asked to sketch the co- and cross- polar radiation patterns in E- and H- planes for several different basic antennas. However, in practice, the question is opposite: you have some specifications on antenna's radiation patterns from customers, and you should choose proper antennas to fulfill the specifications. For each of the following questions, the co-polar radiation patterns in E- and H-planes are given. You should

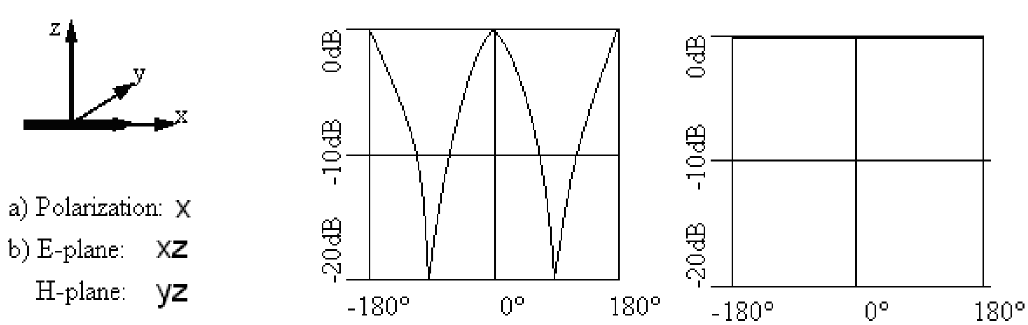
a) Choose a proper antenna that can provide the required radiation patterns. Write the name of the antenna or incremental source; **(1.5 p, 1 p for last 3 radiation patterns)**
b) Sketch a coordinate system and draw your antenna in it, and indicate the polarization. **(1.5 p, 1 p for last 3 radiation patterns)**
c) Explain which planes are E-plane and H-plane. **(1 p)**

Note that there may exist more than one solution for the same radiation pattern requirement.

2.1 Radiation patterns:

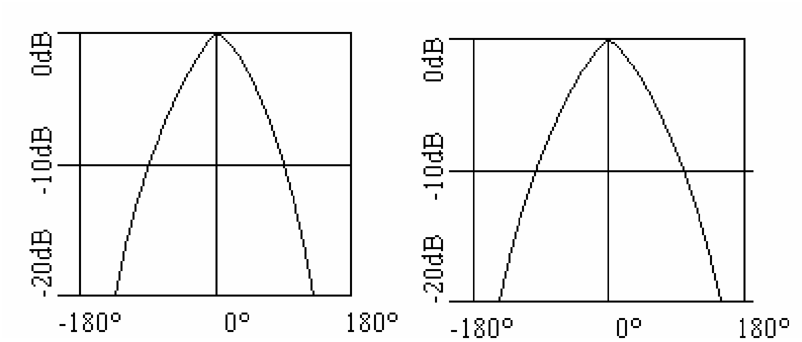


Solution: (4p)
x-directed incremental electric current, dipole.



2.2 Radiation patterns:

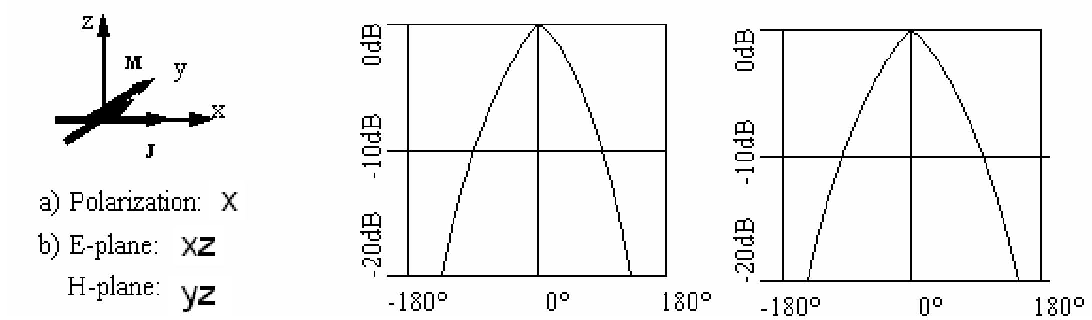
Name:



E-plane H-plane

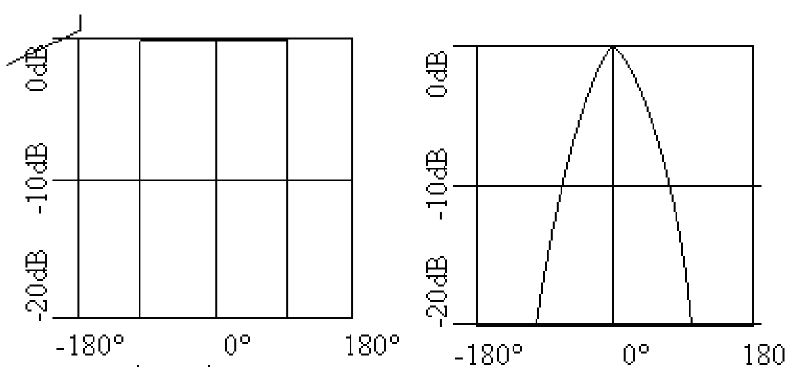
Solution: (4p)

Two crossed incremental electric dipoles located in xy-plane and excited with 90 deg phase difference: Huygen source, eleven antenna, incremental electric current located 0.25 wavelengths above infinite ground plane.



E-plane H-plane

2.3 Radiation patterns:

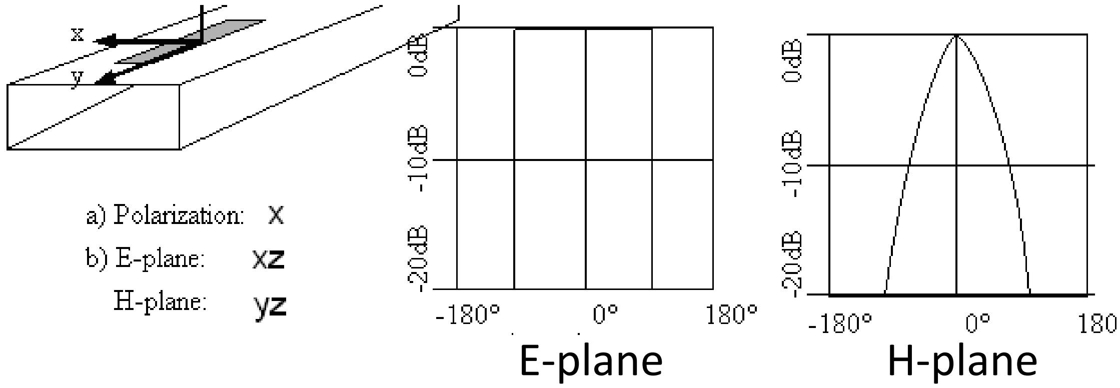


E-plane H-plane

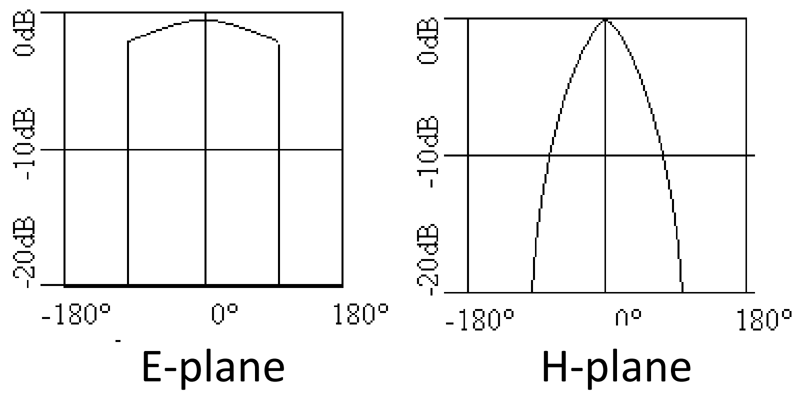
Solution: (4p)

Waveguide slot antenna (assume infinite ground plane), PIFA:

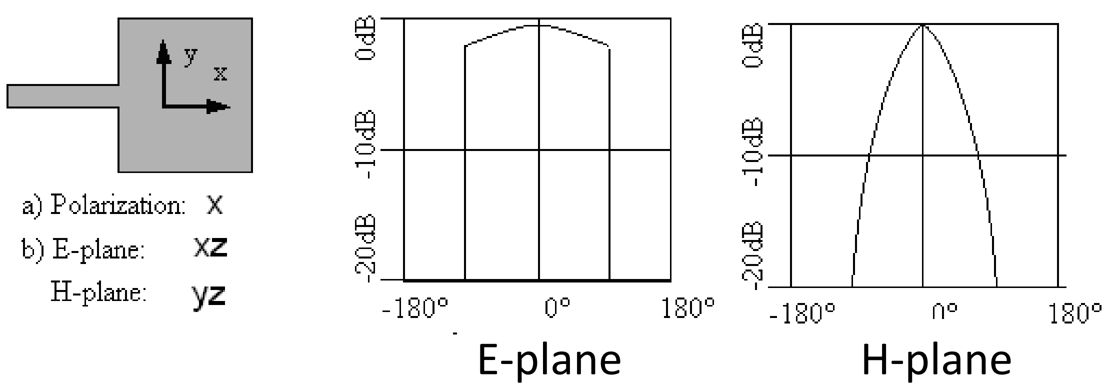
Name:



2.4 Radiation patterns:

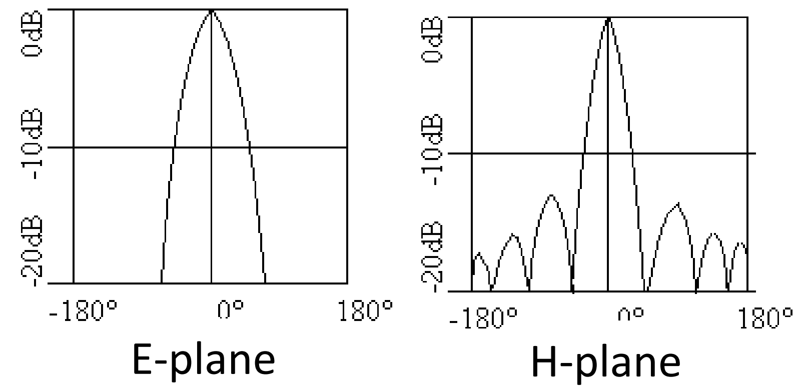


Solution: (4p)
Edge-fed microstrip antenna on high permittivity substrate (top view):



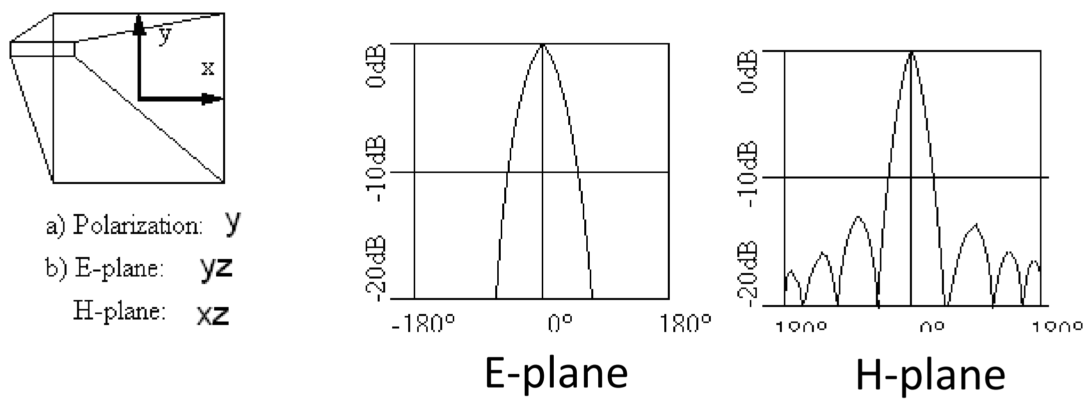
2.5 Radiation patterns:

Name:

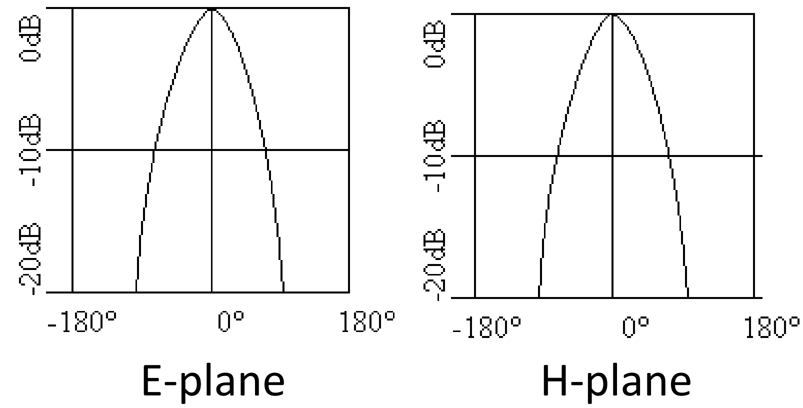


Solution: (3p)

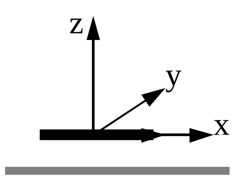
Long pyramidal horn antenna with quadratic aperture fed by TE10 rectangular waveguide mode:



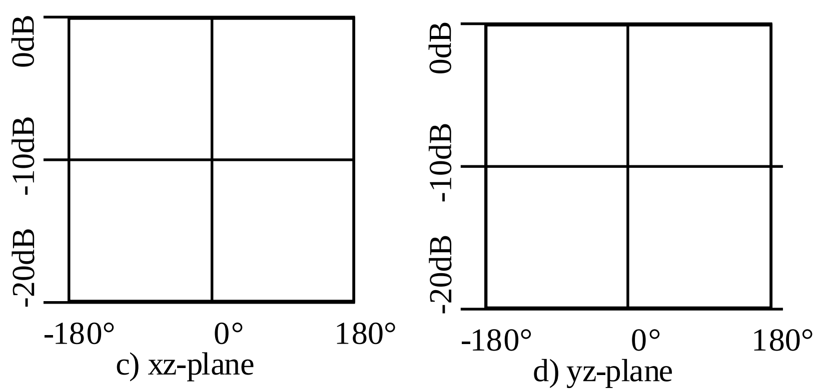
2.6 Radiation patterns:



Solution: (3p)
x-directed incremental electric current located 0.25 wavelengths above infinite ground plane, Huygen source:

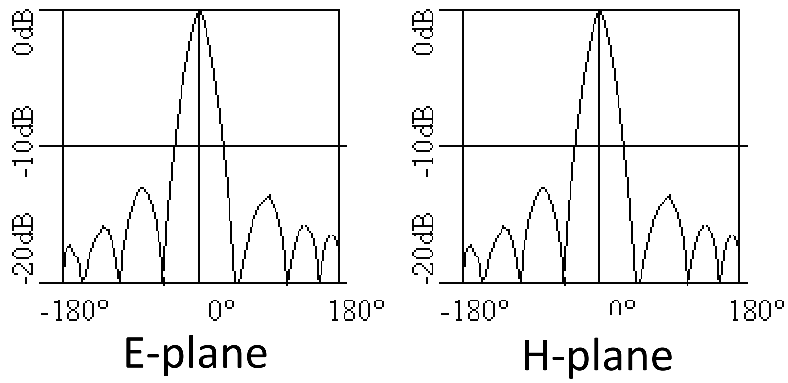


a) Polarization:
b) E-plane:
H-plane:



OR
Long pyramidal horn antenna with quadratic aperture with soft surfaces on all walls, conical horn corrugated.

2.7 Radiation patterns:



Solution: (3p)
Long pyramidal horn antenna with quadratic aperture with hard surfaces on all walls. Small arrays is also a possible answer.

Name:

Name:

PART B (You can use the textbook to solve this problem, but only after PART A has been delivered)

3.0 Diversity antenna (25p)

We have access to two dipole antennas that we would like to use as a diversity antenna in order to improve the performance of a radio unit that is used in an isotropic multipath environment. In the figure below we show the CDF curves for the two antennas measured separately (i.e. without the other antenna present) as well as the CDF when the two antennas are placed close to each other, as they would be when used on the radio, and the CDF when both antenna ports are selection combined. We also know that antenna 2 has an efficiency of 0 dB.

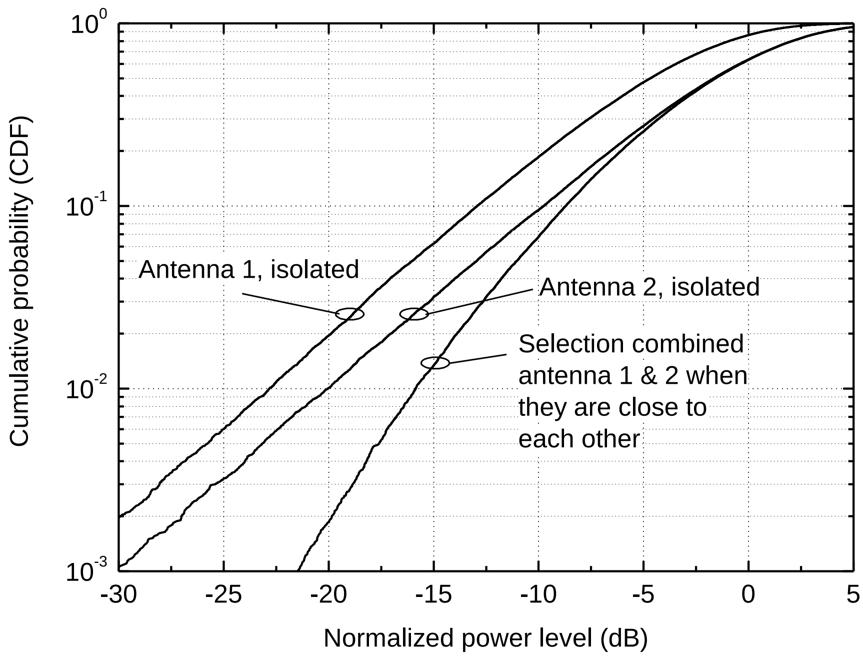


Figure 3.1. CDF:s for the two dipole antennas measured separately without the other antenna present and when placed close to each other when selection combination is used, respectively.

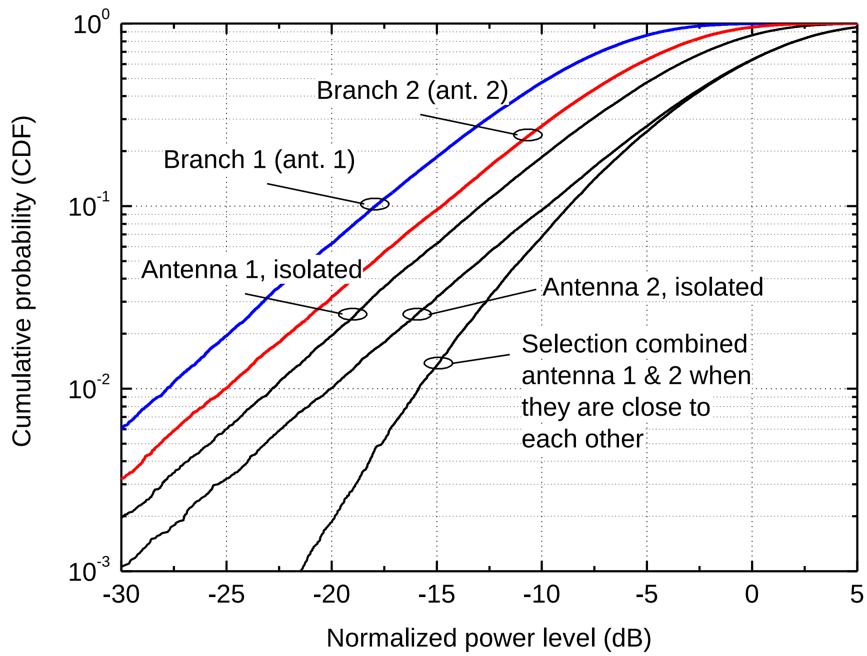
3.1 Why are the CDF curves for antennas 1 and 2 different and what does this difference represents?

Solution: (2p)

The reason for the different CDF curves is that the two antennas have different efficiencies. The horizontal distance between the two curves represents the difference in efficiency, from the figure it can be seen to be 3 dB.

3.2 In the figure, sketch how the CDF curves change for the two ports (branches) when the antennas are placed close to each other. Explain.

Solution: (4p)
See figure.
- The curves for the two branches should be shifted to the left (2p)
- The two curves should be shifted equally much (2p)



3.3 What is the gain of using the two antennas and selection combining if we compare with using only antenna 1? Assume a CDF level of 1%.

Solution: (2p)
From the figure the diversity gain can be seen to be approx. 7 dB.

3.4 What is the gain of using the two antennas and selection combining if we compare with using an <u>ideal reference antenna</u>? Assume a CDF level of 1%.

Solution: (3p)
- From the figure the diversity gain can be seen to be approx. 4 dB (2p)
- It should be realized that antenna 2 represents an ideal reference since it has 0 dB efficiency (1p)

3.5 Assume that we know that the efficiency for each branch decreases by 5 dB when the antennas are closely spaced with the same spacing as for the diversity combined curve. Determine then the absolute value of the complex correlation coefficient between the two branches?

Solution: (3p)
The apparent diversity gain is given by equation 3.12 in the book, i.e.
$G_{app} = 10\sqrt{1-|\rho|^2}$ (note that it is Ok to use other, better, formulas for the apparent diversity gain). Since the efficiency has decreased 5 dB due to that the antennas are closely spaced as compared to when they are far apart the apparent diversity gain is 5+4=9 dB (from d). Thus, we have $G_{app} = 10^{9/10} = 10\sqrt{1-|\rho|^2} \Rightarrow |\rho| = 0.61$

3.6 Now, we have got an offer from an antenna vendor to buy a diversity antenna instead of using the two dipoles. The only information we have is that the antenna is lossless and that it has the equivalent circuit shown in the figure below. The values of the impedances in the figure are
$Z_{11} = Z_{22} = 82.4 + j32.1$; $Z_{12} = Z_{21} = 76.1 - j0.7$ ohm .

What are the effective and apparent diversity gains of this antenna if selection combining and a CDF-level of 1% are assumed?

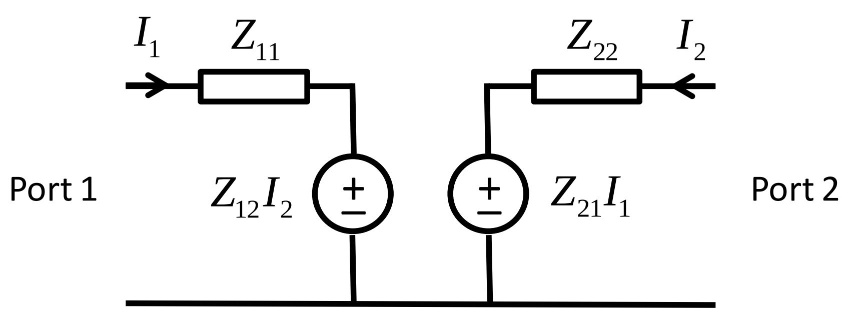
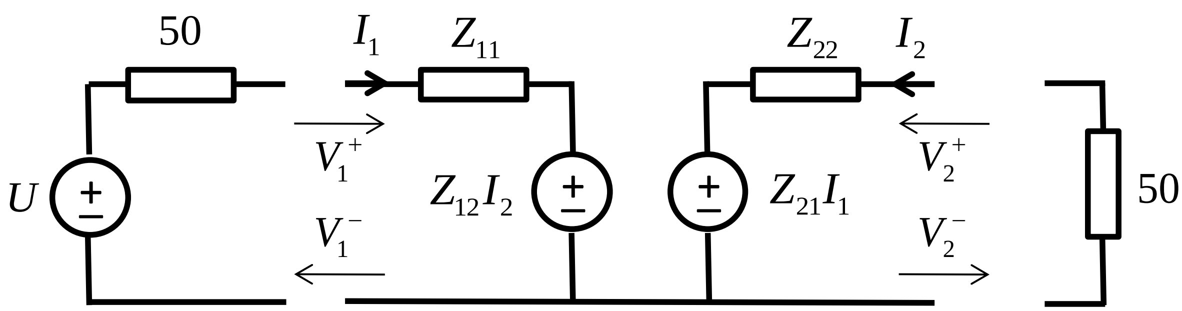


Figure 3.2. Equivalent circuit for diversity antenna.

Solution: (6p)
The apparent diversity gain can be obtained from the knowledge of the correlation coefficient which in turn, since the antenna is lossless, can be calculated from the S-parameters as given by equation 3.10 in the book (note that there is an error in the book, it should be a square root in the denominator, it is however Ok if the expression in the book is used). Thus, the first thing we have to do is to determine the S-parameters. For the given problem
$Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$ so that $S_{11} = S_{22}$ and $S_{12} = S_{21}$.
S_{11} and S_{21} can be determined from the following circuit;



Since port 2 is terminated in 50 ohm $V_2^+ = 0$ and we have the following relations;

$$S_{11} = \frac{V_1^-}{V_1^+} \text{ , } S_{21} = \frac{V_2^-}{V_1^+} \text{ , } V_2^- = V_2$$

We also know that $V_1^+ = \frac{U}{2} \Rightarrow V_1^- = V_1 - \frac{U}{2}$

We can now express the S-parameters in the total voltages as;

$$S_{11} = S_{22} = \frac{2V_1}{U} - 1 \text{ , } S_{12} = S_{21} = \frac{2V_2}{U}$$

The currents in the left and right loops can be determined as;

$$I_1 = \frac{U - Z_{12}I_2}{Z_{11} + 50} \text{ , } I_2 = \frac{-Z_{21}I_1}{Z_{22} + 50}$$

Solving for the currents gives;

$$I_1 = U\frac{Z_{22} + 50}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}} \text{ , } I_2 = -U\frac{Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}}$$

We can now calculate the port voltages as;

$$V_1 = Z_{11}I_1 + Z_{12}I_2 = U\frac{Z_{11}(Z_{22} + 50) - Z_{12}Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}}$$

$$V_2 = -50I_2 = U\frac{50Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}}$$

The S-parameters are now given by;

$$S_{11} = S_{22} = 2\frac{Z_{11}(Z_{22} + 50) - Z_{12}Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}} - 1 = 0.10 + j0.42$$

$$S_{12} = S_{21} = \frac{100Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}} = 0.43 - j0.35$$

Now, when we have the S-parameters we can compute the absolute value of the complex correlation coefficient by using equation 3.10 in the book (or rather the correct version of the equation).

$$|\rho| = \frac{\left|S_{11}^* S_{12} + S_{21}^* S_{22}\right|}{\sqrt{\left(1 - \left(|S_{11}|^2 + |S_{21}|^2\right)\right)\left(1 - \left(|S_{22}|^2 + |S_{12}|^2\right)\right)}} = 0.41$$

The apparent diversity gain is given by equation 3.12 in the book;

$$G_{app} = 10\sqrt{1 - |\rho|^2} = 9.12 = 9.60 \text{ dB}$$

The effective diversity gain is defined as the apparent diversity gain multiplied with the radiation efficiency, equation 3.11. We have by using equation 3.7;

$$G_{eff} = e_{rad} G_{app} = \left(1 - |S_{11}|^2 - |S_{21}|^2\right) 10\sqrt{1 - |\rho|^2} = 4.61 = 6.64 \text{ dB}$$

3.7 If Port 1 is excited with a generator with 50 ohm internal impedance and Port 2 is terminated with 50 ohm, how much power is dissipated in the 50 ohm load at Port 2 and how much power is radiated?

Solution: (5p)

The power terminated in the 50 ohm load at Port 2 is given by; $P_{term2} = P_{avail} |S_{21}|^2$

Where the available power is the power the generator can deliver in a matched load, i.e. $P_{avail} = \frac{|U|^2}{200}$ (where RMS-value for the voltage is assumed).

Thus,

$$P_{term2} = P_{avail} |S_{21}|^2 = |U|^2 \frac{|S_{21}|^2}{200} = \{\text{From (f)}\} = |U|^2 \frac{|0.43 - j0.35|^2}{200} = 1.54 \cdot 10^{-3} |U|^2$$

Since the antenna is lossless the radiated power is given by;

$$P_{rad} = P_{avail} - P_{refl} - P_{term2} = P_{avail}\left(1 - |S_{11}|^2 - |S_{21}|^2\right)$$

$$= \frac{|U|^2}{200}\left(1 - |0.10 + j0.42|^2 - |0.43 - j0.35|^2\right) = 2.52 \cdot 10^{-3} |U|^2$$

Name:

4.0 Dipole Array (25p)

Fig. 4.1 shows an array antenna consisting of four halfwave dipoles.

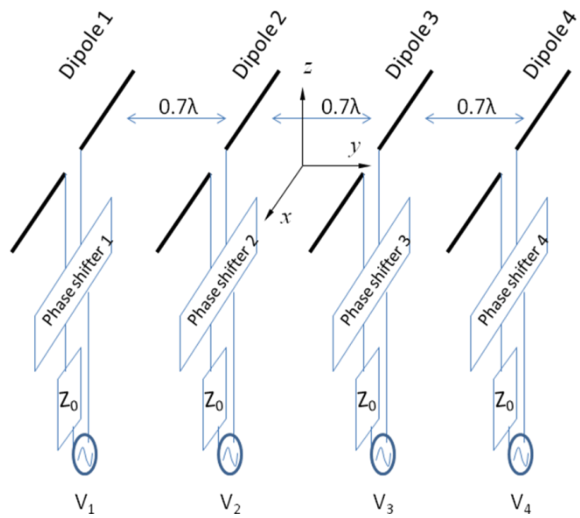
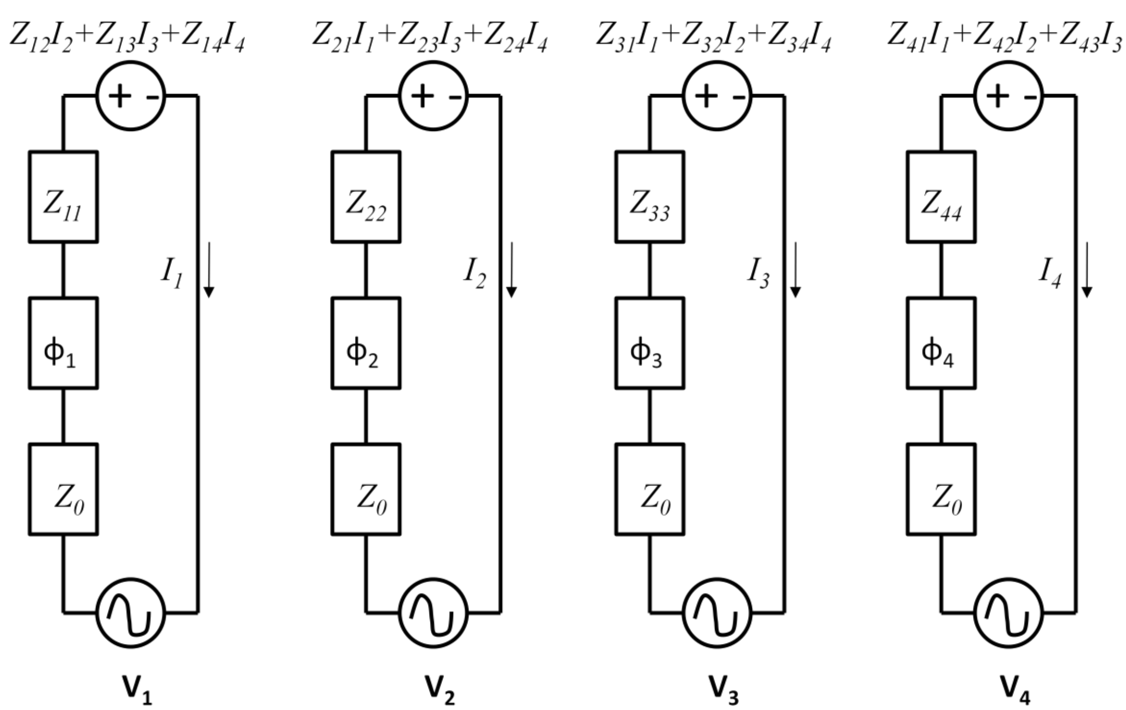


Figure 4.3 A 4-dipole linear array antenna

4.1 Draw the equivalent circuit of the array.

Solution (5p)



4.2 What are the values for the phase shifters when the maximum radiation direction (main beam direction) is along the z-axis? Here we assume uniform excitations (V_1=V_2=V_3=V_4).

Solution (4p)

$\Phi_1= \Phi_2= \Phi_3 =\Phi_4=0$

4.3 Write the expression for the scan impedance of dipole 3 under the condition of 4.2. (Here you do not need to produce any numerical value. It is sufficient to write the expression and explain the procedure of your derivation. All the parameters you use should be known or easy to obtain from formulas or figures. Explain which formulas and figures in the compendium you are using.)

Solution (4p)

We define the followings

$$\mathbf{V}_{ex}=\begin{bmatrix}V_1\\V_2\\V_3\\V_4\end{bmatrix}=\begin{bmatrix}V\\V\\V\\V\end{bmatrix},\ \mathbf{I}=\begin{bmatrix}I_1\\I_2\\I_3\\I_4\end{bmatrix},\ \mathbf{Z}=\begin{bmatrix}Z_{11}+Z_0 & Z_{12} & Z_{13} & Z_{14}\\ Z_{21} & Z_{22}+Z_0 & Z_{23} & Z_{24}\\ Z_{31} & Z_{32} & Z_{33}+Z_0 & Z_{34}\\ Z_{41} & Z_{42} & Z_{43} & Z_{44}+Z_0\end{bmatrix}$$

Where all $Z_{ij}=Z_{ji}$ can be found by using Fig. 10.9 in Per-Simon's book and $\mathbf{V}_{ex}$ is known.

So we have

$$\mathbf{V}_{ex}=\mathbf{ZI} \implies \mathbf{I}=\mathbf{Z}^{-1}\mathbf{V}_{ex}$$

Now $\mathbf{I}$ is solved. Then,

$$Z_{3scan}=\frac{V}{I_3}-Z_0$$

4.4 Write the expression of the embedded radiation field function of dipole 3 when dipole 3 is excited and the rest of the dipoles are terminated with loads (same as the source impedances) Z_0. NOTE that now V_1=V_2=V_4=0. (Here you do not need to produce any numerical value. It is sufficient to write the expression and explain the procedure of your derivation. All the parameters you use should be known or easy to obtain from formulas or figures. Explain which formulas and figures in the compendium you are using.)

Solution (4p)

We have now

$$\mathbf{V}'_{ex}=\begin{bmatrix}V_1\\V_2\\V_3\\V_4\end{bmatrix}=\begin{bmatrix}0\\0\\V\\0\end{bmatrix},\ \mathbf{I}'=\begin{bmatrix}I_1\\I_2\\I_3\\I_4\end{bmatrix},\ \mathbf{Z}=\begin{bmatrix}Z_{11}+Z_0 & Z_{12} & Z_{13} & Z_{14}\\ Z_{21} & Z_{22}+Z_0 & Z_{23} & Z_{24}\\ Z_{31} & Z_{32} & Z_{33}+Z_0 & Z_{34}\\ Z_{41} & Z_{42} & Z_{43} & Z_{44}+Z_0\end{bmatrix}$$

Where all $Z_{ij}=Z_{ji}$ can be found by using Fig. 10.9 in Per-Simon's book and $\mathbf{V}'_{ex}$ is known. So we have

$$\mathbf{I}'=\mathbf{Z}^{-1}\mathbf{V}'_{ex}$$

Now $\mathbf{I}'$ is solved. Then, from (5.11) in Per-Simon's book, we have the radiation function of a halfwave dipole at the origin of the coordinate system as $\mathbf{G}_{dx}(\theta,\varphi)$ with a current of I_0. Note that now $\hat{\mathbf{I}} = \hat{\mathbf{x}}$ so (5.11) should be modified a bit. Then, the embedded radiation function for dipole 3 is

$$\mathbf{G}_{d3\_embedded}(\theta,\varphi) = \mathbf{G}_{dx}(\theta,\varphi)\frac{I_1'}{I_0}e^{jk\mathbf{r}_1\cdot\mathbf{r}} + \mathbf{G}_{dx}(\theta,\varphi)\frac{I_2'}{I_0}e^{jk\mathbf{r}_2\cdot\mathbf{r}} + \mathbf{G}_{dx}(\theta,\varphi)\frac{I_3'}{I_0}e^{jk\mathbf{r}_3\cdot\mathbf{r}} + \mathbf{G}_{dx}(\theta,\varphi)\frac{I_4'}{I_0}e^{jk\mathbf{r}_4\cdot\mathbf{r}}$$

4.5 Write the expression for the radiation field function of the whole array.

Solution (4p)

Almost the same as the above expression but we should use $\mathbf{I}$ instead of $\mathbf{I}'$.

$$\mathbf{G}_{WholeArray}(\theta,\varphi) = \mathbf{G}_{dx}(\theta,\varphi)\frac{I_1}{I_0}e^{jk\mathbf{r}_1\cdot\mathbf{r}} + \mathbf{G}_{dx}(\theta,\varphi)\frac{I_2}{I_0}e^{jk\mathbf{r}_2\cdot\mathbf{r}} + \mathbf{G}_{dx}(\theta,\varphi)\frac{I_3}{I_0}e^{jk\mathbf{r}_3\cdot\mathbf{r}} + \mathbf{G}_{dx}(\theta,\varphi)\frac{I_4}{I_0}e^{jk\mathbf{r}_4\cdot\mathbf{r}}$$

4.6 Now we want to steer the main beam to 30^o from the z-axis. What are now the values of the phase shifters? Are there any grating lobes now? If yes, in which direction?

Solution (4p)

From (10.22), we have

$$\cos\alpha_0 = -\frac{\Delta\Phi}{kd_a}$$

Now $\alpha_0 = 60^o, kd_a = 1.4\pi$ so $\Delta\Phi = -\cos\alpha_0 kd_a = -0.7\pi$

So phase shifters are $0, -0.7\pi, -1.4\pi, -2.1\pi$, respectively.

Now there is a grating lobe, at

$$\alpha_1 = \cos^{-1}\left[\cos 60^o - \frac{\lambda}{d_a}\right] = \cos^{-1}(0.5 - 1.43) = \cos^{-1}(-0.9286) = 158.2^o$$