Solution To Exam in the course Antenna Engineering
2010-05-29

ANTENNA ENGINEERING (SSY100)
(E4) 2009/10 (Period IV)

Saturday 29 May 1400-1800 hours.

Teachers: Adjunct Prof Jan Carlsson, Prof Per-Simon Kildal, Associate Prof Jian Yang
Questions: Jan Carlsson, tel. 0703 665169

The exam consists of 2 parts. Part A is printed on colored paper and must be solved without using the textbook. When you have delivered the colored text and the solutions of Part A (latest 17:00), the textbook can be used for Part B of the exam.

You are allowed to use the following:

For Part A:	**Pocket calculator of your own choice**
For Part B only:	**Mathematical tables including Beta**
	Pocket calculator of your own choice
	Kildal's compendium "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(The textbook can contain own notes and marks on its original printed pages. No other notes are allowed.)

Tentamen består av 2 delar. Del A har tryckts på färgade papper och skall lösas utan att använda läroboken. När du har inlämnat dom färgade arken med uppgifterna för del A och dina svar på dessa uppgifter (senast 17:00), kan du ta fram läroboken för att lösa del B.

Tillåtna hjälpmedel:

För del A:	Valfri räknedosa
För del B:	Matematiska Tabeller inkluderad Beta
	Valfri räknedosa
	Kildals lärobok "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(Boken kan innehålla egna notater skrivna på dom inbundna sidorna. Extra ark med notater tillåts inte.)

PART A (must be delivered before textbook can be used)

1.0 Far field function factors of different antennas (35p)

Note that you do not need to remember the formulas in all details to get scores in the questions below. Try to answer as well as you can. It is easier than it looks.

1.1 Make a sketch of the spherical coordinate system. Write the general expression for the far field of an antenna in the spherical coordinate system. Explain the different factors, and in particular the far field function.

1.2 Explain how the far field function change when the antenna is moved from its original location with the phase reference point in the origin to a point $\vec{r}_0 = x_0\hat{x} + y_0\hat{y} + z_0\hat{z}$.

1.3 Explain how we can use the expression in 1.2 to generate the far field function of an array of equal elements.

1.4 The far field function of linear antennas (straight wire) and planar antennas (planar apertures and arrays) can be separated in two factors that are functions of direction: one vector function, and one scalar function. Which of these functions have the most rapid variation with direction for large antennas? Explain what the scalar function represents for the continuous case (such as long wires and actual apertures), and for the discrete case (linear and planar arrays).

1.5 For each of the cases a, b, c and d below, do the following:
i) Write the expression for the vector function mentioned in 1.4 when you assume y-polarization on axis. ii) Write the name of the source in this function. iii) Write the expressions that determines the form in E-plane, and in H-plane. iv) Explain which planes are E-plane and H-plane in terms of the coordinate system axes. v) Sketch the E- and H-plane patterns.

a) Straight wire antennas.

b) Narrow slots in large flat ground planes.

c) Large apertures in large flat ground planes.

d) Apertures of large reflectors.

1.6 . Write now the general expression for the vector function mentioned in 1.4 for linear and planar array antennas. Assume that the arrays are coinciding with the xy-plane. What is this vector function called? Write the general expression for the vector function when we assume that it is a BOR1 vector function polarized in y-direction with E-plane far field function $G_E(\theta)$ and H-plane far field function $G_H(\theta)$.

1.7 Explain what we mean by co- and cross-polarization, and how we can extract the co- and cross-polar parts of a radiation field function. Write down the two polarization vectors for co-polar y-polarization that are used several places in the compendium (Ludwig's third definition).

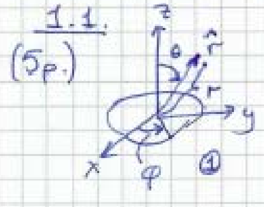
1.8 In order to determine the directivity we need to evaluate the power integral. Express the directivity and explain it in terms of the power integral and the maximum value of the far field function.

①

1.0 Far field function factors of different antennas (35p)

(P.-S. Kildal)

1.1. (5p.)



$$\vec{r} = r\hat{r}$$

$$\hat{r} = \sin\theta(\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}) + \cos\theta\,\hat{z}$$ ①

$$\vec{G} = \underbrace{\frac{1}{r}}_{\text{① divergens factor}} \underbrace{e^{-jkr}}_{\text{phase factor ①}} \underbrace{\vec{G}(\theta,\varphi)}_{\text{complex far field function}}$$

$$\vec{G}(\theta,\varphi) \perp \hat{r} \quad (\text{orthogonal to } \hat{r})$$ ①

1.2. The phase of the far field function changes according to

(3p.)

$$\underbrace{\vec{G}'(\theta,\varphi)}_{\text{New far field function with phase ref. in } \vec{r}_0} = \underbrace{\vec{G}(\theta,\varphi)}_{\text{old one}} \, e^{-jk\vec{r}_0\cdot\hat{r}}$$

(2p., 3p. if correct phase and explained)

1.3. One element $\vec{G}_0(\theta,\varphi)$, located in origin.

(4p) Move element to location $x_1 = \Delta\hat{x}$. Then,

① $$\vec{G}_1(\theta,\varphi) = \vec{G}_0(\theta,\varphi)\, e^{jk\hat{r}_0\cdot\hat{x}\Delta}$$

When they have the same phase ref point we can add the rad. field functions

① $$\vec{G}_{total} = \vec{G}_0 + \vec{G}_1 = \vec{G}_0(\theta,\varphi)\left(1 + e^{jk\hat{r}_0\cdot\hat{x}_0\Delta}\right)$$

Similar for array of many elements. Add all contributions with the phase ref. point.

(in 1.3) ② $$\vec{G}_{total} = \underbrace{\vec{G}_0(\theta,\varphi)}_{\text{element factor}} \underbrace{A(\theta,\varphi)}_{\text{array factor}}, \quad A(\theta,\varphi) = \sum_{n=0}^{N-1} e^{jk\hat{r}_0\cdot\hat{x}_0 (n-1)\Delta}$$

(We accept answers in words only if well explained.)

<u>1.4.</u>
(4_p) $\vec{G}(\theta,\varphi)$ = (Incremental source factor) · (spatial Fourier transform factor along wire or over aperture)

vector function — scalar function

(1) The scalar function varies rapidly.
(2) Represents a spatial Fourier transform for straight wire and planar aperture. Represents the array factor (discrete
(1) Fourier transform, i.e. Fourier series) for linear and planar arrays.

<u>1.5.</u> (8_p)

a) Straight wire antennas:

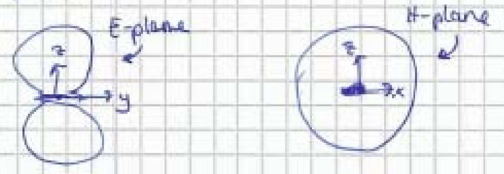
$$\vec{G}_e(\theta,\varphi) = \sin\varphi\cos\theta\,\hat{\theta} + \cos\varphi\,\hat{\varphi}$$

E

Incremental electric current

zy-plane is E-plane

(3) zx-plane is H-plane



b) Narrow slits in large flat ground planes

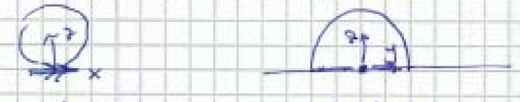
$$\vec{G}_m = \sin\varphi\,\hat{\theta} + \cos\varphi\cos\theta\,\hat{\varphi} \quad \text{for} \quad \theta \leq 90°$$

H

Incremental magnetic current

E- and H-planes as before, zy and zx planes.

(3)



H-plane E-plane

Note, far field function is zero below ground plane.

(3)

(2) c) Large apertures in that ground plane.

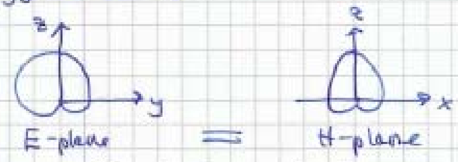
Sames as for b)

d) Apertures of large reflectors

$$\vec{G}_H = \frac{1}{2}(1+\cos\theta)\left(\sin\varphi\,\hat{\theta} + \cos\varphi\,\hat{\varphi}\right)$$

Huygen's source

(2)



E-plane = H-plane

Equal E-and H-planes.

1.6.(2p)

(2) Element factor $\vec{G}_{Element}(\theta,\varphi)$

29 For BOR_1 case $\vec{G}_{Element}(\theta,\varphi) = G_E(\theta)\sin\varphi\,\hat{\theta} + G_H(\theta)\cos\varphi\,\hat{\varphi}$

1.7. (3p)

(1) { Co-polarization: Desired polarization, for specific excitation.
Cross-polarization: Undesired, for same — " — .

(1) { Copolar y-pol. unit vector: $\hat{co} = \hat{y}_{co} = \sin\varphi\,\hat{\theta} + \cos\varphi\,\hat{\varphi}$
Cross — " — polar : $\hat{xp} = \hat{x}_{xp} = -\cos\varphi\,\hat{\theta} + \sin\varphi\,\hat{\varphi}$
(other signs are of course acceptable)

(1) { Copolar rad. field function $G_{co}(\theta,\varphi) = \vec{G}(\theta,\varphi)\cdot\hat{co}(\theta,\varphi)$
Crosspolar — " — $G_{xp}(\theta,\varphi) = \vec{G}(\theta,\varphi)\cdot\hat{xp}(\theta,\varphi)$

1.8 (2p)

$$\text{Directivity} = \frac{4\pi\,|G_{co}(\theta,\varphi)|^2}{P}$$ when maximum is for $\theta=0$.

P is integral of $|G_{co}|^2 + |G_{xp}|^2$ over 4π.

2.0 Antennas in multipath (15p)

2.1 Modern wireless communication systems are designed to work in multipath environments. Explain what we mean by a rich isotropic multipath environment.

2.2 Explain the main characterizing parameter of a single-port antenna in rich isotropic multipath. Use a mobile phone antenna as an example, and explain three different contributions to this characterizing parameter. Express one of them by the S-parameter measured at the port.

2.3 Explain the corresponding characterizing parameter of each port of a multi-port antenna in rich isotropic multipath. Explain the contributions to this factor of a two-port antenna and express two of them in terms of the S-parameters between the ports.

2.4 Explain antenna diversity. Explain how we can define two different diversity gains of a two-port antenna in rich isotropic multipath.

2.5 The far field function is very important for antennas in Line-Of-Sight systems, and in particular we need to minimize the cross-polarization in order to make use of dual polarization. Explain the importance of the radiation field function in rich isotropic multipath. In which way can we now make use of dual polarized antennas and still gain something. What is the characterizing parameter called, and which of the diversity gains in 2.4 does it affect? Write the formula for diversity gain reduction if you remember it.

⑧

2.0 Antennas in multipath
(P.S. Kildal)

2.1 ①The AoA are uniformly distributed over 4π.
(3p) ①The polarization of arriving waves is arbitrary distributed.
①The number of waves is large.
(Three conditions. 1p for each.)

2.2. Radiation efficiency, including mismatch factor,
(3p) i.e. total radiation efficiency.
also referred to as

① The factors: Ohmic losses in antenna itself and chassis of phone when mounted on the phone.
② Ohmic losses in the user's head
Mismatch factor (or reflection eff.)

The latter is
③ $$e_{mismatch} = 1 - |S_{11}|^2$$

2.3. (4p)
① Embedded element efficiency, including mismatch, i.e. total embedded element efficiency.
also called

Three contribution:
① Losses inside antenna and close surroundings like user's head.
② Decoupling efficiency due to absorptions on neighboring ports
Mismatch efficiency.

$$e_{embedded,\ lossless\ case} = 1 - |S_{11}|^2 - |S_{21}|^2$$

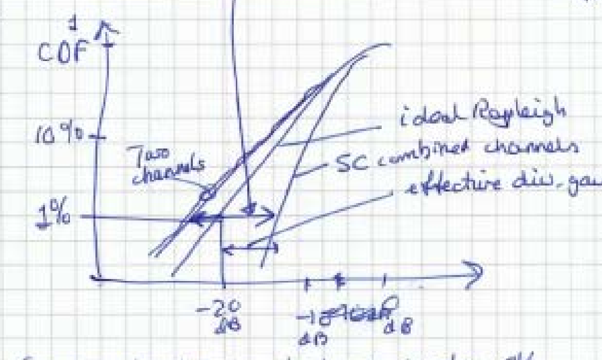
$$e_{mismatch} = 1 - |S_{11}|^2$$

⑤

2.4. Antenna diversity
(3p)

① Effective diversity gain = Apparent diversity gain * Total embedded element efficiency

②



Good explanation by text only is also OK.

2.5. Radiation field function plays a minor roll in
(2p) multipath. Most important is efficiency.

① We can use dual-pol. to achieve very good diversity antenna: Polarization diversity, because the two antennas can be co-located.

① Characterizing parameter in correlation. It effects only apparent diversity gain.

$$ADG = ADG_{max}\sqrt{1-|\rho|^2}$$

complex correlation

Somewhat better formula

$$ADG = ADG_{max}\sqrt{1-0.99|\rho|^2}$$

$ADG_{max} = 10.48$, i.e. 10.2 dB

PART B (You can use the textbook to solve these problems, but only after PART A has been delivered)

3.0 Microstrip patch antenna (30p)

You should design a microstrip line fed rectangular microstrip patch antenna operating at 6 GHz. The substrate is assumed to be very thin and has the relative permittivity $\varepsilon_r = 10.2$.

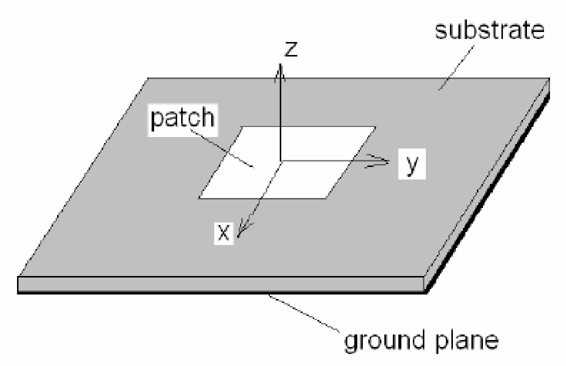


Figure 3.1 Microstrip patch antenna

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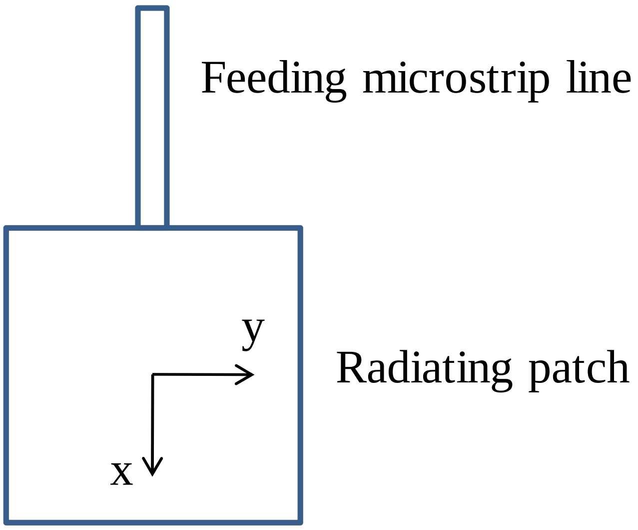
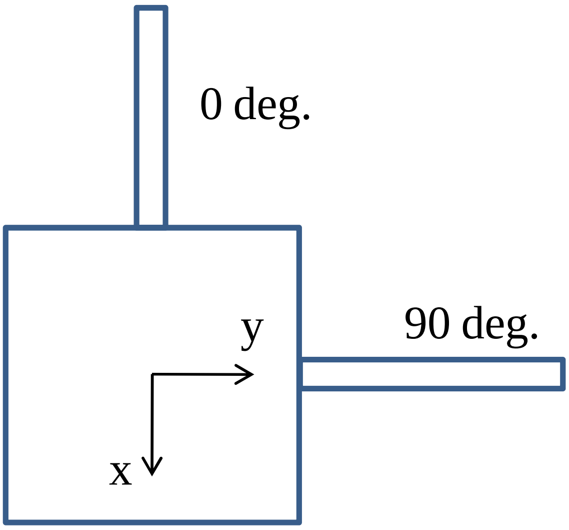
3.1 The antenna should be x-polarized. Sketch how you would excite it with a microstrip line.

Solution: (2p) See figure.



3.2 Sketch how you would excite it for circular polarization using microstrip lines.

Solution: (3p) See figure.



- Microstrip lines should be fed with 90 degrees phase difference
- Amplitude should be the same
- Patch should be square (for 3p all these points are required)

Note: Other solutions are possible.

3.3 Sketch how you would excite if for dual linear polarization using microstrip lines.

Solution: (2p) Same as in 3.2 but microstrip lines should be fed in phase (no requirement of same amplitude).

3.4 For case 3.1 (linear x-pol.), which dimension determines the frequency of operation? What is the resonant length of the patch?

Solution: (3p) The length in the x-direction determines the frequency of operation. The resonant length is approximately given by $l \approx \frac{\lambda_g}{2}$ (1p) where from eq. 6.1 $\lambda_g = \frac{\lambda}{\sqrt{\varepsilon_{eff}}}$ and

$\lambda = \frac{c_0}{f} = \frac{3 \cdot 10^8}{6 \cdot 10^9} = 50 \text{ mm}$. Since the substrate is very thin, i.e. $h << w$, $\varepsilon_{eff} \approx \varepsilon_r$. (1p)

$$\Rightarrow l = \frac{\lambda}{2\sqrt{\varepsilon_r}} = \frac{0.05}{2\sqrt{10.2}} \approx 7.8 \text{ mm } (\approx 0.16\lambda) \text{ (1p)}$$

3.5 Write the expression for the far-field function and sketch the E- and H-plane radiation patterns. Give the $\varphi-\text{angle}$ for the E- and H-planes, respectively.

Solution: (4p) From eq. 6.7 and 6.8 (Note that the patch has not the same orientation as the one in the book):

$$G(\hat{r}) = C_k\left(\hat{y}\times\hat{r}\right)I_m\left(e^{-j\frac{kl}{2}\hat{x}\bullet\hat{r}} + e^{j\frac{kl}{2}\hat{x}\bullet\hat{r}}\right),\quad I_m = 4E_o h\int_{-w/2}^{w/2} e^{-jk(\hat{y}\bullet\hat{r})y'}dy'\,;C_k = \frac{-jk}{4\pi},$$

Where l is the length of the patch and w is the width.

E-plane is given by $\varphi = 0^\circ$ and H-plane by $\varphi = 90^\circ$. E-plane is shown in Fig. 6.5 in book, note that amplitude is almost constant due to the high permittivity. For H-plane amplitude is zero for $\theta = \pm 90^\circ$.

(1p for G, 1p for I_M, 1p for E- and H-plane, 1p for correct sketch)

3.6 For H-plane, express the far-field function as a function of the $\theta-\text{angle}$ in as short form as possible. Explain how you can find the 3-dB beamwidth in the H-plane from this expression. What is the value of the far-field function at $\theta = 60°$? The thickness of the substrate is assumed to be 0.01λ and the width of the patch 0.4λ.

Solution: (4p) From appendix C we have:

$$\begin{cases}\hat{y}\times\hat{r} = \cos\varphi\hat{\theta} - \cos\theta\sin\varphi\hat{\varphi}\\ \hat{x}\bullet\hat{r} = \sin\theta\cos\varphi\,;\ \hat{y}\bullet\hat{r} = \sin\theta\sin\varphi\end{cases}$$

$$\Rightarrow I_M = 4E_0 h\int_{-w/2}^{w/2} e^{jk\sin\theta\sin\varphi y'}dy' = \ldots = 8E_0 h\frac{\sin\left(\frac{kw}{2}\sin\theta\sin\varphi\right)}{k\sin\theta\sin\varphi}$$

$$\Rightarrow \mathbf{G}(\hat{\mathbf{r}}) = \mathbf{G}(\theta,\varphi) = 16E_0 hC_k\left(\cos\varphi\hat{\theta} - \cos\theta\sin\varphi\hat{\varphi}\right)\cos\left(\frac{kl}{2}\sin\theta\cos\varphi\right)\frac{\sin\left(\frac{kw}{2}\sin\theta\sin\varphi\right)}{k\sin\theta\sin\varphi}$$

For H-plane we have $\varphi = 90^\circ$ so;

$$\mathbf{G}(\theta,90^\circ) = -16E_0 hC_k\cos\theta\frac{\sin\left(\frac{kw}{2}\sin\theta\right)}{k\sin\theta}\hat{\varphi} = -16E_0 hC_k\frac{\sin\left(\frac{kw}{2}\sin\theta\right)}{k\tan\theta}\hat{\varphi}$$

$\left|\mathbf{G}(\theta,90^\circ)\right|$ has maximum for $\theta = 0^\circ$ which is given by;

$$\lim_{\theta\to 0}\left|16E_0 hC_k \frac{w}{2}\cos\theta \frac{\sin\left(\frac{kw}{2}\sin\theta\right)}{\frac{kw}{2}\sin\theta}\right| = 16E_0 hC_k \frac{w}{2}$$

The 3-dB beamwidth can be found by searching for the value of θ when $\left|\mathbf{G}\left(\theta_{3dB},90^\circ\right)\right| = \frac{1}{2}\left|\mathbf{G}\left(0^\circ,90^\circ\right)\right| = 4E_0 hC_k w$, the beamwidth is then given by $2\theta_{3dB}$.

$$\left|\mathbf{G}\left(60^\circ,90^\circ\right)\right| = 16E_0 h\left|C_k\right| \frac{\sin\left(\frac{kw}{2}\sin\theta\right)}{k\tan\theta} = ... = 6.5\cdot 10^{-3} E_0$$

(2p for $\mathbf{G}\left(\theta,90^\circ\right)$, 1p for explanation of how to compute beamwidth, 1p for value of $\mathbf{G}\left(60^\circ,90^\circ\right)$)

3.7 Find the value of input impedance of the resonant patch antenna? (same dimensions as in 3.6)

Solution: (2p) The radiation resistance can be read from Fig. 6.7 in the book, $R_{rad} = 150 \text{ ohm}$. Since the patch is resonant the imaginary part of the input impedance is zero (see also Fig. 6.8) and therefore $Z_{in} = R_{rad} = 150 \text{ ohm}$.

(1p for value, 1p for realizing that imaginary part is zero)

3.8 Calculate the matching efficiency (mismatch factor) in dB under the assumption of 50 ohm reference impedance.

Solution: (2p) The matching efficiency is given by: $e_{refl} = 1 - \left|S_{11}\right|^2$ (eq. 3.5 or 2.98).

Where the reflection coefficient is given by: $S_{11} = \frac{Z_{in} - Z_{ref}}{Z_{in} + Z_{ref}} = \frac{150-50}{150+50} = 0.5$

Thus, $e_{refl} = 1 - 0.5^2 = 0.75$

And finally in Decibel, $e_{refl} = 10\cdot\log_{10}\left(0.75\right) \approx -1.25 \text{ dB}$

(1p for correct linear value, 1p for correct dB value)

3.9 Calculate the directivity in dB by using the input impedance and voltage at the radiating edges.

Solution: (6p) In order to compute the directivity we first need to find the radiated power. The radiated power can be computed by the knowledge of the feeding voltage and the radiation resistance as: $P_{rad} = \frac{V^2}{2R_{rad}}$ (1p)

We know that the patch is very thin so the feeding voltage is simply given as the electric field between the patch and the ground plane multiplied (generally an integration is needed) by the height (i.e. the thickness of substrate). Thus we have: $V = E_0 h$ (1p)

And thereby $P_{rad} = \frac{(E_0 h)^2}{2R_{rad}}$.

The directivity is defined in eq. 2.69;

$$D_0 = \frac{4\pi |G_0|^2}{P} = \frac{4\pi |G_0|^2}{2\eta P_{rad}} = \{\eta \approx 120\pi\} = \frac{|G_0|^2}{60 P_{rad}} = \frac{R_{rad} |G_0|^2}{60(E_0 h)^2} \quad (2p)$$

Where $|G_0| = |G_{co}(0,0)| = \{\text{from problem 3.6}\} = |8E_0 h C_k w| = \frac{2E_0 hkw}{\pi}$ (1p)

Finally we have; $D_0 = \frac{R_{rad}}{60(E_0 h)^2}\left(\frac{2E_0 hkw}{\pi}\right)^2 = \frac{R_{rad}}{15}\left(\frac{kw}{\pi}\right)^2 = \frac{4R_{rad}}{15}\left(\frac{w}{\lambda}\right)^2 = \frac{4 \cdot 150}{15} 0.4^2 = 6.4$

In Decibel; $D_0 = 10 \cdot \log_{10}(6.4) \approx 8.1 \text{ dBi}$ (1p)

3.10 In a real antenna we have losses, give two contributions to the loss that reduces the radiation efficiency of the patch antenna.

Solution: (2p) Ohmic losses (i.e. absorption) in the conductive and dielectric parts of the antenna. Conductive loss is given by the conductivity of the metal parts and the loss in the substrate is given by the loss tangent of the dielectric material.

(1p for each contribution)

4.0 Two-dipole array (20p)

Figure 4.1 shows an array of two halfwave dipoles. The self impedance of each dipole and the mutual impedance between them are Z_{11} and Z_{12}, respectively. We know that $Z_{12}=Z_{21}$ due to reciprocity. The spacing between the two dipoles is d, and the location of them is defined in the figure. The wavelength at the operating frequency is λ. Please answer the following questions in terms of parameters Z_{11}, Z_{12}, d, λ, V_1, V_2 and the constants C_k, k and free space impedance η).

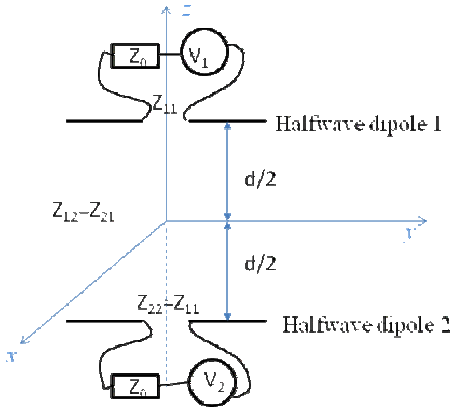
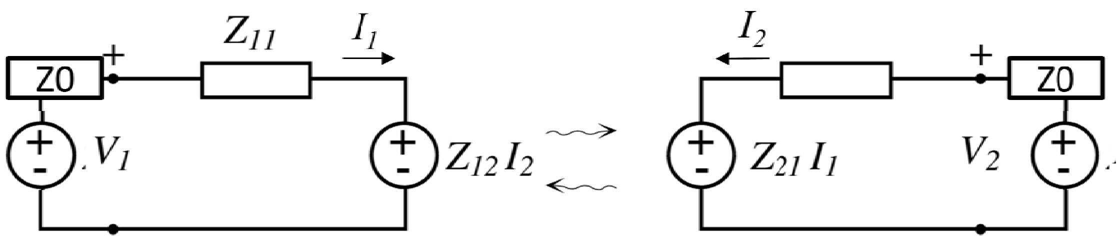


Figure 4.1 two dipole array

4.1 Draw the equivalent circuit of the array.(3p)

A:



The main points are that you should include Z_0 and use voltage source.

4.2 Write the expression for the far field function of dipole 1 alone when it is excited with V_1 without dipole 2 present.(3p)

A:

$$G_1(\theta,\varphi) = C_k\eta\frac{V_1}{Z_{11}+Z_0}\left(\cos\theta\sin\varphi\hat{\boldsymbol{\theta}}+\cos\varphi\hat{\boldsymbol{\varphi}}\right)\tilde{j}(\theta,\varphi)e^{jk\frac{d}{2}\cos\theta}$$

$$\tilde{j}(\theta,\varphi)=\frac{2}{k}\frac{\cos\left(\frac{\pi}{2}\sin\theta\sin\varphi\right)}{1-(\sin\theta\sin\varphi)^2}$$

1.5 points are for using V_1, Z_{11} and Z_0 to express I_1.

1.5 points are for phase change due to antenna moving from the original of the coordinate system.

4.3 Write the expression for the input impedance of dipole 1 when dipole 1 is excited with V_1 and dipole 2 is terminated with load Z_0.(3p)

A:

$$Z_{11emb} = Z_{11} - \frac{Z_{12}^2}{Z_{11} + Z_0}$$

4.4 Write the expression for the far field function of the array and the input impedance of dipole 1 when dipole 1 is excited with V_1 and dipole 2 with V_2.(6p)

A:

Solve the currents from

$$V_1 = \left(Z_{11} + Z_0\right) I_1 + Z_{12} I_2$$

$$V_2 = \left(Z_{11} + Z_0\right) I_2 + Z_{12} I_1$$

$$I_1 = \frac{\left(Z_{11} + Z_0\right) V_1 - Z_{12} V_2}{\left(Z_{11} + Z_0\right)^2 - Z_{12}^2}$$

$$I_2 = \frac{\left(Z_{11} + Z_0\right) V_2 - Z_{12} V_1}{\left(Z_{11} + Z_0\right)^2 - Z_{12}^2}$$

$$\frac{I_2}{I_1} = \frac{\left(Z_{11} + Z_0\right) V_2 - Z_{12} V_1}{\left(Z_{11} + Z_0\right) V_1 - Z_{12} V}$$

$$G(\theta, \varphi) = C_k \eta \frac{\left(Z_{11} + Z_0\right) V_1 - Z_{12} V_2}{\left(Z_{11} + Z_0\right)^2 - Z_{12}^2} \left(\cos\theta \sin\varphi \hat{\boldsymbol{\theta}} + \cos\varphi \hat{\boldsymbol{\varphi}}\right) \tilde{j}\left(\theta, \varphi\right) e^{jk\frac{d}{2}\cos\theta}$$

$$+ C_k \eta \frac{\left(Z_{11} + Z_0\right) V_2 - Z_{12} V_1}{\left(Z_{11} + Z_0\right)^2 - Z_{12}^2} \left(\cos\theta \sin\varphi \hat{\boldsymbol{\theta}} + \cos\varphi \hat{\boldsymbol{\varphi}}\right) \tilde{j}\left(\theta, \varphi\right) e^{-jk\frac{d}{2}\cos\theta}$$

$$\tilde{j}\left(\theta, \varphi\right) = \frac{2}{k} \frac{\cos\left(\frac{\pi}{2} \sin\theta \sin\varphi\right)}{1 - \left(\sin\theta \sin\varphi\right)^2}$$

$$Z_{1scan} = Z_{11} + Z_{12} \frac{\left(Z_{11} + Z_0\right) V_2 - Z_{12} V_1}{\left(Z_{11} + Z_0\right) V_1 - Z_{12} V_2}$$

4.5 We want the shape of the far field function when both dipoles are excited to be the same as the far field function of dipole 1 alone without dipole 2 present, how should we choose V_2 relative to V_1 to achieve this? (5p)

A:

Then, it is required that I_2 should vanish ($I_2=0$) and therefore,

$$V_2 = \frac{Z_{12}}{Z_{11} + Z_0} V_1$$