ESS017 Reglerteknik E ht14
Winter 2014

01/02/15

Student code: ____________________

This exam contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing.

You are allowed to use your cheat sheets, β handbook and calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **The maximum grade** is obtained for 28 points. The passing grade will be set at 15 points or less.
- **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will impede the crediting.
- **Mysterious or unsupported answers** will not receive full credit. A correct answer, unsupported by clear calculations, or clear explanations will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial or even full credit.
- **There are hard and easier questions**, dispatched throughout the exam. The questions are not sorted by order of difficulty but by theme. If you get stuck somewhere, move on to something easier.

Problem	Points	Score
1	10	
2	4	
3	10	
4	11	
Total:	35	

Best of luck to all !!

1. **Controller design**: consider a system modelled by the following transfer function:

$$G(s) = \frac{e^{-\tau s}}{s} \tag{1}$$

   (a) (1 point) What type of controllers ought to be used ? Justify your answer.

   (b) (3 points) Suppose that a PD controller in its ”unfiltered” form:

$$F(s) = K_{\mathrm{P}}(1 + \tau_{\mathrm{D}} s) \tag{2}$$

   is selected. The cross-over frequency $\omega_{\mathrm{c}} = 1.5\,[\mathrm{rad/s}]$ is imposed, and a phase margin $\varphi_{\mathrm{m}} = 20\,\mathrm{deg}$ is required. Propose a design K_{P}, τ_{D}.

   (c) (1 point) Suppose that one wants to use a P controller $F(s) = K_{\mathrm{P}}$. Is it possible to impose specifications on the cross-over frequency and the phase margin as in the previous question ? Justify.

   (d) (2 points) What is the cross-over frequency that can be achieved if the phase margin $\varphi_{\mathrm{m}} = 20\,\mathrm{deg}$ is to be retained ? What is the corresponding K_{P} ?

   (e) (3 points) Suppose that we have an uncertainty $\Delta\tau$ over the time-delay τ. Provide a condition bounding $\Delta\tau$ so that the closed-loop stability is guaranteed for the P controller designed in the previous questions (you will not manage to provide an explicit bound, but you can obtain a condition that must be respected at all frequencies).

2. **Sensitivity & Nyquist**:

   (a) (2 points) The Bode diagram and the Nyquist diagram present the same information about a transfer function in two different forms. What information is that and why do we use two different ways of representing that information in control ? What transfer functions do we represent in the Nyquist diagram, and why that specific function ?

   (b) (2 points) Justify the robust condition $\|\Delta G(j\omega)\| < \|T(j\omega)\|^{-1}$ in the Nyquist diagramm.

3. **State-space**: consider the system:

$$G(s) = \frac{1}{s^2} \tag{20}$$

   (a) (2 points) Write the controllable & observable canonical forms for $G(s)$.

   (b) (4 points) Design a state feedback & a state observer imposing all the closed-loop poles and the poles of the observer at -1

   (c) (4 points) Provide the transfer function of the controller $F(s)$ resulting from combining the state feedback and the state observer. Be careful about the state-space in which you have computed your feedback and observer gain matrix.

4. **Nonlinear system** The Watt regulator (see Fig. 1) is the first controller ever devised. It is made of two masses linked to a vertical spinning axis via two joints. The faster the vertical axis is spinning, the higher the masses are lifted by the centrifugal force. The Watt regulator had a huge impact on the deployment of steam engines in the industry, as it was the only way to accurately control their speed. The masses of the Watt regulator were connected to a tap controlling the steam intake in the engine via a cable, so that the higher the masses were, the less steam was admitted in the engine.

   The Watt regulator dynamics are described by the ordinary-differential equation, which reads as:

$$\ddot{\theta} = -\sin(\theta)\left(\frac{g}{L} - \cos(\theta)\omega^2\right) - \xi\dot{\theta} \tag{33}$$

   Moreover, we will assume that the steam engine has the linear dynamics:

$$\dot{\omega} = -\tau\omega + K(\theta - \theta_0) \tag{34}$$

   In the following, we use the following setup: $L = 0.1\,\mathrm{m}$, $g = 10\,\mathrm{m}\cdot\mathrm{s}^{-2}$, $m = 0.1\,\mathrm{kg}$, $\xi = 50\,\mathrm{s}^{-1}$ and $\omega = 4\pi\,\mathrm{rad/s}$, $\tau = 1\,\mathrm{s}$, $K = 10^2\,\mathrm{s}^{-1}$.

   (a) (1 point) If we look at the Watt regulator as a controller $F(s)$ connected to the steam engine being the system $G(s)$, what are physically the system input u, output y and reference r ?

   (b) (4 points) Provide a non-linear state space representation of (33), and a linear state-space representation for a give steady-state speed ω (you are allowed to provide them in a symbolic form, i.e. without replacing the variables for their numerical values).

   (c) (3 points) Compute the transfer function of the Watt regulator. What type of controller would it be ? Justify your answer, you may have to elaborate a bit.

   (d) (3 points) Is the closed-loop system made by connecting the Watt controller and the steam engine stable ? (Hint: you can apply the Routh-Hurwitz' criteria here, you'll find in your cheat-sheet)

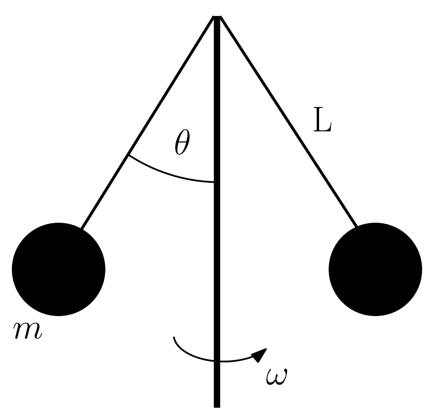

Figure 1: Schematic of the Watt regulator