ESS017 Reglerteknik E ht13
Summer 2014
Exam 2
21/08/14

Student code: ______________________

This exam contains 13 pages (including this cover page) and 5 problems. Check to see if any pages are missing.

You are allowed to use your cheat sheets, β handbook and calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **The maximum grade** is obtained for 33 points, hence you can fail 10 points, and still get the top grade. The passing grade will be given at 13 points.
- **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will impede the crediting.
- **Mysterious or unsupported answers** will not receive full credit. A correct answer, unsupported by clear calculations, or clear explanations will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial or even full credit.
- **Do not get lost in endless computations**. Most of the proposed questions require limited calculations if approached correctly.
- **There are hard and easier questions**, dispatched throughout the exam. The questions are not sorted by order of difficulty but by theme. If you get stuck somewhere, move on to something easier.

Do not write in the table to the right.

Problem	Points	Score
1	7	
2	7	
3	8	
4	8	
5	13	
Total:	43	

Best of luck to all !!

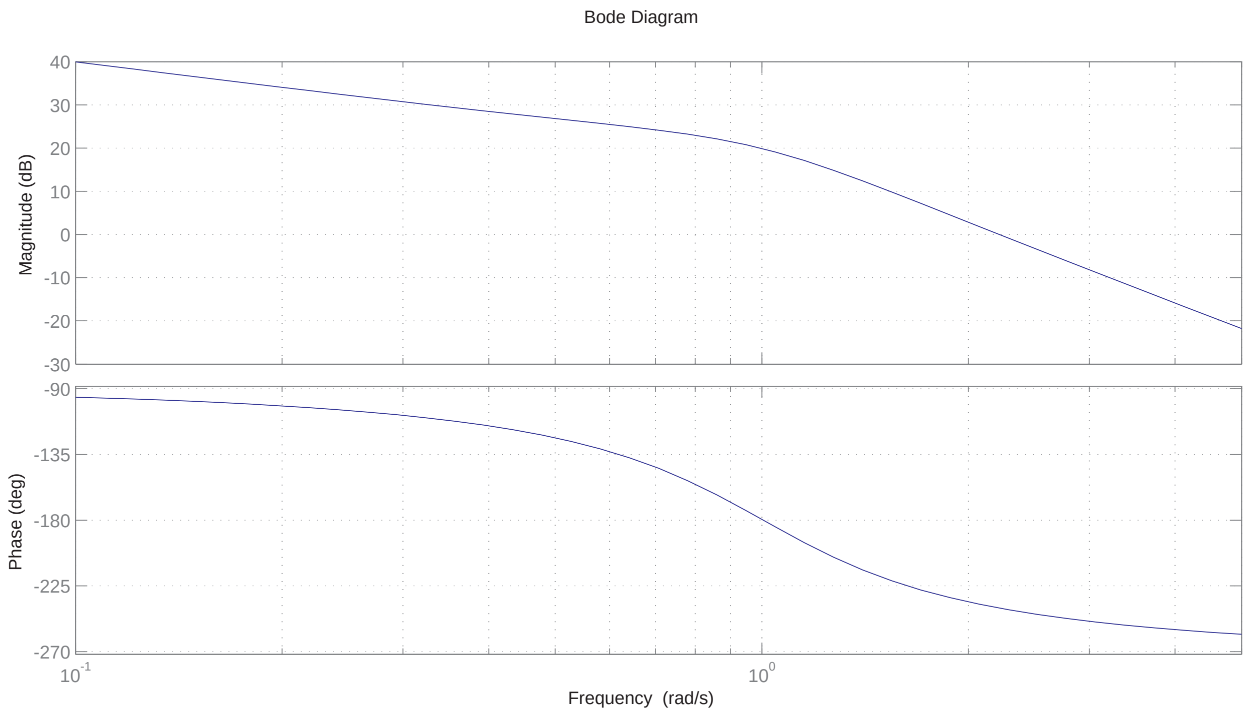

Figure 1: Bode plot of the transfer function $G(s)$ for the DC motor in position control.

1. **Controller design**: we consider a DC motor.The dynamics are described by the ordinary differential equation:

$$J\ddot{\theta}(t) + \xi\dot{\theta}(t) = k_t I(t) + T(t) \tag{1}$$

$$L\dot{I}(t) + RI(t) + k_v\dot{\theta}(t) = u(t) \tag{2}$$

where $\theta(t)$ is the rotor angle, $I(t)$ the motor current, $u(t)$ the motor voltage, and $T(t)$ the motor torque. Coefficient ξ stands for the mechanical friction, J the rotor inertia, L the motor inductance and R its resistance, k_t is the torque constant and k_v the velocity constant.

(a) (2 points) Write down the transfer functions from the voltage $u(t)$ to the velocity $\dot{\theta}$, and the transfer function from the voltage $u(t)$ to the angle θ.

(b) (3 points) We want to achieve position control on the DC motor. Suppose that a PD controller in the *lead* form:

$$F(s) = K_{\mathrm{P}}\left(\frac{1+\tau_{\mathrm{D}}s}{1+\tau_{\mathrm{D}}s/b}\right) \tag{3}$$

is selected. Propose a design K_{P}, τ_{D}, b maximising the control bandwidth while retaining a phase margin of about 60 [deg]. The lead filter shall not exceed a phase advance of 60 [deg]. The Bode plot of the DC motor transfer function $G(s)$ is displayed in Figure 1.

(c) (2 points) Is the PD structure (3) adequate for speed control ? If not, what controller structure would be adequate ? Justify.

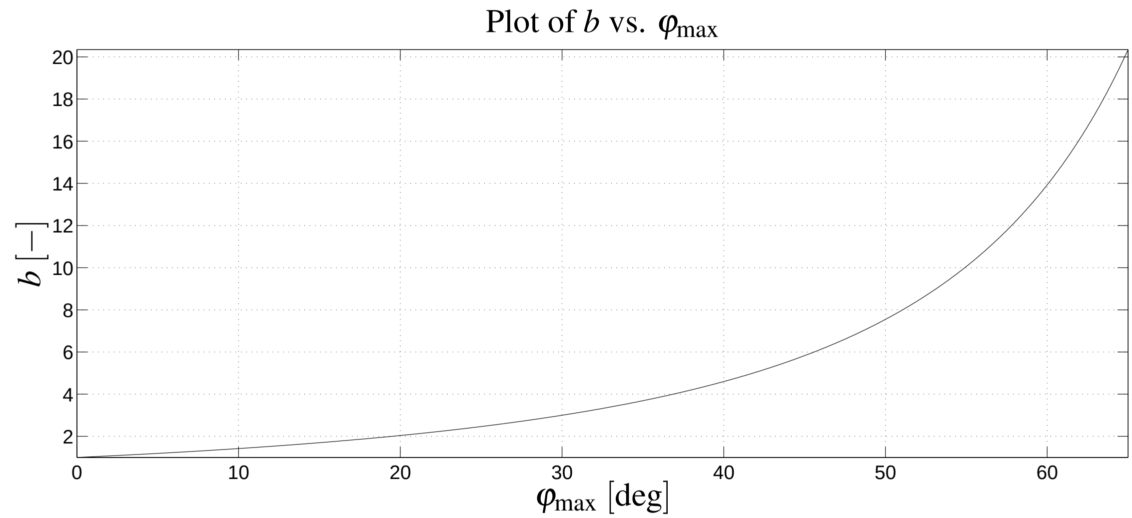

Figure 2: Plot of the formula $b = \frac{1+\sin\varphi_{\max}}{1-\sin\varphi_{\max}}$

Solution:

(a) Assembling the transfer function from u to $\dot{\theta}$ requires basic algebra, but it is a bit tricky. We use:

$$Js^2\theta(s) + \xi s\theta(s) = k_t I(s) + T(s) \tag{4}$$
$$LsI(s) + RI(s) + k_v s\theta(s) = u(s) \tag{5}$$

which yields:

$$I(s) = (Ls+R)^{-1}\left(u(s) - k_v s\theta(s)\right) \tag{6}$$

and

$$Js^2\theta(s) + \xi s\theta(s) = k_t\,(Ls+R)^{-1}\left(u(s) - k_v s\theta(s)\right) + T(s) \tag{7}$$

we then unpack (7) to get $s\theta(s)$ (i.e. the motor speed) on one side and $u(s)$ on the other side. We get:

$$\left[J(Ls+R)s + \xi(Ls+R) + k_t k_v\right]s\theta(s) = k_t u(s) + (Ls+R)T(s) \tag{8}$$

and finally:

$$s\theta(s) = \frac{k_t}{JLs^2 + (JR+\xi L)s + \xi R + k_t k_v}u(s) + \frac{Ls+R}{JLs^2 + (JR+\xi L)s + \xi R + k_t k_v}T(s) \tag{9}$$

Getting the transfer function from u to the angle θ is trivial to get from (9) by multiplying both sides by s^{-1}. We get:

$$\theta(s) = \frac{k_t}{JLs^3 + (JR+\xi L)s^2 + (\xi R + k_t k_v)s}u(s) + \frac{Ls+R}{JLs^3 + (JR+\xi L)s^2 + (\xi R + k_t k_v)s}T(s) \tag{10}$$

(b) One should use the largest possible b. A phase advance of $\varphi_{\max} = 60\,[\text{deg}]$ is allowed, which corresponds to a value of $b \approx 14$ (see Figure 2). The maximal phase advance of the lead filter is best placed at the cross-over frequency, i.e. $\angle F(j\omega_c) = \varphi_{\max} = 60\,[\text{deg}]$. Using the formula (c.f. cheat-sheets):

$$\angle F(j\omega_c) = -180 + \varphi_{\text{margin}} - \angle G_{\text{speed}}(j\omega_c)$$

where $G_{\text{speed}}(s)$ is given by (10) and by the Bode plot 1 (left graph), and the required phase margin is $\varphi_{\text{margin}} = 60\,[\text{deg}]$, we find:

$$\angle G_{\text{speed}}(j\omega_c) = -180 + 60 - 60 = -180\,[\text{deg}]$$

We find by inspection in the bode plot that $\omega_c \approx 1\,[\text{rad/s}]$ fullfills this condition with (approximate answers are accepted here):

$$|G(j\omega_c)| = 20\,[\text{dB}]$$

With $\omega_c = 1\,[\text{rad/s}]$, we can find τ_D so as to set the frequency of the maximum phase advance of the controller $F(s)$, using $\omega_c = \frac{\sqrt{b}}{\tau_D}$ we get:

$$1 = \frac{3.74}{\tau_D} \quad \Rightarrow \quad \tau_D = 3.74$$

Finally, we compute K_P using $|G(j\omega_c)| = 20\,[\text{dB}]$. Indeed, at ω_c we have:

$$|L(j\omega_c)| = |F(j\omega_c)|\,|G(j\omega_c)| = 1 \equiv 0\,[\text{dB}].$$

It follows that $|F(j\omega_c)| = |G(j\omega_c)|^{-1} = -20\,[\text{db}] \equiv 0.1$. We finally use (see the cheat-sheets):

$$|F(j\omega_c)| = \sqrt{b}K_P = 0.0562 \quad \Rightarrow \quad K_P = 0.0325$$

(c) The PD is not an adequate structure for speed control. Indeed the transfer function u to $\dot{\theta}$ is related to the transfer function charted in Figure 1 by a multiplication by s^{-1}. It follows that the gain of the system at low frequency is finite, and not significantly larger that its gain at the cross-over frequency. This makes it difficult to achieve a small static error with a PD controller. A PID would be adequate in this situation, as it would artificially introduce the slope $-20\,[\text{dB/dec}]$ present in the Bode plot 1 and therefore introduce an infinite gain at low frequency.

2. **Nyquist and Bode interpretation**: consider a stable system $G(s)$ in closed-loop with the controller $F(s)$, described in Figure 3 via the Nyquist and Bode plot of its open-loop transfer function $L(s) = F(s)G(s)$, the Bode plot of its sensitivity function $S(s) = \frac{1}{L(s)+1}$ and the Bode plot of the closed-loop transfer function $T(s) = \frac{L(s)}{L(s)+1}$ (or complementary sensitivity function).

Tab 1: Specific values of the Bode plot of $L(s)$

Frequency (rad/s)	Magnitude (db)	Phase (deg)
0	∞	-90
0.246	0	-115
0.894	-13.3	-180

Tab 2: Specific values of the Bode plot of $S(s)$

Frequency (rad/s)	Magnitude (db)	Phase (deg)
0	$-\infty$	90
0.571	3.12	17.8
∞	0	0

(a) (1 point) What is the static error of the closed-loop system ? Justify.

(b) (2 points) At what frequency is the Nyquist curve the closest to the critical point -1 ? What is the distance ? Justify.

(c) (2 points) At what coordinates does the Nyquist curve intersect the real axis ? What frequencies do these points correspond to ? Justify

(d) (2 points) By what ratio can the controller gain be increased before reaching instability ?

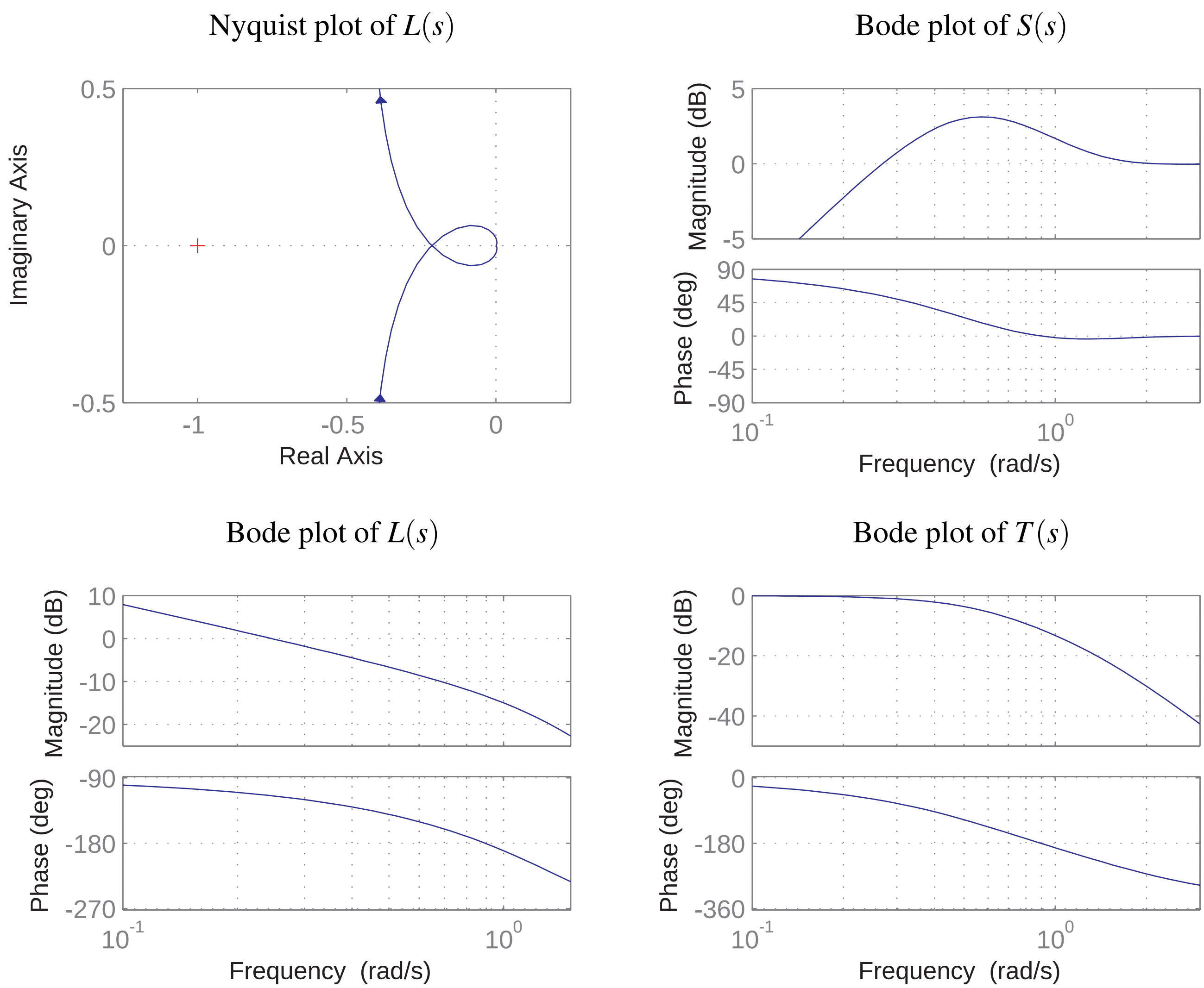

Figure 3: Poles and zeros of the closed-loop $T(s)$, of the open loop transfer function $L(s)$, Bode plot and Nyquist plot of the open loop transfer function $L(s)$.

Solution:

(a) The static error of the closed-loop system is nil. This can be guessed from the Bode plot of the open-loop transfer function $L(s)$ which has a slope at low frequency, and by the gain of the closed-loop transfer function $T(s)$, which has a unitary gain (0 dB) at low frequency. These observations can be confirmed form Table 1, which confirms the infinite gain of $L(s)$ at low frequency, and therefore that $\lim_{s\to 0} T(s) = 1$, which cancels the static error.

(b) We can observe from the Bode plot of the sensitivity function $S(s)$ that it reaches its maximum at about 0.6 [rad/s]. Since

$$S = \frac{1}{1+L} \tag{11}$$

the minimal distance from the Nyquist curve to the critical point -1, i.e.

$$\min_{\omega} \|1 + L(j\omega)\| = \max_{\omega} \|S(j\omega)\| \tag{12}$$

and ω =0.6 rad/s.

(c) The Nyquist curve intersects (in the limit sense) the real axis at the frequency $\omega \to \infty$, which corresponds to $\lim_{s\to\infty} L(s) = 0$. It also intersects the real axis when

$$\angle L(j\omega_c) = -180\,\text{deg} \tag{13}$$

at $\omega_c = 0.894$. At this frequency, we have:

$$\|L(j\omega_c)\| = -13.3\,\text{dB} \equiv 0.22. \tag{14}$$

The second intersection point is then $0.22 + 0j$.

(d) Increasing the controller gain inflates the Nyquist curve. The closed-loop system reaches instability when the "inflated" Nyquist curve intersects the critical point -1. In this case, when the point $0.22+0j$ reaches -1. The controller gain can therefore be increased by a ratio:

$$13.3\,\text{dB} \equiv 0.22^{-1} = 4.62. \tag{15}$$

The closed-loop performance will however suffer before instability is reached.

3. **State-space**: consider the transfer function:

$$G(s) = \frac{1}{s-1} + \frac{1}{s+1} - \frac{2}{s} \tag{16}$$

(a) (2 points) Give a state-space representation in the controllable canonical form. Justify.

(b) (2 points) Is the state-space representation stable ? Is it controllable ? Justify.

(c) (4 points) Design an observer that imposes all the poles of the error dynamics at -1.
Hints: remember that the poles of the state observer $\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})x$ for the state-space representation A, B, C in the observable canonical form are given by the characteristic equation $\lambda^n + \alpha_1 \lambda^{n-1} + \ldots + \alpha_{n-1}\lambda + \alpha_n = 0$ where $\begin{bmatrix} -\alpha_1 & -\alpha_2 & \ldots & -\alpha_n \end{bmatrix}$ is the first column of $A - LC$.

Solution:

(a) We need a transfer function in the form of a single fraction of polynomials, hence we start with rewriting $G(s)$ as:

$$G(s) = \frac{s(s+1) + s(s-1) - 2(s-1)(s+1)}{s(s+1)(s-1)} = \frac{2}{s^3 - s}$$

and then apply the formula for the controllable canonical form (see cheat sheets, and "Reglerteknik" p. 158), which yields:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix} x$$

(b) The transfer function has the poles $-1, +1, 0$ and is therefore unstable, hence the corresponding state-space representation is also unstable. Controllable canonical forms are by construction controllable.

(c) The observable canonical state-space representation of our transfer function reads as:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

The closed-loop poles of the state observer dynamics are given the eigenvalues of the matrix:

$$A - LC \tag{17}$$

where $L = \begin{bmatrix} l1 & l2 & l3 \end{bmatrix}^T$ is the observer gain matrix. Matrix $A - LC$ then reads as:

$$\begin{bmatrix} -l1 & 1 & 0 \\ 1 - l2 & 0 & 1 \\ -l3 & 0 & 0 \end{bmatrix} \tag{18}$$

whose poles are given by the characteristic equation (see hints):

$$\lambda^3 + l_1 \lambda^2 + (l_2 - 1)\lambda + l_3 = 0 \tag{19}$$

We want all solutions of (19) to be $\lambda = -1$, i.e. we want (19) to be identical to:

$$(\lambda + 1)^3 = \lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0 \tag{20}$$

By identification between (19) and (20), we get:

$$l_1 = 3, \qquad l_2 = 4, \qquad l_3 = 1$$

so that the observer gain reads as: $L = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix}$.

4. **Theory**: (thoroughly justified answers are expected here)

   (a) (2 points) Whenever the open-loop transfer function $L(s)$ of a stable system has a relative degree higher or equal to 2, the Bode first integral states that:

$$\int_0^\infty \log|S(j\omega)|d\omega = 0 \tag{21}$$

   Explain in what sense (21) describes a fundamental limitation of control.

   (b) (2 points) Explain in what sense the control of nonlinear systems is more challenging that the control of linear systems.

   (c) (2 points) Explain the difference between the simplified and the full Nyquist criteria. When does the simplified apply ? How does one apply the full Nyquist criteria ?

   (d) (2 points) Explain in detail the meaning and consequences of the inequality (c.f. cheat-sheets):

$$|T(j\omega)| \leq \frac{1}{|\Delta G(j\omega)|}, \quad \forall \omega \geq 0$$

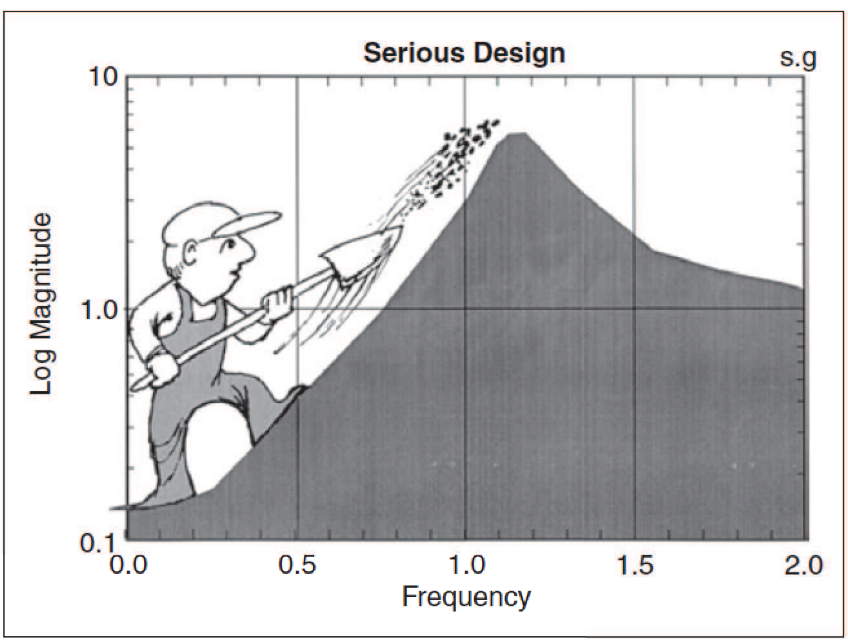

Figure 4: From "Respect the Unstable", G. Stein, IEEE Control System Magazine, 2003.

Solution:

(a) The Bode first integral is a statement on the balance between the area below 0db and above 0db in the log-linear plot of the magnitude of the sensitivity function $|S(j\omega)|$. Good control performances are obtained over the bandwidth where the closed-loop transfer function $T(s) = \frac{L(s)}{1+L(s)}$ has a magnitude close to 0 dB (i.e. close to 1). The closed-loop transfer function is close to 1 where the sensitivity function $S(s) = \frac{1}{1+L(s)}$ has a low magnitude. In order to increase the closed-loop performance one needs to increase the frequency range where $S(j\omega)$ is low. However, the fundamental equation (21) states that the total *area* of the function $S(j\omega)$ that is above 0 dB in a log-linear plot must match the area below 0 dB. The consequence of this principle is illustrated in Fig. 4, where digging the sensitivity function in a given frequency range necessarily increases the sensitivity at other frequencies. For this reason this effect is often referred to as the water-bed effect.

Generally, in control, we dislike large sensitivities. When the sensitivity is large, a) model errors have a strong impact on the closed-loop transfer function, b) the modulus margin, and therefore the robustness is poor.

(b) The control of linear systems can be tackled via the powerful tools available in linear control theory, which are ultimately all derived from the superposition principle, and functional analysis. However, the tools of linear control fail to apply as such to nonlinear systems. An approach often used to control nonlinear system is to develop a linear model that is approximating the nonlinear dynamics at a chosen set point, and deploy the tools of linear control on that linear model instead. Though this approach usually works in practice, it must be carried out with great care. Indeed, the approximate model is valid in a neighbourhood of the chosen set point, and may become fast invalid once the system is moved away from that set point. For that reason, 1) a robust controller shall be designed, so that the approximation introduced by the linear model does not impact too much the control performance and the closed-loop stability, and 2) one must be careful that the closed-loop system does not leave the domain around the set point where the linear model is reasonably good.

(c) The simplified Nyquist criteria applies when the open-loop transfer function $L(s)$ is stable. In that case, it is sufficient to check that the Nyquist curve does not encircle the critical point -1 in the complex plane. If the open-loop transfer function $L(s)$ is unstable, then the simplified criteria does not apply, and the full criteria must be used. The full criteria requires that the Nyquist curve encircles the critical point -1 as many times as $L(s)$ has unstable poles. Note that the direction (clockwise or counterclockwise) in which the Nyquist curve encircles -1 matters, and depends on which direction we span the imaginary axis when constructing the Nyquist contour.

(d) The formula $|T(j\omega)| \leq |\Delta G(j\omega)|^{-1}$, $\forall \omega \geq 0$ establishes an upper bound to the amplitude of the closed-loop transfer function, i.e. $|T(j\omega)|$ based on the amplitude of the relative model error:

$$\Delta G(s) = \frac{G_0(s) - G(s)}{G(s)},$$

in order to keep closed-loop stability. It essentially states that at frequencies ω where the relative model error $|\Delta G(j\omega)|$ is large, then the amplitude of the closed-loop transfer function, i.e. $|T(j\omega)|$, must be small, otherwise stability can be lost.

High closed-loop performance requires that the closed-loop transfer function $|T(j\omega)| \approx 1$ up to high frequencies. Therefore, the proposed inequality imposes a limit to the closed-loop performance, in the sense that whenever $|\Delta G(j\omega)| > 1$, then $|T(j\omega)|$ cannot be close to 1, and the closed-loop performance is degraded. Note that the relative model error is typically high at higher frequencies, where the model gain is small, and where un-modelled dynamics are often present.

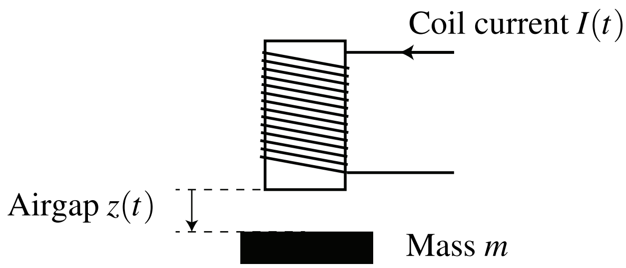

Figure 5: Sketch of the magnetic levitation.

5. **Nonlinear system** Consider a mass suspended in levitation in the magnetic field developed by a electro-magnet (see Figure 5). The evolution of the airgap[1] $z(t)$ is often modelled as:

$$\ddot{z} = g - K\frac{I^2}{z^2} \tag{22a}$$

$$u = RI + \frac{d}{dt}\left(L(z)I\right) \tag{22b}$$

where g is the gravity, R is the coil resistance, I is the coil current, u is the coil voltage, and K a magnetic coefficient. We assume that the impedance $L(z)$ depends linearly on the position z, i.e.:

$$L(z) = L_0 + L_1 z \tag{23}$$

where L_0, L_1 are constants.

(a) (2 points) Provide a nonlinear state-space representation of (22).

Hint: use the chain-rule

$$\frac{d}{dt}\left(L(z)I\right) = \frac{dL(z)}{dt}I + L(z)\frac{dI}{dt} = \frac{dL(z)}{dz}\dot{z}I + L(z)\dot{I} \tag{24}$$

(b) (2 points) Find the steady-state point corresponding to a given airgap z_0.

(c) (2 points) Compute a linear state-space representation approximating the system dynamics at the steady-state corresponding to the reference position $z = z_0$.

(d) (3 points) Is the system controllable ? Justify.

(e) (4 points) Consider the magnetic levitation with the mass dynamics (22a) only, i.e. we us the static approximation $I = \frac{u}{R}$ instead of (22b). Show that it is not possible to stabilise this system with a P controller.

Solution:

(a) We use $\frac{dL(z)}{dz}\dot{z}I + L(z)\dot{I} = L_1\dot{z}I + (L_0 + L_1 z)\dot{I}$ and use it in (22b) to get:

$$\dot{I} = (L_0 + L_1 z)^{-1}\left(u - RI - L_1\dot{z}I\right) \tag{25}$$

[1] distance of the mass from the magnet

A possible nonlinear state-space representation then reads as:

$$\text{state:} \quad x = \begin{bmatrix} z \\ \dot{z} \\ I \end{bmatrix}, \qquad \text{output:} \quad y = z,$$

$$\dot{x} = f(z,\dot{z},I,u) = \begin{bmatrix} \dot{z} \\ g - K\frac{I^2}{z^2} \\ (L_0 + L_1 z)^{-1}(u - RI - L_1\dot{z}I) \end{bmatrix} \tag{26}$$

(b) The steady-state point can be computed from z_0 using $\ddot{z} = \dot{z} = \dot{I} = 0$ in (22a)-(22b) to get:

$$I_0 = \left(\frac{g}{K}\right)^{\frac{1}{2}} z_0, \qquad u_0 = RI_0 = R\left(\frac{g}{K}\right)^{\frac{1}{2}} z_0 \tag{27}$$

(c) We linearise the dynamics (26) at the previously computed steady-state point:

$$\Delta\dot{x} = \underbrace{\frac{\partial f}{\partial x}}_{\equiv A}\Delta x + \underbrace{\frac{\partial f}{\partial u}}_{\equiv B}\Delta u$$
$$\Delta y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{=C}\Delta x$$

where $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial u}$ are evaluated at the point (27). This yields:

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2g}{z_0} & 0 & -\frac{2K}{z_0}\left(\frac{g}{K}\right)^{\frac{1}{2}} \\ 0 & -\frac{L_1 z_0}{L_0+L_1 z_0}\left(\frac{g}{K}\right)^{\frac{1}{2}} & -\frac{R}{L_0+L_1 z_0} \end{bmatrix}, \qquad B = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_0+L_1 z_0} \end{bmatrix} \tag{28}$$

Finally, matrix D is null.

(d) Using the linearisation (28), the controllability matrix reads as:

$$\mathscr{C} = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \frac{1}{L_0 + L_1 z}\begin{bmatrix} 0 & 0 & -\frac{2K}{z_0}\left(\frac{g}{K}\right)^{\frac{1}{2}} \\ 0 & -\frac{2K}{z_0}\left(\frac{g}{K}\right)^{\frac{1}{2}} & \frac{2KR}{z_0(L_0+L_1 z_0)}\left(\frac{g}{K}\right)^{\frac{1}{2}} \\ 1 & -\frac{R}{L_0+L_1 z_0} & \left(\frac{R}{L_0+L_1 z_0}\right)^2 + \frac{2L_1 g}{L_0+L_1 z_0} \end{bmatrix}. \tag{29}$$

Matrix $\mathscr{C}$ is lower-triangular $\mathscr{C}$ and therefore full-rank, hence the state-space representation is controllable. Note that one can make this observation without actually computing $\mathscr{C}$, by assessing its zero entries without actually computing the non-zero entries.

(e) The easiest way to prove this is to compute a state-space representation for the reduced system. We can reuse a lot of the work done previously. The nonlinear state-space representation reads as:

$$\text{state:} \quad x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}, \qquad \text{output:} \quad y = z,$$

$$\dot{x} = f(z,\dot{z},u) = \begin{bmatrix} \dot{z} \\ g - \frac{K}{R^2}\frac{u^2}{z^2} \end{bmatrix} \tag{30}$$

and its linearisation at an arbitrary position z_0 reads as:

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ \frac{2g}{z_0} & 0 \end{bmatrix}, \qquad B = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ -\frac{2K}{Rz_0}\left(\frac{g}{K}\right)^{\frac{1}{2}} \end{bmatrix} \tag{31}$$

A P controller would act on the position z only, and take the generic form $u = -\begin{bmatrix} k & 0 \end{bmatrix} x$, so that the closed-loop matrix of the linearised system reads as:

$$A - BK = \begin{bmatrix} 0 & 1 \\ 2\frac{Rg+Kk}{Rz_0}\left(\frac{g}{K}\right)^{\frac{1}{2}} & 0 \end{bmatrix} \tag{32}$$

Note that it is not even necessary to compute this matrix to get to the result. Indeed, the eigenvalues of a matrix of the form

$$\begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix} \tag{33}$$

are given by the characteristic equation

$$\lambda^2 - \alpha = 0 \tag{34}$$

which takes the solutions $\lambda = \pm\sqrt{\alpha}$. Hence the closed-loop poles are either

1. real if $\alpha > 0$, one negative and one positive, which would yield an exponentially unstable system
2. purely imaginary if $\alpha \leq 0$, yielding an unstable system in practice.

If one computes the closed-loop matrix (32), it can be observed that $\alpha > 0$, yielding an exponentially unstable system.