Exam in

RRY125/ASM510 Modern astrophysics

This is part 2 of the exam. Part 1 is a quiz in Canvas. Do part 1 first.

Time: 6 april 2020, kl. 14.00–16.00. Deadline for handing in solutions: 16.30.
Place: Remote exam in Canvas
Examiner: Magnus Thomasson (phone: 070 − 237 6701)
In case you need to ask the examiner something during the exam, use the phone, not email.

You may use:
All aids are allowed.
However, ***cooperation or help from other people is not permitted***.

Grades:
This exam is only for grade 3.
The maximum number of points is 45 (including the 15 p quiz in Canvas).
Grade 3 requires 25 p.
To get grade 4 or 5, students must get grade 3 on this written exam and take an oral exam on Wednesday 8 April 2020.

Hand-in:
- Hand in your solutions as pdf-files in Canvas. It is good if all pages are in one document. Jpg-files are also allowed, put pdf is preferred. Mark all pages with your name. **Deadline: 16:30**.
- Use a **document scanning app in a cell phone** for transforming your hand-written solutions to digital format. Alternatively, other available scanners can be used. Avoid using the camera app in your cell phone because of low image quality. It is advisable to have good lighting when scanning or taking photos. Examples of free document scanning apps are ***CamScanner*** and ***Genius Scan***, which both are available for iPhone and Android.
- In case of problems with Canvas, you may instead email your solutions to magnus.thomasson@chalmers.se (but this is only if upload to Canvas does not work).

Note: Motivate and explain each answer/solution carefully.

1.

The table below shows properties for six stars located in the same small region of the sky.

Star number	m_B	m_V	M_B	M_V	$B–V$	d (pc)	*parallax* (arcsec)
1		8,0		−2,0	1,4		
2		17,5			1,4	200	
3			−2,5	−2,0		200	
4	23,3			15,0			0,005
5	13,0	12,5				200	
6		21,5	15,0				0,005

a.) Draw a Hertzsprung-Russell diagram for the six stars. **(4 p)**
b.) Do all stars belong to the same star cluster? **(1 p)**
c.) Some of the stars are burning hydrogen in their cores. Choose one of these stars which uses the pp chain, and one which uses the CNO cycle. **(1 p)**
d.) Which of the stars is largest? **(2 p)**
e.) What type of star is star number 6? **(1 p)**
f.) Which star has the lowest surface temperature? **(1 p)**
g.) Two stars have the same or very similar surface temperature. Which two stars? What is the ratio between their radii? **(3 p)**

(Don't forget to show your calculations for question a, and to motivate your answers to questions b-g.)

2.

This problem is about an exoplanet in circular orbit around a star.
The star has a mass of 1.5 solar masses, a radius of $8.4 \cdot 10^5$ km, and a surface temperature of 8100 K.
The orbital period of the exoplanet (in days) is given by your *birthday*:

> Use your birth *month* (2 digits) and your birth *day* (2 digits) and concatenate (join) them to form one number.
> Example: January 8 → 0108 (108 days), December 24 → 1224 (1224 days)

The planet has an albedo (Bond albedo) of 0.5 and a radius of 12000 km.
Calculate *the surface temperature* of the exoplanet! (Disregard possible effects of an atmosphere.) **(8 p)**

3.

Observations of a galaxy clusters show that it has a mean redshift of $z = 0.2$ with a standard deviation of $\sigma_z = 0.004$. The cluster appears to be roughly spherical with an effective radius of 4 arc minutes on the sky.

The cluster contains N galaxies, where N is the same as number as the orbit period in days in problem 2, i.e., use your birth *month* (2 digits) and your birth *day* (2 digits) and concatenate (join) them to form one number; example: April 30 → 0430 (430 galaxies).

Assume that all galaxies are similar, emit most of their energy in the visual band, and that each galaxy has an internal velocity dispersion of 150 km/s and a mass-to-light ratio of 10 in solar units.

a.) Estimate the total dynamical mass of the cluster. **(3 p)**

b.) Estimate the total mass of all the galaxies taken together. **(3 p)**

c.) With new technology, astronomers have managed to study the supermassive black hole in *one* of the cluster galaxies at very high spatial resolution. They find a BH mass of $1 \cdot 10^7\ M_\odot$ and that it is accreting at near-maximum speed. What is the *luminosity* of the AGN compared to the host galaxy? The astronomers now dream of resolving the event-horizon of the black hole. What *angular resolution* (in arc seconds) would such an astronomical telescope require? **(3 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi \nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^{-3} Hz^{-1}] $\qquad n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^{-3} Hz^{-1}] $\qquad U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1}] $\qquad I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\qquad \nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_\nu}{dz} = j_\nu - \alpha_\nu I_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \qquad d\tau_\nu = \alpha_\nu \, dz$

$I_\nu = I_{\nu,\mathrm{bg}} \cdot e^{-\tau_\nu} + S_\nu \cdot (1 - e^{-\tau_\nu}) \qquad T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_\nu} + T_\mathrm{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\ \mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ the Hubble-Lemaître law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} = \frac{a_0}{a}$ redshift

$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ the Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d = \frac{R}{\pi}$ $R = 1$ AU, π = parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ circular velocity

$\theta \approx 1.22\frac{\lambda}{\text{D}}$ resolution of telescope

$N(t) = N_0 e^{-\lambda t}; \ \lambda = \frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\text{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\text{e}} n_{\text{p}}$ recombination and ionization equation

$\frac{L_{\text{I}}}{4 \cdot 10^{10} L_{\text{I},\odot}} \approx \left(\frac{V_{\text{max}}}{200 \text{ km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\text{V}}}{2 \cdot 10^{10} L_{\text{V},\odot}} \approx \left(\frac{\sigma}{200 \text{ km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\text{E}} = \frac{4\pi G M m_{\text{p}} c}{\sigma_{\text{T}}} \approx 1.3 \cdot 10^{31} \frac{M}{\text{M}_\odot} (\text{watt}) \approx 30000 \frac{M}{\text{M}_\odot} \text{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\quad e^{-x} = \frac{1}{e^x}$ $\quad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\quad \lg xy = \lg x + \lg y$ $\quad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}

$c = 2.9979 \cdot 10^8$ m/s

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}

$h = 6{,}62606896 \cdot 10^{-34}$ J s

$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m

1 AU = $1.496 \cdot 10^{11}$ m

1 year = $3.156 \cdot 10^7$ s

1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8

The solar constant (1 AU from the Sun): 1371 W/m^2

$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses: Earth: $5.97 \cdot 10^{24}$ kg, Jupiter: $1.90 \cdot 10^{27}$ kg, Saturn: $5.69 \cdot 10^{26}$ kg, Sun: $1.99 \cdot 10^{30}$ kg

Radii: Earth: 6378 km, Jupiter: 71398 km, Saturn: 60270 km, Sun: $6.96 \cdot 10^5$ km

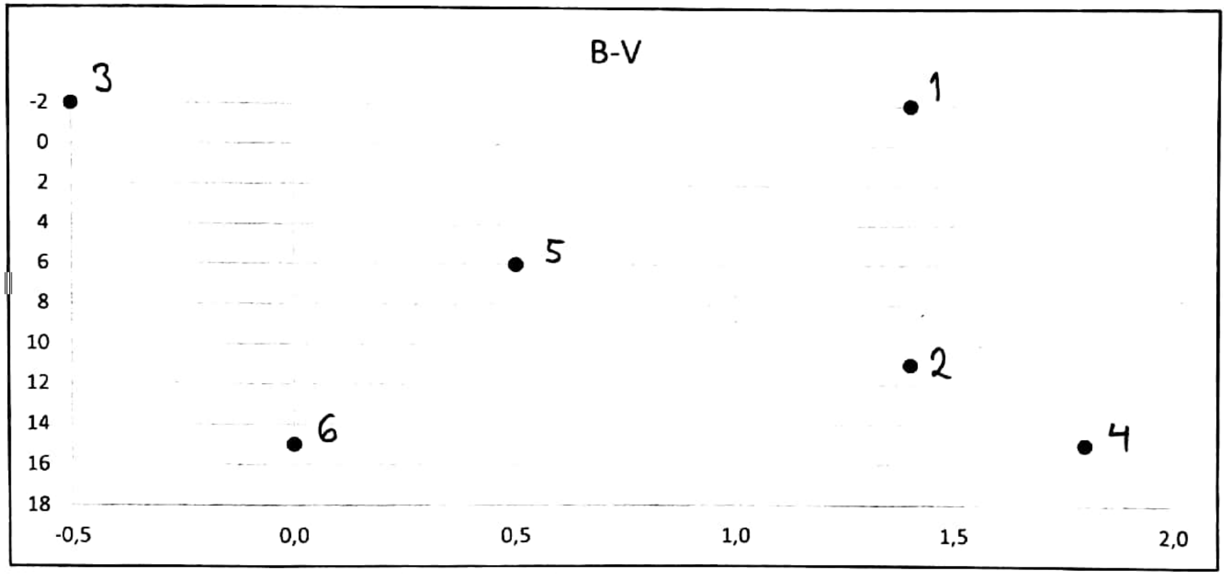
Six stars

Modern astrophysics

6 april 2020

Star no	m_B	m_V	M_B	M_V	B-V	d (pc)	parallax (")
1	9,4	8,0	-0,6	-2,0	1,4	1000,0	0,001
2	18,9	17,5	12,4	11,0	1,4	200,0	0,005
3	4,0	4,5	-2,5	-2,0	-0,5	200,0	0,005
4	23,3	21,5	16,8	15,0	1,8	200,0	0,005
5	13,0	12,5	6,5	6,0	0,5	200,0	0,005
6	21,5	21,5	15,0	15,0	0,0	200,0	0,005

B-V	M_V	d (pc)	
1,4	-2,0	1000,0	red giant
1,4	11,0	200,0	MS
-0,5	-2,0	200,0	MS, hot
1,8	15,0	200,0	MS, cold
0,5	6,0	200,0	MS
0,0	15,0	200,0	white dwarf



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① a) See fig. (M_V vs. B-V)

$m - M = 5 \lg d/10\,pc$. $d = \frac{1}{\text{parallax}}$

$B - V = m_B - m_V = M_B - M_V$

b) No, not #1. (distance)

c.) pp: #2, 4, 5 CNO: #3

d.) #1. Brightest, but cooler than #3 ⇒ must be larger

e.) White dwarf. Hot surface, low luminosity, i.e. small.

f.) #4. Highest B-V (red)

g.) $M_1 - M_2 = -2.5 \lg \frac{F_1}{F_2} = -2.5 \lg \frac{L_1}{L_2} = -2.5 \lg \frac{4\pi R_1^2 \sigma T_1^4}{4\pi R_2^2 \sigma T_2^4} =$

$= -2.5 \lg \frac{R_1^2}{R_2^2} = -5 \lg \frac{R_1}{R_2}$. $M_1 - M_2 = -2 - 11 = -13 \Rightarrow$

$\Rightarrow \frac{R_1}{R_2} = 10^{13/5} = 398.$

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(2) $M_s = 1.5\,M_\odot$, $R_s = 8.4\cdot10^5$ km, $T_s = 8100$ K

$A = 0.5$, $R_p = 12000$ km

$P = 101$ d to 1231 d $\qquad 1\,d = 24\cdot3600\,s = 86\,400\,s$

$$L_s = 4\pi R_s^2 \sigma T_s^4 = 2{,}16\cdot10^{27}\ \text{W}$$

$$\frac{a^3}{P^2} = \frac{GM_s}{4\pi^2} \quad \text{(neglect mass of planet)}$$

$$a = \left(\frac{GM_s}{4\pi^2}P^2\right)^{1/3} = \begin{cases} 7{,}27\cdot10^{10}\ \text{m}\ (0.49\ \text{AU})\ \text{for } P=101\,d \\ 3.85\cdot10^{11}\ \text{m}\ (2.6\ \text{AU})\ \text{for } P=1231\,d \end{cases}$$

Heat balance:

$$\frac{L_s}{4\pi a^2}\cdot\pi R_p^2(1-A) = 4\pi R_p^2\sigma T_p^4$$

$$\left(\frac{L_s}{4\pi a^2} = \text{"solar constant"} = \begin{cases} 32\,500\ \text{W/m}^2 & \text{for } P=101\,d \\ 1160\ \text{W/m}^2 & \text{for } P=1231\,d \end{cases}\right)$$

$$\frac{4\pi R_s^2\sigma T_s^4}{4\pi a^2}\cdot\pi R_p^2(1-A) = 4\pi R_p^2\cdot\sigma T_p^4$$

$$\frac{R_s^2 T_s^4}{a^2}(1-A) = 4T_p^4$$

$$T_p = \left(\frac{R_s}{a}\right)^{1/2}\left(\frac{1-A}{4}\right)^{1/4} T_s = \begin{cases} 518\ \text{K} & \text{for } P=101\,d \\ 225\ \text{K} & \text{for } P=1231\,d \end{cases}$$

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a) $z = 0.2$ $\quad v = H_0 \cdot D$

$1 + z = 1 + \frac{v}{c}$ $\rightarrow$ $v \sim 6 \times 10^4$ km/s.

for $H_0 = 72$ $\rightarrow$ $D \sim 833$ Mpc

$4.'0$ $\rightarrow$ $R \sim 1$ Mpc ; $\sigma \sim 1200$ km/s

$M_{dyn} \sim \frac{\sigma^2 R}{G} \sim 0.33 \times 10^{15} M_\odot$

b) Faber-Jackson : $L_{gal} \sim 0.63 \times 10^{10} L_\odot$

$M/L \sim 10$ $\rightarrow$ $M_{gal} \sim 6 \times 10^{10} M_\odot$

$N_{gal} \sim 430$ $\rightarrow$ $M_{gal}(tot) \sim 2.6 \times 10^{13}$

$L_{gal}(tot) \sim 2.6 \times 10^{12}$

$\frac{M_{dyn}}{L_{gal}(tot)} = \frac{3.3 \times 10^{14}}{2.6 \times 10^{12}} \sim 130 \frac{M_\odot}{L_\odot}$

c) $M_{SMBH} \sim 10^7 M_\odot$. Eddington :

$L_E \sim 3 \times 10^4 \frac{M}{M_\odot} L_\odot = 3 \times 10^{11} L_\odot$

$L_{AGN} >> L_{host}$.

The Schwarzschild radius $r_S \sim \frac{2GM}{c^2} \approx$

$\approx 3 \times 10^{10}$ m $\sim 10^{-6}$ pc

We require a resolution of

~ 0.25 nano arcseconds