Exam in

RRY125/ASM510 Modern astrophysics

Tid: 14 januari 2020, kl. 08.30–12.30
Plats: Johanneberg, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587 (mobil: 070 − 237 6701)
(lärare besöker tentamen ca. kl.09.00 och 11.00)

Tillåtna hjälpmedel:
- Chalmersgodkänd räknedosa, eller annan räknedosa med nollställt minne
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator, or other calculator with cleared memory
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

1.

Choose the most reasonable of the given alternatives for the following (do *not* give a motivation):

(a) Age of the Moon: A) 250 Myr, B) 1.3 Gyr, C) 4 Gyr, D) 13 Gyr

(b) Typical orbit radius of asteroids: A) 0.3 AU B) 3 AU C) 10 AU D) 10^4 AU

(c) Temperature of a Galactic HI cloud: A) 2.8 K, B) 80 K, C) 5800 K, D) 10^6 K

(d) Explanation for absorption lines in the Sun's spectrum:
A) temperature structure of Sun's outer layers
B) very hot and thin extended corona
C) more heavy elements on surface than in centre
D) the degenerate electron gas

(e) Disc galaxy with closely wound spirals: A) Sa, B) Sc, C) SBc, D) E0

(f) Density parameter $\Omega_{M,0}$ (matter) for the present cosmological model:
A) 0.04, B) 0.3, C) 0.7, D) 1.0

(g) Chandrasekhar limit: A) $z = 1100$ B) 1.4 $M_\odot$ C) $\Omega = 1$ D) 2.73 K

(h) Distribution function $f(p)$ for electrons in a white dwarf: A) $2/h^3$
B) Maxwellian
C) $K_1\rho^{5/3}$
D) *pp*-chain

2.

The 20 m diameter radio telescope in Onsala is used to observe a spectral line of CO at a frequency of 115.3 GHz towards a cloud in the disc of the Milky Way. The cloud has a diameter of 10 pc and is located at a distance of 1 kpc.

a.) Will the whole cloud fit into the beam of the telescope? **(1 p)**

b.) At the line frequency (115.3 GHz), the cloud is optically thin, and at all other frequencies it can be assumed to be completely transparent. The temperature of the cloud is 50 K. Will the CO line be seen in emission or absorption against the cosmic microwave background radiation (there are no other background sources)? **(1 p)**

c.) According to an astronomy student who studies the data, the spectral line is observed at a frequency of 116.3 GHz. Is this realistic, or is there a problem with, e.g., the spectrometer? Motivate your answer. **(1 p)**

3.

A large meteorite has been found on the surface of the Earth. Radioactive age dating gives an age of the meteorite of 6.5 Gyr. It is made of a dark material, with a (Bond) albedo of 5 %.

a.) Briefly discuss the age of the meteorite, especially concerning its possible origin. **(1 p)**

b.) Estimate the surface temperature of the meteorite just before it entered Earth's atmosphere. **(1 p)**

4.

We have used spectroscopic observations to detect the presence of a planet in a circular orbit around a star of mass 2 $M_\odot$. The Doppler shift of the spectral lines corresponds to an observed velocity of 45 m/s, and the period is 80 days.

a.) How far from its star is the planet? **(1 p)**

b.) What is the minimum mass of the planet? **(1 p)**

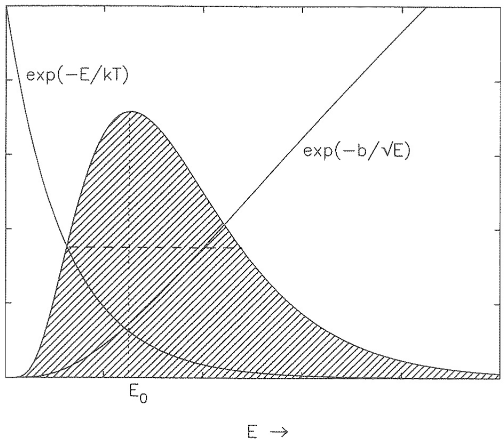
c.) Describe the observational method we require to determine the planet's density. **(1 p)**

5.
The table below shows observed properties for six stars:

Star number	*B*	*V*	*parallax* (arcsec)
1	11,5	10,0	0,100
2	8,5	8,5	0,002
3	8,5	8,0	0,010
4	16,0	14,0	0,167
5	10,5	9,5	0,020
6	15,6	15,0	0,125

a.) Draw a Hertzsprung-Russell diagram for the six stars. Which of the stars is smallest? What type of star is it? **(2 p)**
b.) Estimate the luminosity ratio between the intrinsically brightest star and the intrinsically faintest star in the table. Only consider main sequence stars. Use V band. **(2 p)**

6.
a.) The figure to the right is related to the fusion process. Referring to the figure, discuss (without equations) two basic physical mechanisms involved in calculations of nuclear reaction rates in stars. **(2 p)**
b.) Explain what the CNO cycle and the pp chain are (detailed reactions are not needed). In which stars are the two processes taking place? **(1 p)**



7.
We observe two star clusters with our optical telescope. The clusters are open clusters with the same diameters and they are seen near each other in the Galactic plane. Their angular diameters are x and $4x$ with distance moduli ($m-M$) 15.0 and 12.0 respectively. Find the distances to the clusters and the interstellar extinction coefficient a. **(2 p)**

8.
A quasar is a galaxy where accretion onto a supermassive black hole is giving rise to powerful radio jets at near-relativistic speeds. Consider a jet launched from a distant quasar at speed $0.87c$.
a.) At what angle to the observer must the jet lie to cause maximum superluminal motion? What is the corresponding transversal velocity observed? **(3 p)**
b.) The host galaxy of the quasar resides in a cluster observed to contain 1000 galaxies. Spectral analysis shows the cluster to have a mean redshift of $z = 0.3$ with a standard deviation of $\sigma_z = 0.004$. The effective radius of the cluster is 2.5 Mpc. Estimate the dynamical mass of the cluster. **(1 p)**
c.) At what redshift does the quasar/QSO activity peak and what mechanisms are causing this peak? **(1 p)**

9.

a.) Derive Friedmann's equation using Newtonian physics plus the result from general relativity that the energy per unit mass is $E = -kc^2/2$. Neglect Λ. What does k describe, and which values can k have? **(2 p)**

b.) Express the critical density of the Universe as a function of the Hubble constant. **(1 p)**

c.) Write Friedmann's equation (without the cosmological constant) using only H, k, c, Ω, and a (but not $\dot{a}$). **(1 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi\nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^{-3} Hz^{-1}] $\qquad n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^{-3} Hz^{-1}] $\qquad U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h\nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1}] $\qquad I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h\nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\qquad \nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_v}{dz} = j_v - \alpha_v I_v \qquad S_v = \frac{j_v}{\alpha_v} \qquad d\tau_v = \alpha_v \, dz$

$I_v = I_{v,\mathrm{bg}} \cdot e^{-\tau_v} + S_v \cdot (1 - e^{-\tau_v}) \qquad T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_v} + T_\mathrm{ex} \cdot (1 - e^{-\tau_v})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\ \mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_\mathrm{B}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ the Hubble-Lemaître law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_\mathrm{obs}}{\lambda_\mathrm{em}} = \frac{\nu_\mathrm{em}}{\nu_\mathrm{obs}} = \frac{a_0}{a}$ redshift

$ds^2 = -c^2dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ the Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d = \frac{R}{\pi}$ $R = 1\,\text{AU}, \pi =$ parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ circular velocity

$\theta \approx 1.22\frac{\lambda}{\mathrm{D}}$ resolution of telescope

$N(t) = N_0 e^{-\lambda t};\ \lambda = \frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\mathrm{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\mathrm{e}} n_{\mathrm{p}}$ recombination and ionization equation

$\frac{L_{\mathrm{I}}}{4 \cdot 10^{10} L_{\mathrm{I},\odot}} \approx \left(\frac{V_{\max}}{200\ \text{km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\mathrm{V}}}{2 \cdot 10^{10} L_{\mathrm{V},\odot}} \approx \left(\frac{\sigma}{200\ \text{km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\mathrm{E}} = \frac{4\pi G M m_{\mathrm{p}} c}{\sigma_{\mathrm{T}}} \approx 1.3 \cdot 10^{31}\frac{M}{\mathrm{M}_\odot}(\text{watt}) \approx 30000\frac{M}{\mathrm{M}_\odot}\mathrm{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\quad e^{-x} = \frac{1}{e^x}$ $\quad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\quad \lg xy = \lg x + \lg y$ $\quad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}

$c = 2.9979 \cdot 10^8$ m/s

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}

$h = 6{,}62606896 \cdot 10^{-34}$ J s

$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m

1 AU = $1.496 \cdot 10^{11}$ m

1 year = $3.156 \cdot 10^7$ s

1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8

The solar constant (1 AU from the Sun): 1371 W/m^2

$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses: Earth: $5.97 \cdot 10^{24}$ kg, Jupiter: $1.90 \cdot 10^{27}$ kg, Saturn: $5.69 \cdot 10^{26}$ kg, Sun: $1.99 \cdot 10^{30}$ kg

Radii: Earth: 6378 km, Jupiter: 71398 km, Saturn: 60270 km, Sun: $6.96 \cdot 10^5$ km

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1

(1) a) C b) B c) B d) A e) A f) B g) B h) A

(2) a) Angular size of cloud: $\frac{10\,pc}{1\,kpc} = 0{,}01$ rad

Beam size $\Theta \approx 1{,}22 \frac{\lambda}{D} = 1{,}22 \frac{c/\nu}{D} = 1{,}22 \frac{3\cdot 10^8 / 115.3\cdot 10^9}{20} = 1.6\cdot 10^{-4}$ rad.

No!

b) $T_b = T_{bg} e^{-\tau_\nu} + T_{ex}(1-e^{-\tau_\nu})$

$\Rightarrow$ not line freq, $\tau_\nu = 0 \Rightarrow T_b = T_{bg}$

$\Rightarrow$ Line freq., $\tau_\nu \ll 1 \Rightarrow T_b \approx T_{bg}(1-\tau_\nu) + T_{ex}(1-(1-\tau_\nu)) =$

$= T_{bg}(1-\tau_\nu) + T_{ex}\cdot\tau_\nu = T_b + (T_{ex} - T_{bg})\tau_\nu > T_b$

if $T_{ex} - T_{bg} > 0$ which it is

$\therefore$ emission line

c) $\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{c/\nu - c/\nu_0}{c/\nu_0} = \frac{\nu_0}{\nu} - 1 = \frac{115.3}{116.3} - 1 = -8.6\cdot 10^{-3}$

$\Rightarrow v = -2600$ km/s. Not realistic, too large

(cf. rotation speed ≈ 200 km/s)

(3) a) Age larger than age of the solar system.
Origin outside the solar system.

b.) Heat balance. Assume spherical meteorite, radius R.

$(1-A) S\cdot\pi R^2 = 4\pi R^2 \sigma T^4$, where $S = 1371$ W/m^2

$T = \left(\frac{(1-A)S}{4\sigma}\right)^{1/4} = 275$ K

Modern astro, 14 jan 2020, cont'd

(4.) $m_s = 2 M_\odot$, $V_s = 45$ m/s. $P = 80$ d

a) $\frac{a^3}{P^2} = \frac{G(m_s + m_p)}{4\pi^2} \approx \frac{G m_s}{4\pi^2}$ (assume $m_p \ll m_s$)

$\Rightarrow a = 6.85 \cdot 10^{10}$ m $= 0.46$ AU

b) $V_p = 2\pi a / P = 62\,270$ m/s

$m_s V_s = m_p V_p \Rightarrow m_p = m_s V_s / V_p = 2.88 \cdot 10^{27}$ kg $\approx 1.5 M_{Jup}$ *

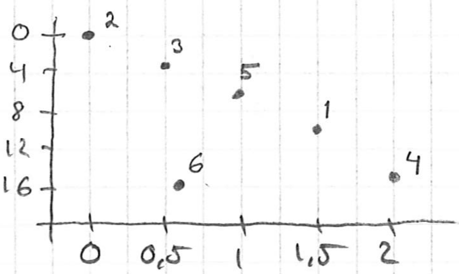
* If seen edge on; so this is a minimum mass

(More accurate: $m_p \sin i = 2.88 \cdot 10^{27}$ kg)

c). Radius needed $\Rightarrow$ transit method

(5.) a) Plot M_V vs B-V. $B = m_B$, $V = m_V$

$M_V = m - 5 \lg \frac{d}{10\,pc}$. $d = 1/\text{parallax}$. $[d] = 1$ pc, $[\text{parallax}] = 1$ arcsec



6 is a white dwarf, smallest (faintest but fairly high surface temperature)

b.) Brightest # 2: $M_2 = 0$ (V-band)

Dimmest # 4: $M_4 = 15,1$ (V-band)

Abs. magn = app. magn. @ 10 pc. Put both stars at 10 pc. The luminosity ratio is then the same as the flux ratio

$M_2 - M_4 = -2,5 \lg \frac{F_2}{F_0} - \left(-2,5 \lg \frac{F_4}{F_0}\right) = \ldots = -2,5 \lg \frac{F_2}{F_4}$

$= -2,5 \lg \frac{L_2}{L_4} \Rightarrow \frac{L_2}{L_4} = 10^{-(M_2 - M_4)/2.5}$

$\frac{L_2}{L_4} = 10^{-(0-15,1)/2,5} = \underline{\underline{1.1 \cdot 10^6}}$

(6.) a) The electrostatic force and tunneling.
One curve from Maxwellian velocity distrib.,
the other from tunneling probability.
Their product (curve with shading) gives an
appreciable reaction rate for energies near E_0.
See fig. 4.4 in the textbook.

b.) See Sect. 4.3 in the textbook

(7.) $\theta_1 = x$, $\theta_2 = 4x$ $\quad \theta = \frac{D}{d}$, D equal $\Rightarrow \frac{d_1}{d_2} = \frac{\theta_2}{\theta_1} = 4$

$m - M = 5 \lg \frac{d}{10\,pc} + ad$, $d_1 = 4 d_2 \Rightarrow$

$$\begin{cases} 15 = 5 \lg \frac{4 d_2}{10\,pc} + a \cdot 4 d_2 & (1) \\ 12 = 5 \lg \frac{d_2}{10\,pc} + a \cdot d_2 & (2) \end{cases}$$

$(1) - (2) \Rightarrow 3 = 5 \lg 4 + 3 a d_2 \Rightarrow a d_2 = -0.0034$ (?)

$(2) \Rightarrow 12 = 5 \lg \frac{d_2}{10\,pc} - 0.0034 \Rightarrow d_2 = 2520$ pc

$\Rightarrow d_1 = 4 d_2 = 10\,060$ pc

$a = \frac{-0.0034}{2524} \approx -1.3 \cdot 10^{-6}$ mag/pc $= -1.3 \cdot 10^{-3}$ mag/pc ≈ 0.

(8) a) See textbook, Sect. 9.4.2. $\beta_{app} = \frac{\beta \sin\theta}{1-\beta\cos\theta}$ $\left(\beta = \frac{v}{c}\right)$

θ for max β_{app}: $\frac{d\beta_{app}}{d\theta} = 0 \Rightarrow \cos\theta = \beta$, $\theta = \arccos\beta$

$$\beta_{app}^{max} = \frac{\beta\sqrt{1-\cos^2\theta}}{1-\beta\cos\theta} = \{\cos\theta = \beta\} = \frac{\beta\sqrt{1-\beta^2}}{1-\beta^2} = \frac{\beta}{\sqrt{1-\beta^2}}$$

Here: $\beta = 0.87 \Rightarrow \theta = \arccos\beta = 29,5^\circ$

$$\beta_{app}^{max} = \frac{0.87}{\sqrt{1-0.87^2}} = 1,76 \ ; \ v_\perp = 1,76\,c$$

b) $\sigma_z = 0.004 \Rightarrow \Delta v = \sigma_z \cdot c = 1,2 \cdot 10^6$ m/s

$$E_{kin} = \frac{M(\Delta v)^2}{2}, \quad E_{pot} = -\frac{GM^2}{2R}, \quad 2E_{kin} + E_{pot} = 0$$

$$\Rightarrow 2 \cdot \frac{M(\Delta v)^2}{2} - \frac{GM^2}{2R} = 0 \Rightarrow M = \frac{2R(\Delta v)^2}{G}$$

$$\Rightarrow M = 3.33 \cdot 10^{45}\ \text{kg} = 1.67 \cdot 10^{15}\, M_\odot$$

c.) $z \approx 2$ See the textbook Sect. 11.8.1.

(9) a) See the textbook Sect. 10.4
k describes curvature, $k = -1, 0, +1$

b) Critical density $\leftrightarrow k = 0$
$\frac{\dot a}{a} = H$, $k = 0$, $\Lambda = 0$ in the Friedmann eq:

$$H^2 + 0 = \frac{8\pi G}{3}\rho + 0 \Rightarrow \rho = \rho_{crit} = \frac{3H^2}{8\pi G}$$

c.) $\frac{\dot a}{a} = H$. $\rho = \Omega\rho_{crit} = \Omega\frac{3H^2}{8\pi G}$, so Friedmann's eq:

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\Omega\frac{3H^2}{8\pi G} \Leftrightarrow H^2 + \frac{kc^2}{a^2} = \Omega H^2$$

$$\Leftrightarrow \frac{kc^2}{a^2} = H^2(\Omega - 1)$$