Exam in

RRY125/ASM510 Modern astrophysics

Tid: 24 april 2019, kl. 14.00–18.00
Plats: Maskinsalar, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587 (mobil: 070 − 237 6701)
(lärare besöker tentamen ca. kl.14.30 och 16.30)

Tillåtna hjälpmedel:
- Chalmersgodkänd räknedosa, eller annan räknedosa med nollställt minne
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator, or other calculator with cleared memory
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

1.

Choose the most reasonable of the given alternatives for the following (do *not* give a motivation):

(a) Surface pressure at Mars (Earth = 1):	A) 100,	B) 5,	C) 0.5,	D) 0.01
(b) Material in Mercury's core:	A) C+N+O,	B) Rock,	C) Fe+Ni,	D) H+He
(c) Typical orbit radius of asteroids:	A) 0.3 AU	B) 3 AU	C) 50 AU	D) 10^4 AU
(d) Temperature of Galactic coronal gas:	A) 2.8 K,	B) 80 K,	C) 6000 K,	D) 10^6 K
(e) Disc galaxy without spiral arms:	A) S0,	B) Sa,	C) SBc,	D) E7
(f) Source of the "21 cm line":	A) CMBR,	B) H_2,	C) HII,	D) HI

(g) Two stars with apparent magnitudes 8 and 9, resp., form a binary system. What is the total apparent magnitude for the binary? A) 1, B) 7.6, C) 9.4, D) 17

(h) Ingredient in modern theory for the formation of the planets in the Solar system:

A) gravitational waves
B) tidal forces from spiral arms
C) planetary migration
D) accretion of material from the ISM **(4 p)**

2.

The textbook describes an important result concerning radiative transfer in stellar atmospheres as follows:

"… the specific intensity of radiation at a frequency ν coming out of a stellar atmosphere is approximately equal to the Planck function at a depth of the atmosphere where the optical depth for that frequency ν equals unity."

Use this result to explain the formation of stellar *absorption* lines. Under what circumstance can instead *emission* lines be seen in a stellar spectrum? **(2 p)**

3.

Assume that you have discovered a planetary system where the most massive planet is a Saturn-sized planet in circular orbit (radius 8 AU) around a Sun-like star.

a.) What is the maximum Doppler shift of the Calcium H line (396.847 nm)? **(1 p)**

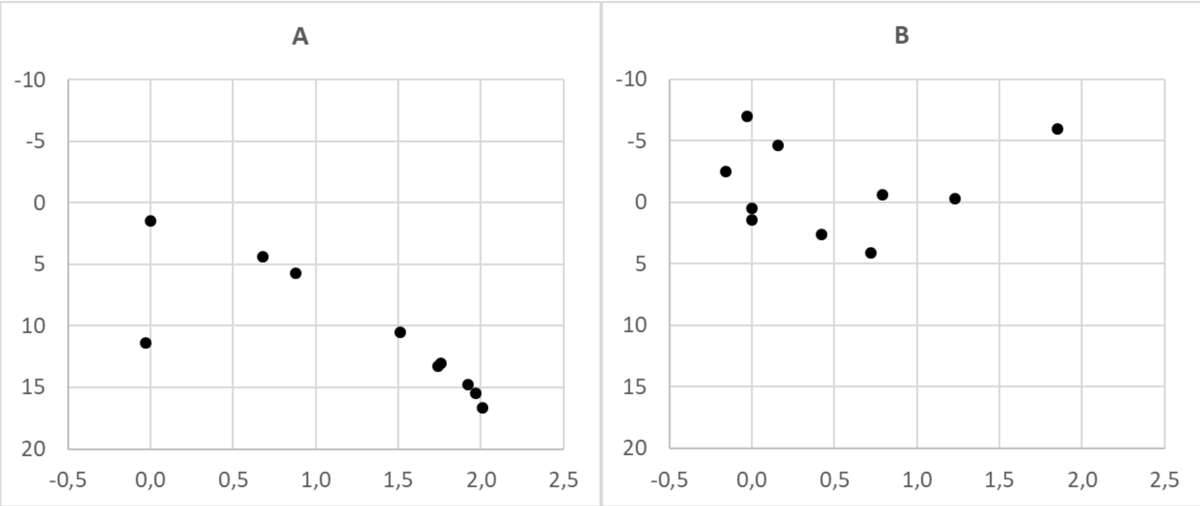
b.) How far from the centre of the star is the centre of mass of the system? **(1 p)**

c.) What is a "hot Jupiter" and explain why there seem to be so many of them? **(1 p)**

4.

A 5 solar mass star has moved off the main sequence and is on the "subgiant branch" on its way to the giant branch. Estimate the Kelvin-Helmholtz (KH) timescale for the star (T_{final} = 5010 K, L = 1000 L_{Sun}). Compare the KH time scale to the time (350 000 yr) the star spends on the subgiant branch. Why is there a discrepancy with the KH time? **(3 p)**

5.
The figures below show Hertzsprung-Russell diagrams for the brightest stars and the nearest stars, resp.
a.) Which of the diagrams shows the nearest stars, A or B? Which quantities are given on the two axes (same for both diagrams)? **(1 p)**
b.) Explain why one of the diagrams contains mainly main sequence stars, while the other diagram contains only a few main sequence stars. **(2 p)**



6.
This question is about white dwarfs. The relativistic equation of state for a degenerate electron gas can be written $P = K\rho^{4/3}$, where the constant $K \approx 10^{10}$ if we use SI units.
a.) Use this equation of state, and one or several stellar structure equations, to make a *rough estimate* of the mass of a star composed of this type of matter. **(2 p)**
b.) What is this type of stars called? What is the mass you calculated called (your result may differ from the result of detailed calculations)? How has this type of stars indirectly been very important for modern cosmology? **(2 p)**

7.
In certain active galaxies, changes in the position of substructures may be detected over intervals as short as a year. For a quasar at a redshift of $z = 0.2$, one component is seen to move away from the nucleus with an apparent angular velocity of 1.74 milliarcseconds per year.
a.) What is the corresponding apparent, projected linear velocity of this component? **(1 p)**
b.) What is the minimum matter ejection velocity required in order to produce the apparent superluminal speed observed in a.)? **(3 p)**

8.

a.) Express the critical density of the Universe as a function of the Hubble constant *H*. **(1 p)**

b.) For a matter dominated flat Universe with $\Omega = 1$ and $\Lambda = 0$, calculate age of the Universe as a fraction of the present age (i.e., t/t_0) corresponding to a redshift of $z = 1$. (You may make a reasonable assumption of how the density depends on the scale factor). **(3 p)**

c.) Very early in the history of the Universe $a(t)$ was proportional to $t^{1/2}$. Show that at these very early times, the density parameter $\Omega = \rho/\rho_c$ was very close to 1. (You may *not* assume that $k = 0$.) **(3 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi \nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^{-3} Hz^{-1}] $\quad$ $n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^{-3} Hz^{-1}] $\quad$ $U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1}] $\quad$ $I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\quad$ $\nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_\nu}{dz} = j_\nu - \alpha_\nu I_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \qquad d\tau_\nu = \alpha_\nu \, dz$

$I_\nu = I_{\nu,\mathrm{bg}} \cdot e^{-\tau_\nu} + S_\nu \cdot (1 - e^{-\tau_\nu}) \qquad T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_\nu} + T_\mathrm{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\ \mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ — the Hubble-Lemaître law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} = \frac{a_0}{a}$ — redshift

$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ — Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ — the Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d = \frac{R}{\pi}$ $R = 1$ AU, π = parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ circular velocity

$\theta \approx 1.22\frac{\lambda}{\text{D}}$ resolution of telescope

$N(t) = N_0 e^{-\lambda t};\ \lambda = \frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\text{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\text{e}} n_{\text{p}}$ recombination and ionization equation

$\frac{L_{\text{I}}}{4 \cdot 10^{10} L_{\text{I},\odot}} \approx \left(\frac{V_{\text{max}}}{200\ \text{km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\text{V}}}{2 \cdot 10^{10} L_{\text{V},\odot}} \approx \left(\frac{\sigma}{200\ \text{km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\text{E}} = \frac{4\pi G M m_{\text{p}} c}{\sigma_{\text{T}}} \approx 1.3 \cdot 10^{31} \frac{M}{\text{M}_\odot} (\text{watt}) \approx 30000 \frac{M}{\text{M}_\odot} \text{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\quad e^{-x} = \frac{1}{e^x}$ $\quad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\quad \lg xy = \lg x + \lg y$ $\quad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}

$c = 2.9979 \cdot 10^8$ m/s

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}

$h = 6{,}62606896 \cdot 10^{-34}$ J s

$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m

1 AU = $1.496 \cdot 10^{11}$ m

1 year = $3.156 \cdot 10^7$ s

1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8

The solar constant (1 AU from the Sun): 1371 W/m^2

$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses: Earth: $5.97 \cdot 10^{24}$ kg, Jupiter: $1.90 \cdot 10^{27}$ kg, Saturn: $5.69 \cdot 10^{26}$ kg, Sun: $1.99 \cdot 10^{30}$ kg

Radii: Earth: 6378 km, Jupiter: 71398 km, Saturn: 60270 km, Sun: $6.96 \cdot 10^5$ km

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(1.) a) D b) C c) B d) D e) A f) D g) B h) C

(2.) See textbook, p. 45–46
Emission lines if the temperature of the stellar atmosphere increases outwards.

(3.) a) $m_s V_s = m_p V_p$. $V_p = \sqrt{\frac{G(m_s + m_p)}{a}}$, with $a = 8$ AU.

$m_s = 1\ M_\odot \gg m_p \Rightarrow V_p = 10\,531$ m/s $\Rightarrow V_s = 3.01$ m/s

$\frac{\Delta\lambda}{\lambda_0} = \frac{V_s}{c} \Rightarrow \Delta\lambda = 3.99 \cdot 10^{-6}$ nm

b.) $\left.\begin{array}{l} m_s r_s = m_p r_p \\ a = r_s + r_p \end{array}\right\}$ $r_s = \frac{a}{1 + \frac{m_s}{m_p}} = 3.42 \cdot 10^{8}$ m $= 342\,000$ km

c.) See lecture notes

(4.) $\tau_{KH} \approx \frac{E_g}{L} \approx \frac{GM^2}{RL}$. $L = 4\pi R^2 \sigma T^4 = 1000\ L_{sun}$

$L_{sun} = 3.92 \cdot 10^{26}$ W (Physics Handbook, or calculate from, e.g., the solar constant)

$R = \sqrt{\frac{1000\ L_{sun}}{4\pi\sigma T^4}} = 2.96 \cdot 10^{10}$ m (using $T = 5010$ K)

$\Rightarrow \tau_{KH} = 5.7 \cdot 10^{11}$ s $= 18\,000$ years

But only the core collapses → gives different timescale.
See also lecture notes.

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(5.) a) Nearest stars: A

M (absolute magnitude) on y-axis

B−V (colour index) on x-axis.

b) Nearest stars can be assumed to be "typical", i.e., mainly on main sequence (esp. the lower part). Brightest stars: several giants, bright but less common due to short lifetime; can be bright even if at large distances.

(6.) a) Simplify $\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$ to $\frac{P}{R} = \frac{GM}{R^2}\rho$

Use $P = K\rho^{4/3}$ and $\rho = M/(\frac{4\pi}{3}R^3)$ $\{$or $\rho \approx \frac{M}{R^3}\}$

$\Rightarrow M = \left(\frac{K}{G}\right)^{3/2}\left(\frac{3}{4\pi}\right)^{1/2} = 9\cdot 10^{29}$ kg $= 0{,}45\, M_\odot$

b) White dwarf (a type of compact star).

Chandrasekhar limit. Supernovae Type Ia (mass transfer in binary system to a white dwarf, mass > Chandrasekhar limit $\Rightarrow$ explosion).

Used for distance measurements.

(7) a) $v = zc = H_0 \ell \Rightarrow \ell = \frac{zc}{H_0} = 833$ Mpc (distance)

$\frac{\Delta\varphi}{\Delta t} = 1.74$ mas/yr $= 2.67\cdot 10^{-16}$ rad/s

Apparent $v_\perp = \ell \cdot \frac{\Delta\varphi}{\Delta t} = 6{,}87\cdot 10^9$ m/s $= 22{,}9\, c$

($\beta_{app} = 22{,}9$)

b) see next page

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(7).b) See textbook, p. 282: $\beta_{app} = \frac{\beta \sin\theta}{1-\beta\cos\theta}$ $(\beta = \frac{v}{c})$

θ for max β_{app}: $\frac{d\beta_{app}}{d\theta} = 0 \Rightarrow \cos\theta = \beta$, $\theta = \arccos\beta$

$$\beta_{app}^{max} = \frac{\beta\sqrt{1-\cos^2\theta}}{1-\beta\cos\theta}\{\text{with } \cos\theta=\beta\} = \frac{\beta\sqrt{1-\beta^2}}{1-\beta^2} = \frac{\beta}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{\beta_{app}^{max}}{\sqrt{1+\beta_{app}^{max\,2}}} = \frac{22,9}{\sqrt{1+22,9^2}} = \underline{\underline{0,999}}$$

(8) a) Friedmann eq. with $k=0$ & $\Lambda=0$ & $\frac{\dot{a}}{a} = H$

$$\Rightarrow H^2 + 0 = \frac{8\pi G}{3}\rho_{crit} + 0 \Rightarrow \rho_{crit} = \frac{3H^2}{8\pi G}$$

b) $\Omega = 1 \Rightarrow k=0$. $\rho = \rho_0\left(\frac{a_0}{a}\right)^3$. Friedmann eq. $\Rightarrow$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_0\left(\frac{a_0}{a}\right)^3 \Rightarrow \dot{a}^2 = \frac{8\pi G}{3}\rho_0 a_0^3 \cdot \frac{1}{a} \Rightarrow$$

$$\frac{da}{dt} = \sqrt{\frac{8\pi G}{3}\rho_0 a_0^3} \cdot \frac{1}{a^{1/2}} \Rightarrow \int_0^a a^{1/2}da = \sqrt{}\int_0^t dt \Rightarrow$$

$$\left.\begin{aligned} &\frac{2}{3}a^{3/2} = \sqrt{}\, t \Rightarrow a = \left(\frac{3}{2}\sqrt{}\, t\right)^{2/3} = K t^{2/3} \\ &a = a_0 \,\&\, t = t_0 \text{ (now)} \Rightarrow K = a_0 t_0^{-2/3} \\ &1+z = \frac{a_0}{a}.\quad z = 1 \Rightarrow a = \frac{1}{2}a_0 \end{aligned}\right\} \Rightarrow$$

$$\frac{1}{2}a_0 = a_0 t_0^{-2/3} \cdot t^{2/3} \Rightarrow \frac{t}{t_0} = \left(\frac{1}{2}\right)^{3/2} = 0.35$$

c) See textbook ch. 10.7. Friedmann's eq. with $\dot{a}/a = H$ and ρ_{crit} from a):

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\Omega\frac{3H^2}{8\pi G} = \Omega H^2 \Rightarrow$$

$$\frac{kc^2}{a^2} = H^2(\Omega - 1) \Rightarrow \frac{kc^2}{a^2H^2} = \Omega - 1 \Rightarrow \frac{kc^2}{\dot{a}^2} = \Omega - 1.$$

$$a \propto t^{1/2} \Rightarrow \dot{a} \propto t^{-1/2} \Rightarrow |\Omega - 1| \propto t \Rightarrow \begin{cases} t \to 0 \\ \Rightarrow \Omega \to 1 \end{cases}$$