190114 Modern astrophysics, tenta 15 januari 2019

1

(1) a) A b) B c) B d) C e) D f) B g) D h) D

(2) $T_b = T_{bg} e^{-\tau_\nu} + T_{ex}(1 - e^{-\tau_\nu})$

Off spectral line freq.: $\tau_\nu = 0 \Rightarrow T_b = T_{bg}$

On spectral line freq: $\tau_\nu > 0$

No spectral lines observed: $T_b = T_{bg}$ $\Bigg\} \Rightarrow$

$$T_{bg} = T_{bg} e^{-\tau_\nu} + T_{ex}(1 - e^{-\tau_\nu}) \Rightarrow T_{bg} = T_{ex}$$

(3) $m_s = 2 M_\odot$, $V_s = 45$ m/s, $P = 80$ d

Assume circular orbit.

a) $m_p \ll m_s \Rightarrow \frac{a^3}{P^2} = \frac{G m_s}{4\pi^2} \Rightarrow a = 6.85 \cdot 10^{10}$ m $= 0.46$ AU

b) $V_p = 2\pi a / P = 62\,270$ m/s

$m_s V_s = m_p V_p \Rightarrow m_p = m_s V_s / V_p = 2.88 \cdot 10^{27}$ kg $= 1.5\, M_{Jup}$ *

(if seen edge-on, m_p is minimum mass)

c) Radius needed $\Rightarrow$ transit method

d) $m_s V_s = m_p V_p$ with $m_s = 2 M_\odot$, $V_s = 1$ m/s, $m_p = 1\, M_\oplus$

$\Rightarrow V_p = m_s V_s / m_p = 666\,667$ m/s

$V_p = \sqrt{G m_s / a} \Rightarrow a = \frac{G m_s}{V_p^2} = 5.97 \cdot 10^8$ m $= 0.004$ AU

{ * More accurate: $m_p \sin i = 2.88 \cdot 10^{27}$ kg,

$\sin i < 1$, so $2.88 \cdot 10^{27}$ is the minimum mass }

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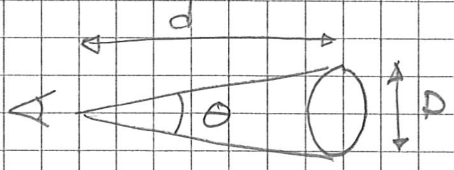
2



(4) $\theta_1 = y$, $\theta_2 = 5y$

$\theta = \frac{D}{d}$, D equal $\Rightarrow \frac{d_1}{d_2} = \frac{\theta_2}{\theta_1} = 5$

Distance moduli $(m-M)_1 = 15.0$, $(m-M)_2 = 11.0$

$m - M = 5 \lg \frac{d}{10\,pc} + ad$, $d_1 = 5d_2 \Rightarrow$

$$\begin{cases} 15.0 = 5 \lg \frac{5d_2}{10\,pc} + a \cdot 5d_2 & (1) \\ 11.0 = 5 \lg \frac{d_2}{10\,pc} + a \cdot d_2 & (2) \end{cases}$$

$(1) - (2) \Rightarrow 4.0 = 5 \lg 5 + 4ad_2 \Rightarrow ad_2 = 0.1263$

$(2) \Rightarrow 11.0 = 5 \lg \frac{d_2}{10\,pc} + 0.1263 \Rightarrow d_2 = 1495\ pc$

$d_1 = 5d_2 = 7477\ pc$

$a = 0.1263 / 1495 = 8.45 \cdot 10^{-5}$ mag/pc $= 0.084$ mag/kpc

(5) A: 3 B: 1 C: 14 D: 7/9/13 E: (8)/9 F: (8)/10

(6) $P(t) = P_0 e^{-\lambda t} \Rightarrow P_0 = P(t) e^{\lambda t}$

$D(t) = D_0 + (P_0 - P(t)) = D_0 + P(t)(e^{\lambda t} - 1)$

$\frac{D(t)}{S(t)} = \frac{D_0}{S_0} + \frac{P(t)}{S(t)}(e^{\lambda t} - 1)$ $\{S(t) = S_0\}$

$e^{\lambda t} - 1 =$ slope of the curve $= \frac{0.75 - 0.70}{1.0 - 0.0} = 0.05$

$\Rightarrow e^{\lambda t} = 1.05 \Rightarrow t = \frac{\ln 1.05}{\lambda} = \frac{\ln 1.05}{\ln 2} \cdot t_{1/2} = 3.4$ Gy

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3 7. $\sigma = 250\,\mathrm{km/s}$, $R = 60\,\mathrm{kpc}$, $m = +9.4$

a) Virial theorem: $2 \cdot \frac{M\sigma^2}{2} - \frac{GM^2}{2R} = 0 \implies$

$M = 2R\sigma^2/G = 3.5 \cdot 10^{42}\,\mathrm{kg} = 1.7 \cdot 10^{12}\,M_\odot$

Faber-Jackson: $\frac{L_V}{2 \cdot 10^{10}\,L_{V,\odot}} = \left(\frac{\sigma}{200\,\mathrm{km/s}}\right)^4 \implies$

$L_V = 2 \cdot 10^{10} \cdot \left(\frac{250}{200}\right)^4 = 4.88 \cdot 10^{10}\,L_{V,\odot}$

If the flux from the Sun at 10 pc is $F_\odot$:

$M = -2.5 \lg \frac{4.88 \cdot 10^{10} F_\odot}{F_0} = -2.5 \lg(4.88 \cdot 10^{10}) - 2.5 \lg \frac{F_\odot}{F_0} =$

$= -26.72 + M_\odot = -26.72 + 4.8 = -21.92$

$m - M = 5 \lg \frac{d}{10\,\mathrm{pc}} \implies d = 10 \cdot 10^{(m-M)/5}\,\mathrm{pc} = 18.4\,\mathrm{Mpc}$

b.) Assume Eddington luminosity:

$L_E = 1.3 \cdot 10^{31} \frac{M}{M_\odot}\,\mathrm{W} = 5 \cdot 10^{39}\,\mathrm{W}$.

$\implies M = 3.8 \cdot 10^{8}\,M_\odot$ (mass of SMBH)

Event horizon radius $R = \frac{2GM}{c^2} = 1.1 \cdot 10^{12}\,\mathrm{m} = 7.5\,\mathrm{AU}$

Orbit radius < event horizon $\implies$ impossible

8. See textbook and lecture notes

9. See textbook and lecture notes.

k describes curvature. $k = -1, 0, +1$

$\Omega = \frac{\rho}{\rho_c}$, ρ_c = critical density

$k = 0 \longleftrightarrow \Omega = 1$, $k = -1 \longleftrightarrow \Omega < 1$, $k = +1 \longleftrightarrow \Omega > 1$

Exam in

RRY125/ASM510 Modern astrophysics

Tid: 15 januari 2019, kl. 08.30–12.30
Plats: Maskinsalar, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587 (mobil: 070 − 237 6701)
(lärare besöker tentamen ca. kl.09.00 och 11.00)

Tillåtna hjälpmedel:
- Chalmersgodkänd räknedosa, eller annan räknedosa med nollställt minne
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator, or other calculator with cleared memory
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

1.
Choose the most reasonable of the given alternatives for the following (do *not* give a motivation):

(a) Surface pressure at Venus (*Mars = 1*):	A) 10000	B) 100	C) 0.01	D) 0.0001
(b) Temperature of a Galactic HI cloud:	A) 2.5 K,	B) 80 K,	C) 8000 K,	D) 10^6 K
(c) Colour index $B–V$ for a red supergiant:	A) +4.8,	B) +1.8,	C) –0.2,	D) –13.7
(d) Central temperature of a main sequence star of spectral class A:	A) 10^4 K,	B) $9{\cdot}10^5$ K,	C) $2{\cdot}10^7$ K,	D) 10^9 K
(e) Chandrasekhar limit:	A) $z = 1100$	B) 2.73 K	C) $\Omega = 1$	D) 1.4 $M_\odot$
(f) Disc galaxy with open spiral arms:	A) Sa,	B) Sc,	C) S0,	D) E0

(g) The relativistic equation of state in a white dwarf:
A) $P = nkT$ B) $f(p) = 2/h^3$ C) $n_e = n_0 e^{-E/kT}$ D) $P = K_2\rho^{4/3}$

(h) Ingredient in modern theory for the formation of the planets in the Solar system:
A) hot Jupiters
B) tidal forces from nearby star
C) accretion of material from the ISM
D) planetary migration **(4 p)**

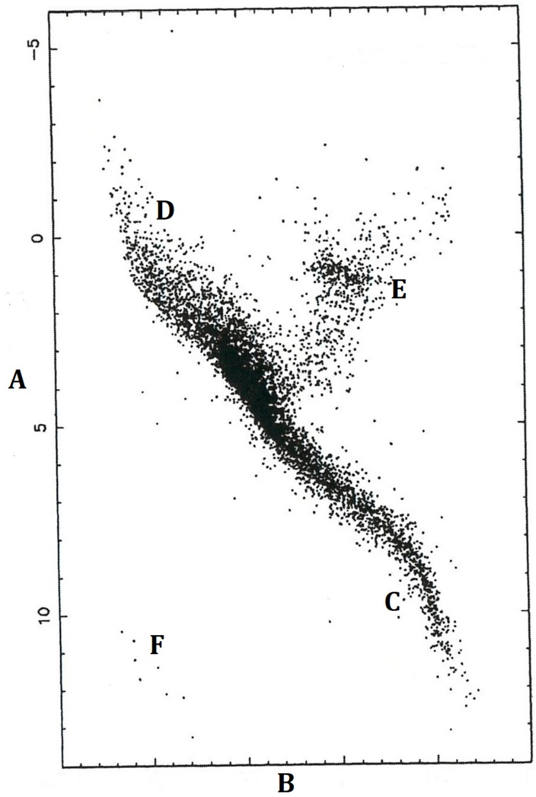
2.
A radio telescope observes a cosmic gas cloud seen towards a continuum background source with temperature T_{bg}. Assume that the cloud is completely transparent at all frequencies except for some frequencies corresponding to spectral lines of the gas in the cloud. The excitation temperature of the gas cloud is T_{ex}. Under what circumstance will the observed spectrum contain *neither* emission lines *nor* absorption lines? **(2 p)**

3.
We have used spectroscopic observations to detect the presence of a planet around a star of mass 2 $M_\odot$. The Doppler shift of the spectral lines corresponds to an observed velocity of 45 m/s, and the period is 80 days.
a.) How far from its star is the planet? **(1 p)**
b.) What is the minimum mass of the planet? **(1 p)**
c.) Describe the observational method we require to determine the planet density. **(1 p)**
d.) If the accuracy of our velocity determinations is 1 m/s, what is the maximum distance of an Earth-like planet from the central star astronomers could detect? **(1 p)**

4.
Two open star clusters, which are seen near each other in the galactic plane, have angular diameters y and $5y$, and distance moduli 15.0 and 11.0, respectively. Assuming their actual diameters are equal, find their distances and the interstellar extinction coefficient a. **(2 p)**

5.
The figure to the right shows the HR diagram of nearby stars. Six areas in the diagram are marked with letters (A-F). For each letter A-F, choose a correct description from the list (number 1-14) below.
- Give only one number per letter A-F.
- Each number may only be used once.
(Not all numbers will be used. There might be several correct solutions.) **(2 p)**

1. Surface temperature scale
2. Central temperature scale
3. Absolute magnitude scale
4. Apparent magnitude scale
5. Mass scale
6. Radius scale
7. Only young stars
8. Only old stars
9. Large stars
10. White dwarfs
11. Black holes
12. Stars very similar to the Sun
13. CNO cycle reactions in the core
14. pp chain reactions in the core

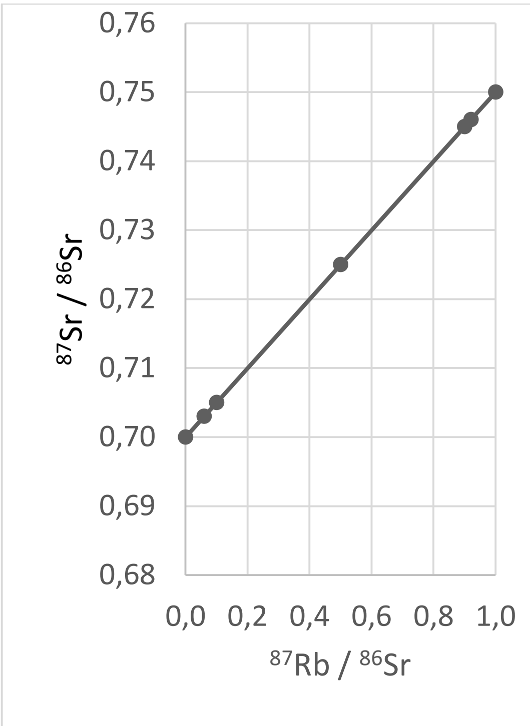


6.
Isotope ratios in samples of a meteorite have been measured in order to determine the age of the meteorite.

The rubidium isotope ^{87}Rb ("parent") decays to the strontium isotope ^{87}Sr ("daughter") with a half-life of 48.8 Gyr. The strontium isotope ^{86}Sr is stable.

The figure to the right shows the isotope ratio (number of atoms) $^{87}Sr/^{86}Sr$ plotted against $^{87}Rb/^{86}Sr$.

Calculate the age of the meteorite! **(4 p)**



7.

The velocity dispersion of the elliptical galaxy NGC 4472 is 250 km/s, and its radius is 60 kpc. We also know that its apparent magnitude is +9.4, and we can assume that there is no extinction.

a.) Calculate the distance and mass of NGC 4472. **(2 p)**

b.) NGC 4472 hosts an active galactic nucleus (AGN) powered by a supermassive black hole (SMBH) The AGN has a luminosity of $5 \cdot 10^{39}$ W. Astronomers claim to have discovered a star orbiting the SMBH at a distance of 0.5 light minutes. Is this likely to be true given the mass of the SMBH? **(2 p)**

8.

As the Universe expands and ages it also evolves.

a.) Describe and compare the methods you need to study the evolution of QSOs and the inter-galactic medium. **(2 p)**

b.) What is the evolutionary behaviour of QSOs and how can we explain this? **(2 p)**

c.) How can we use today's abundance of deuterium to address issues on big bang nucleo-synthesis? **(1 p)**

9.

Derive Friedmann's equation using Newtonian physics plus the result from general relativity that the energy per unit mass is $E = -kc^2/2$. Neglect Λ.

What does k describe, and which values can k have? Define the density parameter Ω. How are k and Ω related? **(3 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1+m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi \nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT}-1)}$ [m^{-3} Hz^{-1}] $\qquad$ $n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT}-1)}$ [J m^{-3} Hz^{-1}] $\qquad$ $U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT}-1)}$ [W m^{-2} Hz^{-1}] $\qquad$ $I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT}-1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\qquad$ $\nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_v}{dz} = j_v - \alpha_v I_v$ $\qquad$ $S_v = \frac{j_v}{\alpha_v}$ $\qquad$ $d\tau_v = \alpha_v \, dz$

$I_v = I_{v,\mathrm{bg}} \cdot e^{-\tau_\nu} + S_v \cdot (1 - e^{-\tau_\nu})$ $\qquad$ $T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_\nu} + T_\mathrm{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\,\mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ the Hubble-Lemaître law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} = \frac{a_0}{a}$ redshift

$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ the Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d = \frac{R}{\pi}$ $R = 1$ AU, π = parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ circular velocity

$\theta \approx 1.22\frac{\lambda}{\mathrm{D}}$ resolution of telescope

$N(t) = N_0 e^{-\lambda t};\ \lambda = \frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\mathrm{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\mathrm{e}} n_{\mathrm{p}}$ recombination and ionization equation

$\frac{L_{\mathrm{I}}}{4 \cdot 10^{10} L_{\mathrm{I},\odot}} \approx \left(\frac{V_{\max}}{200\ \mathrm{km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\mathrm{V}}}{2 \cdot 10^{10} L_{\mathrm{V},\odot}} \approx \left(\frac{\sigma}{200\ \mathrm{km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\mathrm{E}} = \frac{4\pi G M m_{\mathrm{p}} c}{\sigma_{\mathrm{T}}} \approx 1.3 \cdot 10^{31}\frac{M}{\mathrm{M}_\odot}(\text{watt}) \approx 30000\frac{M}{\mathrm{M}_\odot}\mathrm{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\qquad e^{-x} = \frac{1}{e^x}$ $\qquad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\qquad \lg xy = \lg x + \lg y$ $\qquad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}
$c = 2.9979 \cdot 10^8$ m/s
$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}
$h = 6{,}62606896 \cdot 10^{-34}$ J s
$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m
1 AU = $1.496 \cdot 10^{11}$ m
1 year = $3.156 \cdot 10^7$ s
1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8
The solar constant (1 AU from the Sun): 1371 W/m^2
$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses:	Earth: $5.97 \cdot 10^{24}$ kg,	Jupiter: $1.90 \cdot 10^{27}$ kg,	Sun: $1.99 \cdot 10^{30}$ kg
Radii:	Earth: 6378 km,	Jupiter: 71398 km,	Sun: $6.96 \cdot 10^5$ km.