Exam in

RRY125/ASM510 Modern astrophysics

Tid: 8 januari 2018, kl. 08.30–12.30
Plats: Maskinsalar, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587 (mobil: 070 − 237 6701)
(lärare besöker tentamen ca. kl.09.00 och 11.00)

Tillåtna hjälpmedel:
- Typgodkänd räknedosa, eller annan räknedosa med nollställt minne
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator, or other calculator with cleared memory
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

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1.
Choose the most reasonable of the given alternatives for the following (do *not* give a motivation):

	A	B	C	D
(a) Common element in Venus' core	A) H	B) C	C) Si	D) Fe
(b) Typical orbit radius of asteroids:	A) 0.3 AU	B) 3 AU	C) 10 AU	D) 10^4 AU
(c) Typical average number density of a molecular cloud:	A) 300 cm^{-3}	B) 10^{-3} cm^{-3}	C) 10^6 cm^{-3}	D) 1 AU^{-3}
(d) Radius of a white dwarf:	A) 10^6 km	B) 10^4 km	C) 10 km	D) 1 AU
(e) Radius of a red giant:	A) 10^6 km	B) 10^4 km	C) 10 km	D) 1 AU
(f) Object with Seyfert activity:	A) supergiant	B) Sa galaxy	C) E galaxy	D) pulsar
(g) Cosmological parameter $\Omega_{\Lambda,0}$:	A) 0.04	B) 0.3	C) 0.7	D) 1.0

(h) Distribution function $f(p)$ for electrons in a white dwarf: A) $2/h^3$
B) Maxwellian
C) $K_1\rho^{5/3}$
D) *pp*-chain

(4 p)

2.
Two important results from radiative transfer theory are:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}),\text{ and}$$

$$I_\nu(\tau_\nu = 0, \mu = 1\ ) \approx B_\nu(\tau_\nu = 1).$$

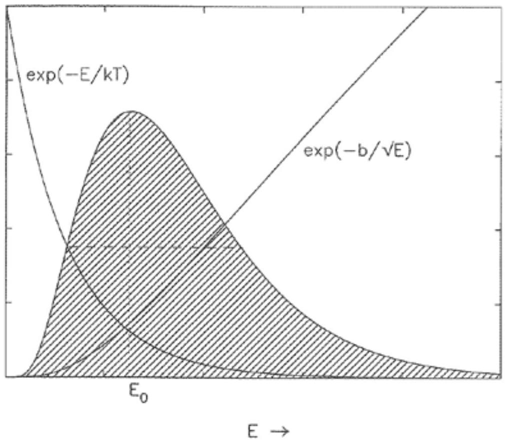
Explain the following using the result(s) above:
a.) Why most stellar spectra show absorption lines. **(2 p)**
b.) The type of spectrum of a cold and optically thin nearby gas cloud observed towards the continuum emission from a bright and hot quasar. **(1 p)**

3.
Two open star clusters are found near each other in the Galactic plane by an optical telescope. They have angular diameters *y* and 3.5*y*, and distance moduli 15.3 and 11.5, respectively. Assuming their actual diameters are equal, calculate their distances and the interstellar extinction coefficient *a*. **(2 p)**

4.
One possible form of the Hertzsprung-Russell diagram has the absolute magnitude on one axis, and the logarithm of the surface temperature on the other axis.
a.) Draw such an HR diagram and indicate the position of the Sun and the main sequence. **(1 p)**
b.) Derive a relation between the absolute magnitude and the logarithm of the surface temperature for stars with the same radius. Sketch the relation in the HR diagram. **(2 p)**

Hints: Assume for simplicity that the magnitudes are bolometric, i.e., all wavelengths are included. It might be best to use the logarithm with base 10.

5.
The figure to the right is related to the fusion process. Referring to the figure, discuss (without equations) two basic physical mechanisms involved in calculations of nuclear reaction rates in stars. **(2 p)**



6.
If the Earth were moved to the orbit of Venus (radius 0.72 AU), what would its average surface temperature become? Assume that the Earth has an albedo of 0.35. Why is the surface temperature of Venus much higher than this, although its albedo is about 0.7? **(2 p)**

7.
Two important methods of exoplanet detection are the Doppler and Transit methods. Describe the main principles of each method (helpful figures are welcome) and describe the main advantages and disadvantages of the methods. **(4 p)**

8.
In certain active galaxies, changes in the position of substructures may be detected over intervals as short as a year. For the quasar 3C 273 at a redshift of $z = 0.158$, one component is seen to move away from the nucleus with an apparent angular velocity of 2.2 milliarcseconds per year.
a.) What is the corresponding apparent, projected linear velocity of this component? **(1 p)**
b.) What is the minimum matter ejection velocity required in order to produce the apparent superluminal speed observed in a)? **(2 p)**
c.) A band of rouge researchers are studying the supermassive black hole (SMBH) of another AGN with a luminosity of 10^{39} W. They claim to have detected a star orbiting the SMBH at a distance of 0.5 light minutes. Is this likely to be true given the mass of the SMBH? **(2 p)**

9.
The neutrino background has been suggested as a candidate for dark matter (DM) - describe briefly how the neutrino could fit with the hot dark matter (HDM) scenario and give at least one argument against why HDM is a good model for DM. **(2 p)**

10.
Calculate how the scale factor a depends on time t after the big bang for a matter-dominated flat universe without a cosmological constant. Express your solution in a_0 and H_0. **(3 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi\nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^{-3} Hz^{-1}] $\qquad n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^{-3} Hz^{-1}] $\qquad U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h\nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1}] $\qquad I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h\nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\qquad \nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_\nu}{dz} = j_\nu - \alpha_\nu I_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \qquad d\tau_\nu = \alpha_\nu \, dz$

$I_\nu = I_{\nu,\mathrm{bg}} \cdot e^{-\tau_\nu} + S_\nu \cdot (1 - e^{-\tau_\nu}) \qquad T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_\nu} + T_\mathrm{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\,\mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ Hubble's law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} = \frac{a_0}{a}$ redshift

$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d = \frac{R}{\pi}$ $R = 1$ AU, π = parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ circular velocity

$\theta \approx 1.22\frac{\lambda}{\text{D}}$ resolution of telescope

$N(t) = N_0 e^{-\lambda t};\ \lambda = \frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\text{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\text{e}} n_{\text{p}}$ recombination and ionization equation

$\frac{L_{\text{I}}}{4 \cdot 10^{10} L_{\text{I},\odot}} \approx \left(\frac{V_{\max}}{200\ \text{km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\text{V}}}{2 \cdot 10^{10} L_{\text{V},\odot}} \approx \left(\frac{\sigma}{200\ \text{km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\text{E}} = \frac{4\pi GMm_{\text{p}}c}{\sigma_{\text{T}}} \approx 1.3 \cdot 10^{31}\frac{M}{\text{M}_\odot}(\text{watt}) \approx 30000\frac{M}{\text{M}_\odot}\text{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\quad e^{-x} = \frac{1}{e^x}$ $\quad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\quad \lg xy = \lg x + \lg y$ $\quad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}

$c = 2.9979 \cdot 10^8$ m/s

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}

$h = 6{,}62606896 \cdot 10^{-34}$ J s

$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m

1 AU = $1.496 \cdot 10^{11}$ m

1 year = $3.156 \cdot 10^7$ s

1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8

The solar constant (1 AU from the Sun): 1371 W/m^2

$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses:	Earth: $5.97 \cdot 10^{24}$ kg,	Jupiter: $1.90 \cdot 10^{27}$ kg,	Sun: $1.99 \cdot 10^{30}$ kg
Radii:	Earth: 6378 km,	Jupiter: 71398 km,	Sun: $6.96 \cdot 10^5$ km.

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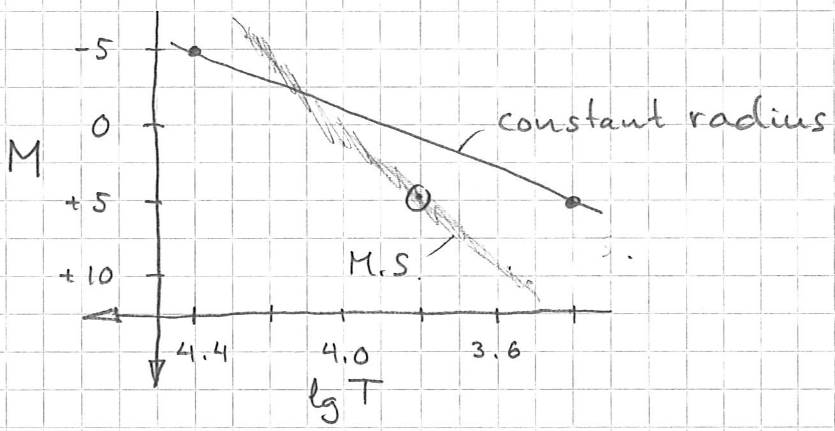
(1.) a) D b) B c) A d) B e) D f) B g) C h) A

(2.) a) 2nd equation. Emission at line frequency from shallower depths with cooler temperatures. See textbook, section 2.4.3

b) 1st equation. Absorption lines (1st term dominates, $I_\nu(0)$ large)

(4) a)



b) $L = \sigma T^4 4\pi R^2 \Rightarrow \lg L = 4 \lg T + 2 \lg R + \text{const.}$

Thus, for some radius $\lg L = 4 \lg T + \text{const.}$

Let the stars be at distance $d = 10$ pc, then $M = -2.5 \lg F/F_0$. F is proportional to L, so $M = -2.5 \lg L + \text{const}$. With $\lg L$ above,

$M = -2.5 \cdot 4 \lg T + \text{const} = -10 \lg T + \text{const}$.

These are straight lines in the HR diagram (one for each radius); see fig. above

(5) The electrostatic force and tunneling.
One curve from Maxwellian velocity distribution, the other from tunneling probability.
Their product (curve w. shading) gives an appreciable reaction rate for energies near E_0.
See fig. 4.4 in the textbook.

(6.) Absorbed = emitted. R = radius of planet,
r = distance from Sun in AU,
$S_\oplus$ = solar constant @ 1 AU (1371 W/m²)
A = albedo

$$\frac{S_\oplus}{r^2} \cdot \pi R^2 \cdot (1-A) = \sigma T^4 \cdot 4\pi R^2 \Rightarrow T = \left(\frac{S_\oplus (1-A)}{r^2 \cdot 4\sigma}\right)^{1/4}$$

$\Rightarrow T = 295$ K (with $A = 0.35$ & $r = 0.72$)
Venus: strong greenhouse effect.

(10.) Friedmanns eq. with $k=0$ & $\rho_M = \rho_{M,0}\left(\frac{a_0}{a}\right)^3$:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_{M,0}\left(\frac{a_0}{a}\right)^3 \Rightarrow \frac{da}{dt} = \sqrt{\frac{8\pi G}{3}\rho_{M,0}a_0^3}\; a^{-1/2}$$

$$\Rightarrow \int a^{1/2}\,da = \sqrt{}\int dt \Rightarrow \frac{2}{3}a^{3/2} = \sqrt{\frac{8\pi G}{3}\rho_{M,0}a_0^3}\cdot t$$

But $\rho_{M,0} = \rho_c$ (critical, since $k=0$)

$H_0 = \frac{\dot{a}}{a_0}$ in Friedmanns eq. $\Rightarrow \rho_c = \frac{3H_0^2}{8\pi G}$, so

$$\frac{2}{3}a^{3/2} = \sqrt{H_0^2 a_0^3}\; t,$$ or

$$\frac{a}{a_0} = \left(\frac{3}{2}H_0 t\right)^{2/3}$$

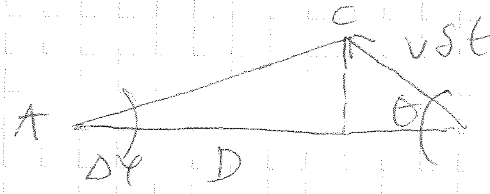
3) use $m - M = 5 \log \frac{D}{10pc} + A$; $A = aD$

use relations between sizes and solve for D_1, D_2 & a

7) See lecture notes

8) The observer infers a transverse velocity

a) $\beta_{app} = \frac{V_T}{c} = \frac{1}{c} \cdot D \frac{\Delta\varphi}{\Delta t}$



$D\Delta\varphi$ app. distance moved by jet $\rightarrow V_T$

Hubbles law $\rightarrow$ D from z

b) $\beta_{app} = \frac{V \sin\theta}{1 - \beta\cos\theta} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}$

angle at which β_{app} is maximized

$\beta_{app} = \beta \frac{\sin\theta}{1 - \beta\cos\theta} = f(\beta, \theta)$; $\frac{\partial \beta_{app}}{\partial \theta} = 0 \rightarrow$

$\rightarrow \theta_{max} = \cos^{-1}\beta$; insert back into expression for β_{app} : $\beta_{app}^{max} = \beta \frac{\sin\theta_{max}}{1 - \beta\cos\theta_{max}} = \frac{\beta}{\sqrt{1-\beta^2}} \rightarrow$

$\rightarrow \frac{V}{c} = \sqrt{\frac{\beta_{app}^{max\,2}}{1 + \beta_{app}^{max\,2}}}$

c) Eddington luminosity $\rightarrow$ min. mass of SMBH; calculate size of event horizon (EH)
Planet would be inside EH $\rightarrow$ not possible

9) See lecture notes and text book