Exam in

RRY125/ASM510 Modern astrophysics

Tid: 9 januari 2017, kl. 08.30–12.30
Plats: Samhällsbyggnad, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587 (mobil: 070 - 237 6701)
(lärare besöker tentamen ca. kl.09.00 och 11.00)

Tillåtna hjälpmedel:
- Typgodkänd räknedosa, eller annan räknedosa med nollställt minne
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator, or other calculator with cleared memory
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

1.

Choose the most reasonable of the given alternatives for the following (do *not* give a motivation):

(a) Surface pressure at Venus (*Mars = 1*):	A) 10000	B) 100	C) 0.01	D) 0.0001
(b) Main element in Jupiter (by mass)	A) H	B) Si	C) C	D) O
(c) Mass of an asteroid	A) 0.1 kg	B) 10^{13} kg	C) 10^{23} kg	D) 10^{30} kg
(d) Pulse period of a pulsar:	A) 3 μs	B) 1 s	C) 2 min	D) 1 hour
(e) Abs. magnitude of supernova Type Ia	A) +19	B) 0	C) −5	D) −19
(f) Object with superluminal motion	A) supergiant	B) Sc galaxy	C) quasar	D) pulsar
(g) Disc galaxy with closely wound spirals:	A) Sa,	B) Sc,	C) SBc,	D) E0
(h) Makes nuclear fusion possible in stars	A) neutrino oscillations			
	B) magnetism			
	C) tunnelling			
	D) electron degeneracy pressure			

(4 p)

2.

The textbook describes an important result concerning radiative transfer in stellar atmospheres as follows:

"… the specific intensity of radiation at a frequency *ν* coming out of a stellar atmosphere is approximately equal to the Planck function at a depth of the atmosphere where the optical depth for that frequency *ν* equals unity."

Most stellar spectra show absorption lines, but under what circumstance can instead *emission* lines be seen in a stellar spectrum? Explain using the result above. **(2 p)**

3.

We have used spectroscopic observations to detect the presence of a planet around a star of mass 2 $M_{\odot}$. The Doppler shift of the spectral lines corresponds to an observed velocity of 45 m/s, and the period is 80 days.

a.) How far from its star is the planet? **(1 p)**

b.) What is the minimum mass of the planet? **(1 p)**

c.) Describe the observational method we require to determine the planet density. **(1 p)**

4.

For four stars, the following apparent magnitudes and distances have been observed:

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1: B = 13.9 V = 13.9 d = 6000 pc
2: B = 15.5 V = 14.0 d =   40 pc
3: B = 12.5 V = 12.5 d =   20 pc
4: B = 11.0 V =  9.5 d =  800 pc
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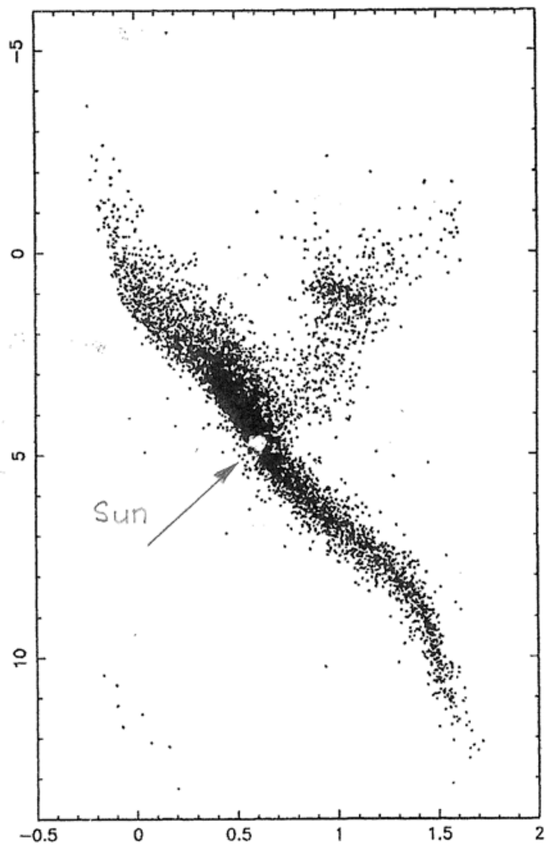
a.) Explain what the pp chain and the CNO cycle are (detailed reactions are not needed). **(1 p)**

b.) Pick one star which produces energy in its core with the pp-chain, and one which does not produce energy through hydrogen burning in its core. Briefly motivate your answer. **(1 p)**

c.) Which of the stars has the largest radius, and which has the smallest radius? Briefly motivate your answer. **(1 p)**

d.) In which of the stars is degeneracy pressure important, and what is that type of star called? **(1 p)**

The figure to the right can be useful: it shows the HR diagram of nearby stars (incl. the Sun).



5.

a.) Briefly describe the two main stellar populations of the Milky Way. **(1 p)**

b.) Our Galaxy has multiple phases of the interstellar medium (ISM) which is often divided into five different constituents ranging from the coldest molecular (H_2) phase to the hottest coronal gas. How are the two stellar populations distributed in relation to the H_2 and coronal gas? Why? **(2 p)**

6.

A spherical galaxy cluster is observed to contain 1000 galaxies. Spectral analysis show the cluster to have a mean redshift of z=0.03 with a standard deviation of σ_z=0.004. The effective radius of the cluster is 2.5 Mpc. Each galaxy has an apparent magnitude of +14.7.

a.) Estimate the dynamical mass of the cluster. **(1 p)**

b.) Estimate the mass to light ratio of the cluster in solar units. **(2 p)**

c.) You would like to detect emission from molecular gas in individual galaxies in the cluster through observing the 115 GHz CO line. Assuming that each galaxy has a typical diameter of 20 kpc, how large a radio telescope do you need for a cluster-galaxy to fit in the telescope beam? **(1 p)**

7.
The cosmic microwave background (CMB) photons we detect on Earth originate from the "Last Scattering Surface" (LSS).
a.) Why is there an LSS − how did it arise? **(1 p)**
b.) Describe briefly the physics behind the Sunyaev-Zeldovich effect. Outline a method to use the SZ effect to measure the Hubble constant. **(2 p)**

8.
This problem is about finding the epoch (time after the big bang) of matter-radiation equality.
a.) Write expressions for how the matter density and the radiation density, resp., depends on the scale factor. Then use them to find an expression for a_{eq}/a_0, the ratio between the scale factor at the epoch of matter-radiation equality and the scale factor at the present epoch. **(1 p)**
b.) Calculate the numerical values of the matter density and the radiation density at the present epoch. Assume that the matter density is equal to the critical density, and use Friedmann's equation to derive an expression for it. The radiation density can be written $\rho_{R,0}=1.68 a_B T_0^4/c^2$, where the constant $a_B=7.6\cdot 10^{-16}$ in SI units, and T_0 is the present value of the temperature of the CMBR. **(3 p)**
c.) Solve Friedmann's equation (i.e., find $a(t)$) for the time *before* matter-radiation equality. Make a reasonable simplification of Friedmann's equation first, and explain why you do it. (Neglect the cosmological constant.) Then, using results from above, calculate the numerical value of the epoch (time after the big bang) of matter-radiation equality. **(3 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi \nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^{-3} Hz^{-1}] $\qquad n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^{-3} Hz^{-1}] $\qquad U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1}] $\qquad I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\qquad \nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_\nu}{dz} = j_\nu - \alpha_\nu I_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \qquad d\tau_\nu = \alpha_\nu \, dz$

$I_\nu = I_{\nu,\mathrm{bg}} \cdot e^{-\tau_\nu} + S_\nu \cdot (1 - e^{-\tau_\nu}) \qquad T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_\nu} + T_\mathrm{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\ \mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ Hubble's law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} = \frac{a_0}{a}$ redshift

$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c}=\frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d=\frac{R}{\pi}$ $R=1\,\mathrm{AU}, \pi=$ parallax angle ($R=1$ and $[\pi]=''$ gives d in pc)

$E_{\mathrm{kin}}=\frac{mv^2}{2}$ kinetic energy

$E_{\mathrm{pot}}=-\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\mathrm{kin}}=\frac{M(\Delta v)^2}{2}, \quad E_{\mathrm{pot}}=-\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\mathrm{kin}}+E_{\mathrm{pot}}=0$ the virial theorem

$V_c=\sqrt{\frac{GM}{R}}$ circular velocity

$\theta\approx 1.22\frac{\lambda}{\mathrm{D}}$ resolution of telescope

$N(t)=N_0e^{-\lambda t};\ \lambda=\frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\mathrm{e}}}{dt}=N_{\mathrm{star}}\frac{q}{V}-\alpha n_{\mathrm{e}}n_{\mathrm{p}}$ recombination and ionization equation

$\frac{L_{\mathrm{I}}}{4\cdot 10^{10}L_{\mathrm{I},\odot}}\approx\left(\frac{V_{\max}}{200\ \mathrm{km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\mathrm{V}}}{2\cdot 10^{10}L_{\mathrm{V},\odot}}\approx\left(\frac{\sigma}{200\ \mathrm{km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\mathrm{E}}=\frac{4\pi GMm_{\mathrm{p}}c}{\sigma_{\mathrm{T}}}\approx 1.3\cdot 10^{31}\frac{M}{\mathrm{M}_\odot}(\mathrm{watt})\approx 30000\frac{M}{\mathrm{M}_\odot}\mathrm{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\quad e^{-x} = \frac{1}{e^x}$ $\quad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\quad \lg xy = \lg x + \lg y$ $\quad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}

$c = 2.9979 \cdot 10^8$ m/s

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}

$h = 6{,}62606896 \cdot 10^{-34}$ J s

$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m

1 AU = $1.496 \cdot 10^{11}$ m

1 year = $3.156 \cdot 10^7$ s

1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8

The solar constant (1 AU from the Sun): 1371 W/m^2

$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses:	Earth: $5.97 \cdot 10^{24}$ kg,	Jupiter: $1.90 \cdot 10^{27}$ kg,	Sun: $1.99 \cdot 10^{30}$ kg
Radii:	Earth: 6378 km,	Jupiter: 71398 km,	Sun: $6.96 \cdot 10^5$ km.

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(1) a) A b) A c) B d) B e) D f) C g) A h) C

(2.) The temperature of the stellar atmosphere increases outwards. See textbook p. 45-46.

(4.) a) Hydrogen fusion reactions. See textbook, sect. 4.3

b) Calculate B-V & M_V. $M_V = V - 5\lg\frac{d}{10pc}$

Star 1:	B-V = 0.0	M_V = 0.0	(CNO)
2:	1.5	11.0	pp; lower main seq.
3:	0.0	11.0	no H-burning in core, wd
4:	1.5	0.0	no H-burning in core; giant

c) $L = 4\pi R^2 \sigma T^4$.

Star 4 is very luminous but "cold" → large radius

Star 3 is very dim but "hot" → small radius

d) Star 3: white dwarf

(8.) a) $\rho_M = \rho_{M,0}\left(\frac{a_0}{a}\right)^3$. $\rho_R = \rho_{R,0}\left(\frac{a_0}{a}\right)^4$

$\rho_M = \rho_R \Rightarrow \frac{a_{eq}}{a_0} = \frac{\rho_{R,0}}{\rho_{M,0}}$

b) Matter: Critical density when k=0 and Λ=0. Friedmann's eq. with $\frac{\dot{a}}{a} = H_0$ gives the present critical density:

$H_0^2 = \frac{8\pi G}{3}\rho_c \Rightarrow \rho_c = \frac{3H_0^2}{8\pi G} (= \rho_{M,0})$

$H_0 = 72$ km/s/Mpc $\Rightarrow \rho_{M,0} = 9.7\cdot10^{-27}$ kg m^{-3}

Radiation: $\rho_{R,0} = 1{,}68\, a_B T_0^4/c^2$ with $T_0 = 2.7$ K

$\Rightarrow \rho_{R,0} = 7.5\cdot10^{-31}$ kg m^{-3}

c.) Before equality: radiation dominates.

Friedmann's eq with $\Lambda = 0$:

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3} \rho_{R,0} \left(\frac{a_0}{a}\right)^4$$

For small a, the R.H.S. dominates over $\frac{kc^2}{a^2}$, which can be ignored:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho_{R,0} \left(\frac{a_0}{a}\right)^4 \Rightarrow \left(\frac{da}{dt}\right)^2 a^2 = \frac{8\pi G}{3} \rho_{R,0} a_0^4$$

$$\Rightarrow \int_0^{a_{eq}} a\, da = \sqrt{\frac{8\pi G}{3} \rho_{R,0}}\; a_0^2 \int_0^{t_{eq}} dt \Rightarrow$$

$$\Rightarrow \frac{1}{2} a_{eq}^2 = \sqrt{}\; a_0^2\, t_{eq} \Rightarrow \frac{a_{eq}}{a_0} = \sqrt{2} \left(\frac{8\pi G}{3} \rho_{R,0}\right)^{1/4} t_{eq}^{1/2}$$

This $\frac{a_{eq}}{a_0}$ should be equal to $\frac{\rho_{R,0}}{\rho_{M,0}}$.

$\rho_{R,0}$ & $\rho_{M,0}$ from b., solve for t_{eq}:

$$t_{eq} = \left(\frac{\rho_{R,0}}{\rho_{M,0}}\right)^2 \cdot \frac{1}{2} \left(\frac{3}{8\pi G \rho_{R,0}}\right)^{1/2} = \ldots = 1{,}46 \cdot 10^{11}\, s$$

$$= 4600 \text{ years}$$

3a) Kepler's 3rd $\frac{a^3}{P^2} = M$ → $a \sim 0.46$ AU

b) $m_p \sin i = \left[\frac{m_*^2 P V_*^3}{2\pi G}\right]^{1/3}$ → $m_p \approx 3 \times 10^{27}$ kg

c) See lecture notes

5) See course book & lecture notes

6) a) Use Hubbles law, redshift $z = \frac{V_{obs}}{c}$ $DH_0 = V_{obs}$

→ $D = 125$ Mpc $\sigma \sim 1199$ km/s $\sim \Delta V$

Spherical cluster, assume it is relaxed — use virial theorem to get $M_{dyn} \sim \frac{(\Delta v)^2 R}{G} \sim 10^{15} M_\odot$

b) 1000 galaxies with apparent magnitude $14.7 = m$

$M_{bol} + 5 \lg \frac{D}{10pc} = m$ → $M_{bol} \sim -20.78$

$M_{bol} - M_{bol\odot} = -2.5 \lg\left(\frac{L}{L_\odot}\right)$ → $L_{galaxy} = 1.7 \times 10^{10} L_\odot$

→ $\frac{M}{L_{tot}} \sim \frac{10^{15}}{1.7 \times 10^{13}} \sim 60$

c) each galaxy: diam $d = 20$ kpc, Observe at $\nu \sim 115$ GHz → $\lambda - 2.6$ mm

D of cluster 125 Mpc → $1'' \sim 606$ pc

20 kpc $= 33'' \sim 33 \times 4.84 \times 10^{-6}$ rad

$\theta \sim 1.22 \frac{\lambda}{D_{tel}}$ → $D_{tel} \sim 20$ m

7) See course book & lecture notes