Exam in

RRY125/ASM510 Modern astrophysics

Tid: 13 december 2011, kl. 14.00–18.00
Plats: Maskinsalar, Chalmers
Ansvarig lärare: Susanne Aalto, ankn. 5506, mobil 0702-520152, och
Magnus Thomasson ankn. 8587
(lärare besöker tentamen ca. kl.14.30 och 16.30)

Tillåtna hjälpmedel:
- Typgodkänd räknedosa (andra räknedosor måste ha nollställt minne)
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator (other calculators must have cleared memory)
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

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1.

Choose the most reasonable of the given values for the following (do not give a motivation):

(a) Mass of an asteroid:	A) 1 kg,	B) 10^{13} kg,	C) 10^{24} kg,	D) 10^{27} kg
(b) Age of the Moon:	A) 500 Myr,	B) 1 Gyr,	C) 4 Gyr,	D) 10 Gyr
(c) Diameter of a red giant (ly=light year):	A) 1 ly,	B) 1 AU,	C) 10^{6} km,	D) 1 pc
(d) Surface temperature of a blue star:	A) 20000 K,	B) 6000 K,	C) 3000 K,	D) 10^{7} K
(e) Redshift of a nearby cluster of galaxies:	A) 3.0,	B) –0.5,	C) 0.01,	D) 0.0001
(f) Distance to a quasar (ly=light year):	A) 10^{5} ly,	B) 10^{6} AU,	C) 10 Gpc,	D) 3 Gpc

(3 p)

2.

Assume that a Jupiter-sized planet is in a circular orbit (radius 10 AU) around a Sun-like star.

a.) What is the maximum possible Doppler shift of the Calcium H line (396.847 nm)? **(1 p)**

b.) How far from the centre of the star is the centre of mass of the system? **(1 p)**

3.

With a radiotelescope, you observe a cosmic gas cloud. There is no radio source behind the cloud, except for the cosmic microwave background radiation (temperature 2.7 K). The cloud contains a molecule (which can be seen as a two level system) with an excitation temperature of 50 K. At the transition frequency of the molecule, the optical depth is 0.7. At other frequencies, it is negligible.

Sketch a graph of the brightness temperature as a function of frequency (i.e., the spectrum), and give values of relevant brightness temperatures in the graph.

Hint: The brightness temperature is proportional to the specific intensity. **(2 p)**

4.

Calculate the surface temperature of the side of Mercury facing the Sun (i.e., the hot side). Assume that Mercury always turns the same side towards the Sun (this is almost true). Mercury's Bond albedo (reflectivity) is 0.06. Assume Mercury's distance to the Sun is constant, 0.387 AU. **(2 p)**

5.

a.) Draw a schematic HR (Hertzsprung-Russel) diagram. Use bolometric magnitude and colour index on the axes. **(3 p)**

b.) Describe how you can use an HR diagram to determine age and distance to a globular cluster in the Milky Way. **(1 p)**

c.) The QSO number density has changed with the age of the Universe and it was highest around $z = 2$. How can we explain this? **(1 p)**

6.

a.) Describe and compare the stellar populations, rotation curves, radial surface brightness distribution and interstellar medium of elliptical and spiral galaxies. **(2 p)**

b.) Hubble's "tuning fork" diagram goes from "early" to "late" types – explain (briefly) why this is misleading. **(1 p)**

7.
Two open star clusters, which are seen near each other in the galactic plane, have angular diameters y and $3y$, and distance moduli ($m - M$) 15.0 and 12.0, respectively. Assuming their actual diameters are equal, find their distances and the interstellar extinction coefficient a.
(2 p)

8.
The cosmic microwave background (CMB) photons we detect on Earth originate from the "Last Scattering Surface" (LSS).
a.) Why is there an LSS – how did it arise? **(1 p)**
b.) The CMB photons may interact with galaxy clusters before they reach us. Describe briefly the physics behind the Sunyaev-Zeldovich effect. **(1 p)**

9.
A supernova of Type Ia is observed in a distant spiral galaxy. It has an apparent magnitude of +18.7, and it is known that such supernovae have absolute magnitude −19.3. The HI line from gas in the galaxy is observed with the 25 m diameter radiotelescope in Onsala at a frequency of 1295.9 MHz.
a.) What is a supernova Type Ia? Why are such supernovae particularly useful for measurements of very large cosmic distances? **(2 p)**
b.) From the information given, make a rough estimate of the age of the Universe (i.e., calculate the Hubble time). **(2 p)**
c.) Does the galaxy fill the beam of the telescope (i.e., is its angular size smaller or larger than the telescope beam)? **(1 p)**
d.) Which important discovery about the Universe was made by observing supernovae, and awarded with the Nobel prize 2011? **(1 p)**

10.
Two problems where you need to use the Friedmann equation (without the cosmological constant):
a.) Derive an expression (which will include H_0) for the critical density of the Universe. **(1 p)**
b.) For a "flat" Universe dominated by ordinary matter, derive how the scale factor depends on time. (You do not have to *derive* how the density depends on the scale factor.) **(2 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi \nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^-3 Hz^-1] $\qquad n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^-3]

$U_\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^-3 Hz^-1] $\qquad U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^-3]

$I_\nu = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^-2 Hz^-1] $\qquad I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^-2]

$I_\nu = \frac{2 h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^-2 Hz^-1 sr^-1] $\qquad \nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_\nu}{dz} = j_\nu - \alpha_\nu I_\nu \quad S_\nu = \frac{j_\nu}{\alpha_\nu} \qquad d\tau_\nu = \alpha_\nu \, dz$

$I_\nu = I_{\nu,\mathrm{bg}} \cdot e^{-\tau_\nu} + S_\nu \cdot (1 - e^{-\tau_\nu}) \qquad T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_\nu} + T_\mathrm{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\ \mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Cosmology

$v = H_0 \cdot d$	Hubble's law
$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \frac{\nu_{\text{em}}}{\nu_{\text{obs}}} = \frac{a_0}{a}$	redshift
$ds^2 = -c^2dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$	Robertson-Walker metric
$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$	Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$	the Doppler effect
$d = \frac{R}{\pi}$	$R = 1$ AU, π = parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)
$E_{\text{kin}} = \frac{mv^2}{2}$	kinetic energy
$E_{\text{pot}} = -\frac{GMm}{R}$	potential energy for a point mass m orbiting a point mass M
$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$	(energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)
$2E_{\text{kin}} + E_{\text{pot}} = 0$	the virial theorem
$V_c = \sqrt{\frac{GM}{R}}$	circular velocity
$\theta \approx 1.22\frac{\lambda}{\text{D}}$	resolution of telescope
$N(t) = N_0e^{-\lambda t};\ \lambda = \frac{\ln 2}{t_{1/2}}$	radioactive decay
$\frac{dn_{\text{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\text{e}}n_{\text{p}}$	recombination and ionization equation
$\frac{L_{\text{I}}}{4 \cdot 10^{10} L_{\text{I},\odot}} \approx \left(\frac{V_{\max}}{200\ \text{km/s}}\right)^4$	(the Tully-Fisher relation)
$\frac{L_{\text{V}}}{2 \cdot 10^{10} L_{\text{V},\odot}} \approx \left(\frac{\sigma}{200\ \text{km/s}}\right)^4$	(the Faber-Jackson relation)

$$L_{\text{E}} = \frac{4\pi GMm_{\text{p}}c}{\sigma_{\text{T}}} \approx 1.3 \cdot 10^{31}\frac{M}{\text{M}_\odot}(\text{watt}) \approx 30000\frac{M}{\text{M}_\odot}\text{L}_\odot$$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\qquad e^{-x} = \frac{1}{e^x}$ $\qquad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\qquad \lg xy = \lg x + \lg y$ $\qquad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$) , $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67\cdot10^{-11}$ N m^2 kg^{-2}

$c=2.9979\cdot10^8$ m/s

$\sigma = 5.67\cdot10^{-8}$ W m^{-2} K^{-4}

$h = 6{,}62606896 \cdot 10^{-34}$ J s

$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857\cdot10^{16}$ m

1 AU = $1.496\cdot10^{11}$ m

1 year = $3.156\cdot10^7$ s

1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8

The solar constant (1 AU from the Sun): 1371 W/m^2

$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses:	Earth: $5.97\cdot10^{24}$ kg,	Jupiter: $1.90\cdot10^{27}$ kg,	Sun: $1.99\cdot10^{30}$ kg
Radii:	Earth: 6378 km,	Jupiter: 71398 km,	Sun: $6.96\cdot10^5$ km.

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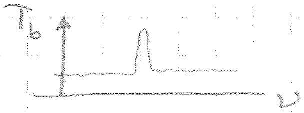
(1) a) B b) C c) B d) A e) C f) D

(3) $T_b = T_{bg} e^{-\tau_\nu} + T_{ex}(1-e^{-\tau_\nu})$



at line: $\tau_\nu = 0.7 \Rightarrow T_b = 2.7e^{-0.7} + 50(1-e^{-0.7}) = \underline{26.5\text{ K}}$

off line: $\tau_\nu = 0.0 \Rightarrow T_b = T_{bg} = \underline{2.7\text{ K}}$

(4) $\begin{cases} P_{in} = \dfrac{S_\oplus}{a^2}(1-A)\pi R^2 \\ P_{ut} = \sigma T^4 \, 2\pi R^2 \end{cases}$, $a = 0.387$ AU, $S_\oplus = 1371\ \text{W/m}^2$, $A = 0.06$ (assume blackbody rad.)

$P_{in} = P_{ut} \Rightarrow \dfrac{S_\oplus}{a^2}(1-A) = \sigma T^4 \, 2 \Rightarrow T = \underline{525\text{ K}}$

(9.) a) Exploding white dwarfs in binary systems

2p Very bright, and all have the same luminosity.

b.) $m - M = 5 \lg \dfrac{d}{10\text{pc}} + A$, assume $A = 0$

$18.7 - (-19.3) = 5 \lg \dfrac{d}{10\text{pc}} \Rightarrow d = 3.98 \cdot 10^8$ pc

2p $1 + \dfrac{v}{c} = \dfrac{\nu_{em}}{\nu_{obs}} \Rightarrow v = \left(\dfrac{\nu_{em}}{\nu_{obs}} - 1\right) c = \left(\dfrac{1420.4}{1295.9} - 1\right) c = 28\,800\ \dfrac{\text{km}}{\text{s}}$

$v = H_0 d \Rightarrow H_0 = \dfrac{v}{d} = \dfrac{28\,800}{398} = 72.4\ \dfrac{\text{km/s}}{\text{Mpc}}$

$t = \dfrac{1}{H_0} = \dfrac{1}{72.4 \cdot 10^3} \cdot 10^6 \cdot 3.0857 \cdot 10^{16}\ \text{s} = 4.26 \cdot 10^{17}\ \text{s} = \underline{13.5\text{ Gyr}}$

c) $\theta = 1.22 \dfrac{\lambda}{D} = 1.22 \dfrac{0.21}{25} = 0.01$ rad

1p @ 398 Mpc : $0.01 \cdot 398$ Mpc $= 3.98$ Mpc > galaxy size

No, does not fill the beam

1p d.) Accelerating universe (dark energy)

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(10) Friedmann eq. without cosmol. const.

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho$$

a) Critical case: $k=0$

$$H_0 = \frac{v}{d} = \frac{\dot{a}}{a}$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_c \implies \rho_c = \frac{3}{8\pi G}\frac{\dot{a}^2}{a^2} = \frac{3H_0^2}{8\pi G}$$

b) $$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \text{, so } \frac{da}{dt} = \sqrt{\frac{8\pi G}{3}\rho}\; a$$

$$\rho = \rho_c\left(\frac{a_0}{a}\right)^3 \text{, so}$$

$$\frac{da}{dt} = \sqrt{\frac{8\pi G}{3}\rho_c a_0^3}\; a^{-1/2} \implies$$

$$\int_0^a a^{1/2}\,da = \sqrt{\frac{8\pi G}{3}\rho_c a_0^3}\int_0^t dt$$

$$\frac{2}{3}a^{3/2} = \sqrt{\frac{8\pi G}{3}\rho_c a_0^3}\; t$$

$$a(t) = \left(\frac{3}{2}\right)^{2/3}\left(\frac{8\pi G}{3}\rho_c a_0^3\right)^{1/3} t^{2/3} = (6\pi G\rho_c)^{1/3} a_0 t^{2/3}$$

Or note that $\frac{8\pi G}{3}\rho_c = H_0^2$, so

$$a(t) = \left(\frac{3}{2}\right)^{2/3}\left(H_0^2\right)^{1/3} a_0 t^{2/3} \text{, or}$$

$$\frac{a(t)}{a_0} = \left(\frac{3}{2}H_0 t\right)^{2/3}$$