Exam in Course Radar Systems and Applications (RRY080)

16th January 2010, 14:00-18:00.

Additional materials: Beta, Physics Handbook (or equivalent), Dictionary and Electronic calculator (it is free-of-choice but no computers are allowed).

Leif Eriksson (tel: 4856) and Anatoliy Kononov (tel 1844) will visit around 15:00 and 17:00.

Read through all questions before this time to check that you have understood the questions.

Answers may only be given in English.

Clearly show answers (numbers and units!) to each part of question just after the corresponding solution.

Begin each question on a new sheet of paper!

Each question carries a total of ten points. Where a question consists of different parts, the number of points for each part of the question is indicated in parentheses. **In most cases it should be possible to answer later parts of the question even if you cannot answer one part.**

Grades will be awarded approximately as <20 = Fail, 20-29 = 3, 30-39 = 4, ≥40 = 5
(exact boundaries may be adjusted later).

Review of corrected exam papers will take place on the two following occasions:

Monday the 1st of February, 10:00-11:45
Friday the 5th of February, 14:00-15:00

1. Basic terms & principles

a) The elevation gain pattern of a radar antenna is approximated by $G = G_o \exp\left(-900\theta^2\right)$, where θ is the elevation angle in radians. Compute the 3 dB beamwidth expressed in degrees. (2p)

b) What is the bistatic RCS of a metallic sphere with radius r >> wavelength? (2p)

c) Compute the Doppler frequency shift from a satellite moving with relative velocity of 5 km/s observed by a radar operating with frequency at f = 15 GHz. (2p)

d) Compute the distance to the radar horizon for a marine radar located 30 m above sea level. Assume the Earth radius is R_E = 6378 km and take into account refraction by air in normal atmospheric conditions. (2p)

e) A radar system operates with a pulse repetition frequency of 10 kHz. Compute the maximum unambiguous range. (2 p)

2. Repeat pass InSAR for moving objects.

A satellite SAR interferometer operates at altitude h_0=800 km, incidence angle $\theta = 23°$ and with a horizontal antenna baseline B (between the two passes over the area) of 200 m. The frequency of the SAR is 10 GHz. Assume a flat earth geometry.

a) Draw one or two figures that explain the geometry of the interferometer. The altitude, incidence angle and antenna baseline must be indicated in the figure/s. The parallel and perpendicular components of the baseline should also be clearly marked. (2 p)

b) Determine the fringe spacing (distance for one full fringe) on a horizontal flat surface in the original slant range image. Hint: One full fringe corresponds to $\Delta\phi = 2\pi$ (3 p)

c) Two radar reflectors have been deployed in the neighborhood of each other in order to investigate the sensitivity to small motion detection. The SAR interferometer images the two reflectors on two occasions and one of the reflectors is moved by 1cm vertically between the two over passes. Determine the relative change of the phase difference between the reflectors. (3 p)

d) Determine the maximum vertical displacement of a reflector that can be measured without phase ambiguities (2 p)

3. Radar scattering

A monostatic radar system with a network analyzer and an antenna is used in the laboratory exercise to measure the backscatter and dielectric properties of different objects and materials. The aperture of the antenna is 20 cm and the distance between the antenna and the object is kept at 2 m. For the following questions, assume that the frequency is 10 GHz.

a) Explain the difference between near field and far field and calculate the minimum distance for the far field (according to the Fraunhofer criterion) if the target has a characteristic horizontal size of 30 cm. (3 p)

b) Determine the width of the illuminated zone (the foot print) limited by the antenna gain. (1 p)

c) Determine the width of the first Fresnel zone. (1 p)

d) At high frequency, the radar cross section of a trihedral reflector is given by,

$$\sigma_{trihedral} = \frac{4\pi a^4}{3\lambda^2}$$

where *a* is the trihedral base length.

How large must a metallic sphere be to have the same radar cross section as a trihedral. (2 p)

e) At this frequency, what should be the characteristic roughness height of a surface to fulfill the Rayleigh criterion for a smooth surface? (2 p)

f) In addition to the roughness, what else affect the scattering from a surface of sand? (1 p)

4. Processing of radar signals

A search radar is to detect a target with the probability of detection $P_D = 0.9$ by using one received pulse to take a decision on the presence of the target. The radar is designed to provide the range resolution $\delta R = 75\,\mathrm{m}$. The radar wavelength is $\lambda = 0.03\,\mathrm{m}$ and the transmitted pulse width is $T = 50 \cdot 10^{-6}\,\mathrm{s}$.

a) Compute the minimum signal-to-noise ratio (SNR, dB) required to ensure the specified probability of detection for a Swerling I target if the probability of false alarm $P_{FA} = 10^{-6}$. (1 p)

b) Assume the transmitted pulse is an LFM (linear-frequency modulated) signal. Write an analytical expression for the complex envelope of this signal and compute its bandwidth B and compression ratio CR. Explain the physical meaning of the compression ratio. (2 p)

c) The radar observes two zero-Doppler point targets having the same angular coordinates but separated in range by a distance of D m. Draw the envelopes of corresponding echo signals at the output of the LFM matched filter for the two cases: (i) D = 150 m and (ii) D = 50 m. Ignore the receiver noise. How could you characterize the possibility to distinguish the targets in case (i) and (ii) taking into account the radar resolution ability? (3 p)

d) Estimate the maximum range measurement error ΔR due to the range-Doppler coupling effect for LFM signals if the target radial velocity $|V_r| \leq V_{r\max}$, where $V_{r\max}$ = 150 m/s. Illustrate this effect by using a contour plot of the LFM ambiguity function. (4 p)

5. Radar measurements of Venus

In 1959 and 1961, the MIT Lincoln Laboratory Millstone radar made comprehensive measurements of the Earth-Venus distance. The data and subsequent analysis led to the determination of the astronomical unit (mean distance between the Sun and the Earth) with unprecedented accuracy (+/- 300 km). Similar and consistent results were obtained during the same time period by the NASA JPL Goldstone radar.

Millstone radar parameters used in 1961

Peak transmit power, P_{peak}	2.5 MW
Average transmit power, P_{avg}	0.15 MW
Operating frequency, f	440 MHz
Effective aperture of the antenna, A_e	207 m^2
System noise temperature, T_s	240 K
Total system losses, L	1 dB
Pulse lengths	0.5 ms; 2 ms; 4 ms

Assume the propagation speed is the speed of light in vacuum, i.e. 299792458 m/s, and that Venus has a mean radar cross section of 1.7×10^{13} m^2 (15% of its projected area). Assume the time delay is 283 s which was obtained on 10 April 1961 when the distance between Earth and Venus was at its minimum. Require 95% probability of detection at 0.0001 false-alarm probability in the analysis.

a) Determine the required coherent integration time assuming that Venus is a steady target. (4p)

b) Real measurements show that Venus radar echoes are time-varying and have a Doppler bandwidth of about 0.5 Hz. Assume a Swerling case 1, i.e. coherent integration of a steady target during a dwell time of 2 s (reciprocal of the Doppler bandwidth) followed by incoherent integration of a target with randomly varying radar cross section σ according to the probability density function $p(\sigma) = (1/\overline{\sigma})\exp(-\sigma/\overline{\sigma})$. Determine the required total integration time for detection. (4p)

c) The NASA JPL Goldstone radar also made measurements during the same time period. It operated using a higher frequency of 2388 MHz, system noise temperature of 64 K, average transmit power of 13 kW and total system losses of 1 dB. Did the Goldstone radar need more or less integration time to detect Venus compared to the Millstone radar? Motivate your answer. Assume the same effective aperture area as the Millstone radar antenna. The expected attenuation through the entire atmosphere as a function of frequency for different elevation angles is given in the graph below. (2p)

Auxiliary Formulas

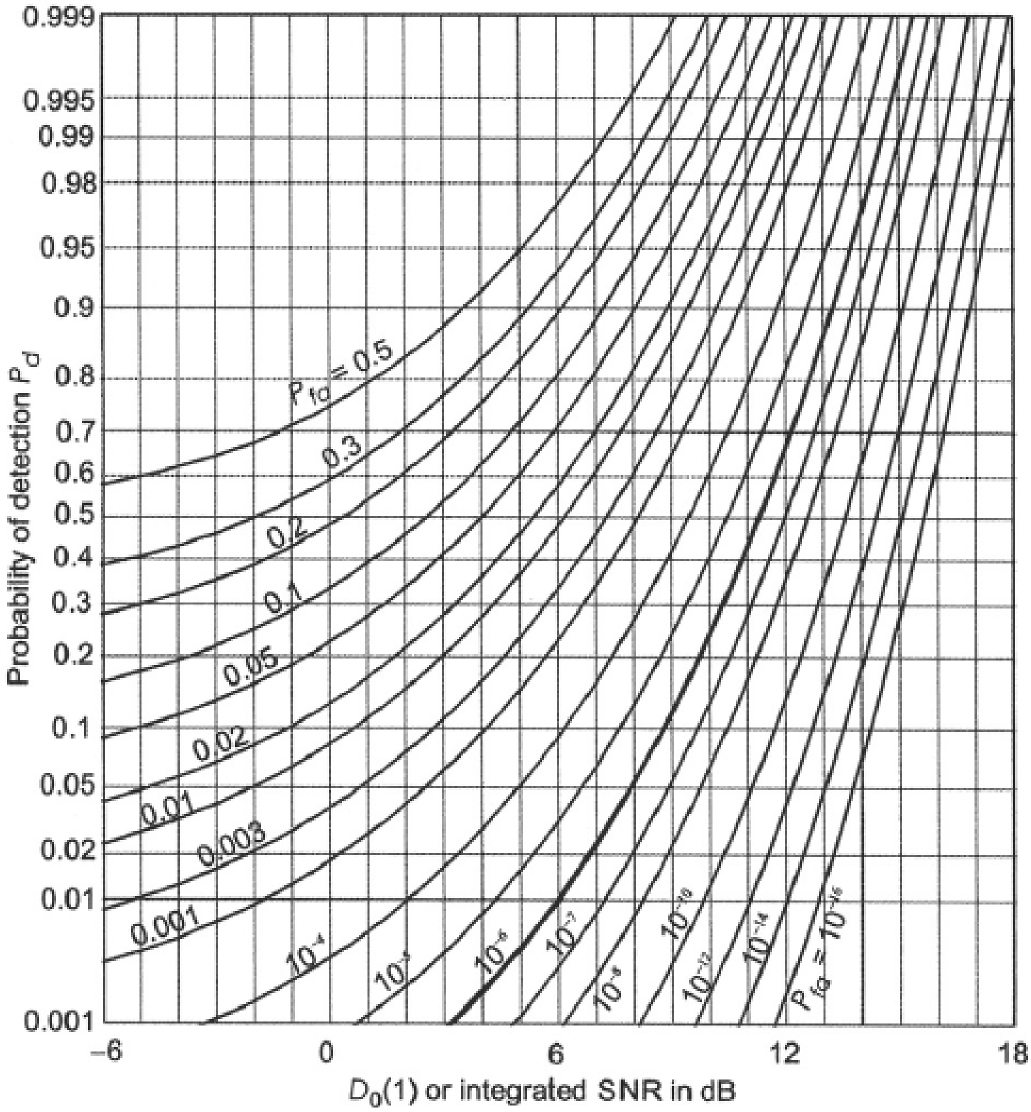
$$L_f(n_e) = \left[L_f(1)\right]^{1/n_e} \qquad (4.15)$$

$$L_f(n_e)[\text{dB}] = \frac{1}{n_e}\cdot\left(10\cdot\lg L_f(1)\right) = \frac{L_f(1)[\text{dB}]}{n_e}$$

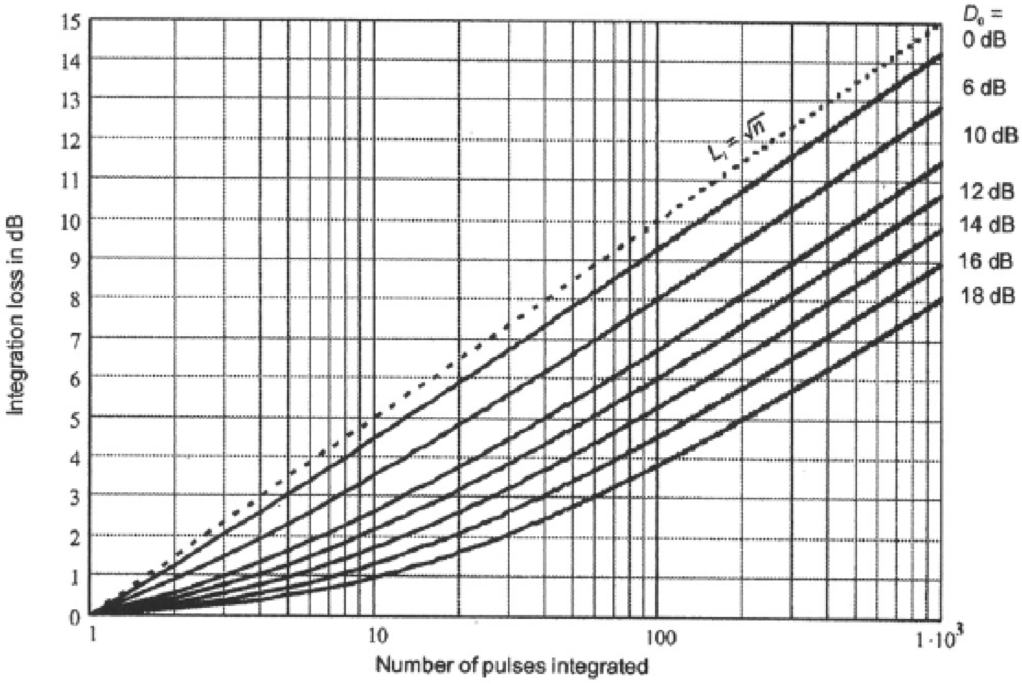
$$n_e = \begin{cases} 1 & \text{for Swerling-1} \\ n & \text{for Swerling-2} \\ 2 & \text{for Swerling-3} \\ 2n & \text{for Swerling-4} \end{cases}$$

$n_e \to \infty$ for nonfluctuating model (Swerling - 0 or 5) and $L_f(1) = 1$ or $L_f(1)[\text{dB}] = 0$

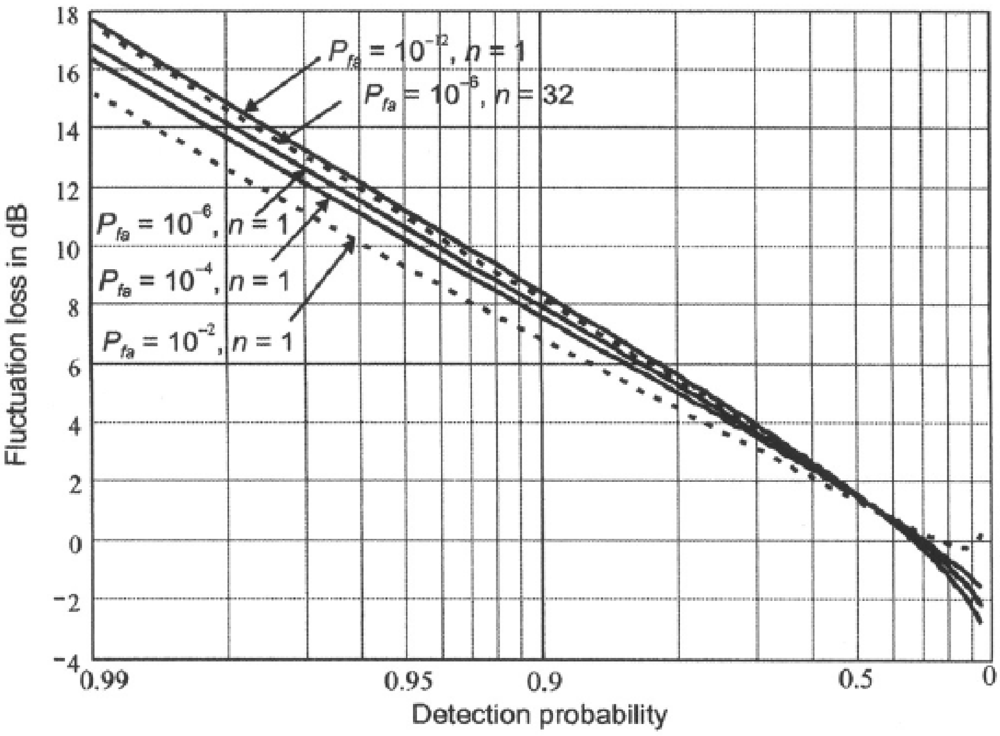
$$SNR_{\text{req}} = D_e(n, n_e) = \frac{D_0(1)L_i(n)L_f(n_e)}{n} \qquad (4.16)$$



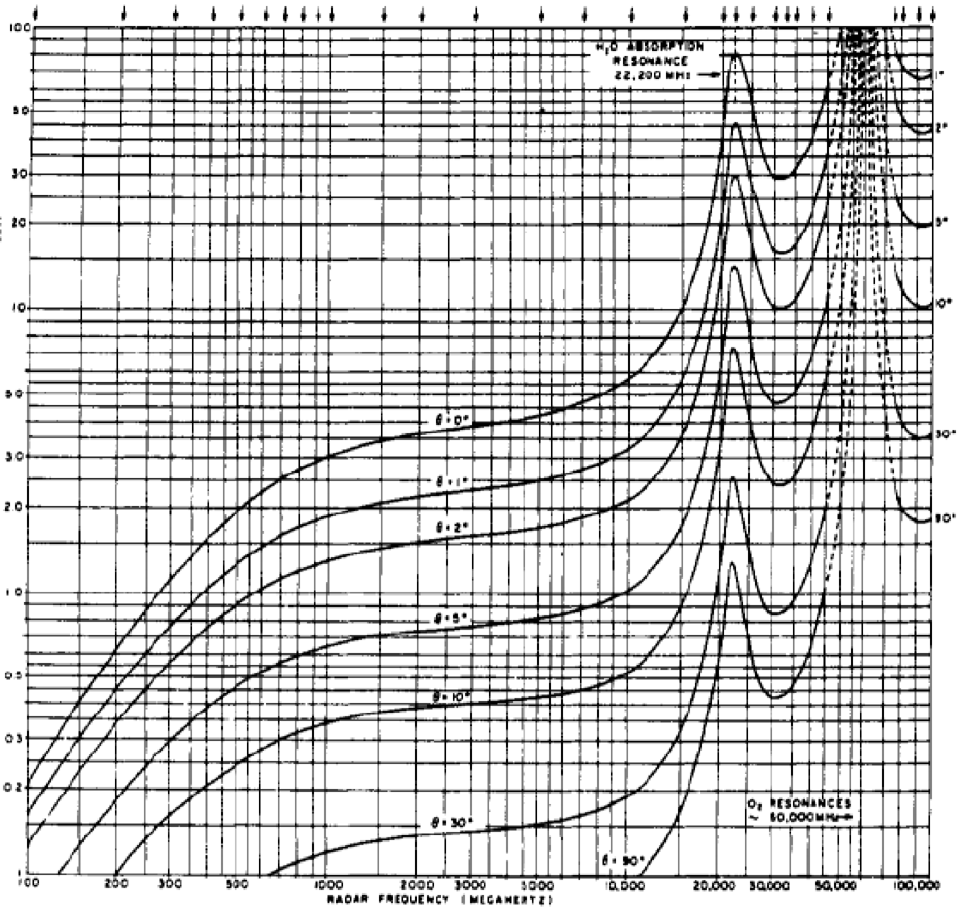
Detectability factor for a steady target
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 45]



Integration loss versus number of pulses integrated after envelope detection
for different values of output detectability factor $D_0(1)$
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 52]



Fluctuation losses for case 1 target (Swerling-1 model)
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 61]



Attenuation by entire atmosphere (dB)

Formulas which may be used at examination of the course Radar Systems and Applications (equation and page numbers refer to the course book by Sullivan)

Radar equation

$$SNR = \frac{P_{peak} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.23)$$

$$SNR = \frac{P_{avg} G^2 \lambda^2 \sigma t_{dwell}}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.24)$$

Antennas

$$A_e = \frac{G\lambda^2}{4\pi} \qquad (1.18)$$

$$A_e = A\eta \qquad \text{(p. 15)}$$

$$R_{far} = \frac{2L^2}{\lambda} \qquad \text{(p. 43)}$$

System noise

$$k_B T_{sys} = k_B \left[T_{ant} / L_{radar} + T_{radar} \left(1 - 1/L_{radar}\right) + T_{rcvr} \right] \qquad (2.10)$$

$$T_{rcvr} = (F-1) T_0 \quad , F > 1 \qquad (2.11)$$

Radar Cross Section

$$\sigma = \lim_{R \to \infty} 4\pi R^2 \frac{|\mathbf{E_s}|^2}{|\mathbf{E_i}|^2} \qquad (3.1)$$

$$\sigma = 4\pi \frac{A^2}{\lambda^2}$$ (Broadside RCS of large metallic flat plate, p. 69-70)

Radar clutter

$$\sigma_c = \sigma^0 A \qquad \text{(p. 87)}$$

$$\sigma_c = \eta V \qquad \text{(p. 88)}$$

Envelope detection of targets embedded in noise (single-pulse decision)

a) Nonfluctuating (steady) target

$$P_D \approx \frac{1}{2} \left[erfc \left(\sqrt{\ln\left(\frac{1}{P_{FA}}\right)} - \sqrt{SNR + \frac{1}{2}} \right) \right] \qquad \text{(4.9) corrected}$$

b) Fluctuating target

$$P_D = P_{FA}^{\;1/(1+SNR)} \qquad (4.13)$$

Matched filter

$$h(t) = Ku^*(t_m - t) \quad \Leftrightarrow \quad H(\omega) = KU^*(\omega)\exp(-j\omega t_m) \qquad (4.32)$$

Ambiguity function

$$\left|\chi(\tau, \nu)\right| = \left|\int_{-\infty}^{+\infty} u(t)u^*(t-\tau)\exp(j2\pi\nu t)\,dt\right| \qquad \text{(p. 116)}$$

Doppler frequency (this formula takes into account the sign of Doppler frequency)

$$f_d = -\frac{2v}{\lambda} = -\frac{2\cdot(dR/dt)}{\lambda} \qquad (1.32)$$

Maximum unambiguous velocity interval

$$\Delta v_u = \frac{f_R \lambda}{2} \qquad \text{(p. 22)}$$

Inverse SAR

$$\delta_{rpn} = \frac{c}{2B} \qquad (6.4)$$

$$\delta_{crpn} = \frac{\lambda}{2\Delta\phi} \qquad (6.5)$$

SAR

$$\delta_r \approx \frac{c}{2B} \qquad \text{(p. 216)}$$

$$\delta_{cr} \approx \frac{\lambda}{2\Delta\theta} \qquad (7.1)$$

$$CNR = \frac{P_{avg} G^2 \lambda^3 \sigma^0 \delta_r}{2(4\pi)^3 R^3 k_B T_{sys} L V \cos\psi} \qquad (7.46)$$

Solutions to

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d) Compute the distance to the radar horizon for a marine radar located 30 m above sea level. Assume the Earth radius is R_E = 6378 km and take into account refraction by air in normal atmospheric conditions. (2p)

e) A radar system operates with a pulse repetition frequency of 10 kHz. Compute the maximum unambiguous range. (2 p)

Solution 1

a) $\theta_{3dB} = 2\sqrt{\frac{\ln 2}{900}}\ rad = 3.2^o$

b) $\sigma = \pi r^2$

c) $f_d = \frac{2v_{rel}}{\lambda} = 500\ kHz$

d) $d \approx \sqrt{2h\frac{4R_E}{3}} = 22.6\ km$

e) $R_a = \frac{c}{2f_a} = 15\ km$

2. Repeat pass InSAR for moving objects.

A satellite SAR interferometer operates at altitude h_0=800 km, incidence angle θ = 23° and with a horizontal antenna baseline B (between the two passes over the area) of 200 m. The frequency of the SAR is 10 GHz. Assume a flat earth geometry.

a) Draw one or two figures that explain the geometry of the interferometer. The altitude, incidence angle and antenna baseline must be indicated in the figure/s. The parallel and perpendicular components of the baseline should also be clearly marked. (2 p)

b) Determine the fringe spacing (distance for one full fringe) on a horizontal flat surface in the original slant range image. Hint: One full fringe corresponds to $\Delta\phi = 2\pi$ (3 p)

c) Two radar reflectors have been deployed in the neighborhood of each other in order to investigate the sensitivity to small motion detection. The SAR interferometer images the two reflectors on two occasions and one of the reflectors is moved by 1cm vertically between the two over passes. Determine the relative change of the phase difference between the reflectors. (3 p)

d) Determine the maximum vertical displacement of a reflector that can be measured without phase ambiguities (2 p)

Solution 2

a)

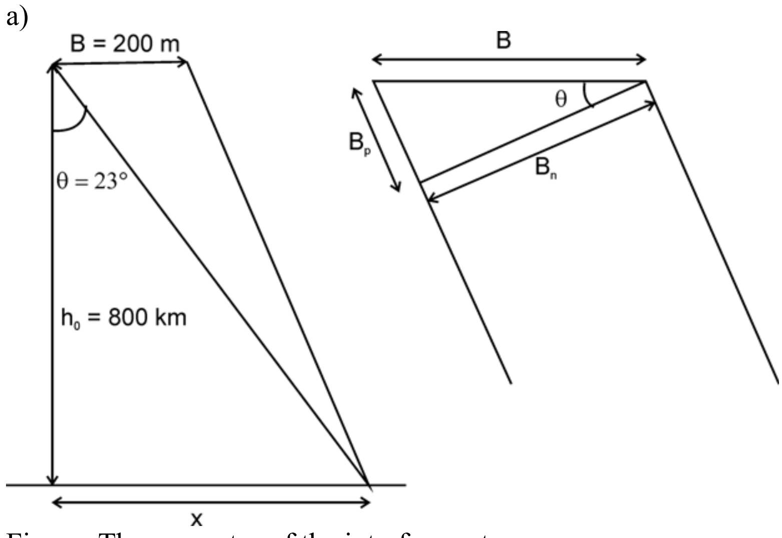


Figure: The geometry of the interferometer.

b) The baseline B is small compared to the altitude of the satellite orbits h_0, so we can assume that the rays from the two passes are parallel. From the figure it can be seen that a change in incidence angle θ will cause a change in B_p, where the phase difference, ϕ , between the satellite passes is related to B_p by

$\phi = 2kB_p$ (1)

and

$B_p = B\sin\theta$ (2)

For a small angle $d\theta$:

$dB_p = B\cos\theta d\theta = B_n d\theta$ (3)

$$d\phi = 2kB_n d\theta = \frac{4\pi}{\lambda} B_n d\theta \Rightarrow d\theta = \frac{\lambda \cdot d\phi}{4\pi B\cos\theta} \quad (4)$$

We want to relate this to a distance in the slant range image, dr

$$r = \frac{h_0}{\cos\theta} \quad \Rightarrow \quad \frac{dr}{d\theta} = \frac{h_0 \sin\theta}{\cos^2\theta} \quad \Rightarrow \quad dr = \frac{h_0 \sin\theta}{\cos^2\theta} d\theta \quad (5)$$

$$\Rightarrow \quad dr = \frac{h_0 \sin\theta}{\cos^2\theta} \frac{\lambda \cdot d\phi}{4\pi B\cos\theta} = \frac{\lambda \cdot h_0 \sin\theta}{4\pi B\cos^3\theta} d\phi \quad (6)$$

For one full fringe, $d\phi = 2\pi$, we have the distance in the slant range image

$$dr = \frac{\lambda \cdot h_0 \sin\theta}{2B\cos^3\theta} = 30.0m \quad (7)$$

The distance in the ground range image for one full fringe is

$$dx = \frac{\lambda h_0}{2B\cos^3\theta} = 76.9m \quad (8)$$

c) In the first image the phase difference between the radar reflectors is zero.
In the second image the phase difference = $2k\Delta R$, where ΔR is the change in range between the reflectors.

$\Delta R = \Delta z \cos\theta$ (9)

=> Interferometric phase difference between the reflectors = $2k\Delta z \cos\theta$ = 3.86 radians = 220° (=0.61 of a full fringe).
When the reflectors are separated horizontally, a phase as calculated in part a (i.e. related to horizontal separation Δx) is also present.

d) Phase ambiguity occurs when $\Delta\phi \geq 2\pi$

$2k\Delta z \cos\theta = 2\pi \rightarrow \Delta z = 0.0163\ m$

3. Radar scattering

A monostatic radar system with a network analyzer and an antenna is used in the laboratory exercise to measure the backscatter and dielectric properties of different objects and materials. The aperture of the antenna is 20 cm and the distance between the antenna and the object is kept at 2 m. For the following questions, assume that the frequency is 10 GHz.

a) Explain the difference between near field and far field and calculate the minimum distance for the far field (according to the Fraunhofer criterion) if the target has a characteristic horizontal size of 30 cm. (2 p)

b) Determine the width of the illuminated zone (the foot print) limited by the antenna gain. (1 p)

c) Determine the width of the first Fresnel zone. (1 p)

d) At high frequency, the radar cross section of a trihedral reflector is given by,

$$\sigma_{trihedral} = \frac{4\pi a^4}{3\lambda^2}$$

where a is the trihedral base length.

How large must a metallic sphere be to have the same radar cross section as a trihedral. (2 p)

e) At this frequency, what should be the characteristic roughness height of a surface to fulfill the Rayleigh criterion for a smooth surface? (2 p)

f) In addition to the roughness, what else affect the scattering from a surface of sand? (1 p)

Solution 3

a) In the far field the distance between the antenna and the target is great enough that the antenna may be regarded as a point source. The wave fronts striking the target are essentially planar. In the near-field the wave front is no longer planar (see e.g. Sullivan page 43).

The Fraunhofer criterion implies that the far field exist when $R>R_{far}=2L^2/\lambda$, where L is the characteristic length of the target. In our case R_{far}=6m

b) $\theta_B=\lambda/D = 0.15$ rad

$r/h=\tan(\theta/2)$

r = 0.1503 m, i.e. width = 0.3 m

c) radius of footprint = $(h^2 + (h+\lambda/2)^2)^{0.5}$ = 2.839 m, i.e width = 5.678 m

d) $\pi r^2 = \dfrac{4\pi a^4}{3\lambda^2}$ $\qquad r = \sqrt{\dfrac{4a^4}{3\lambda^2}} = \dfrac{2a^2}{\lambda\sqrt{3}} = 38.49a^2$

e) Rayleigh criterion for a smooth surface: $2ks\cos\theta<\pi/8$
where s is the characteristic roughness height.

$s < 0.0009401$ m

f) The moisture of the sand.

4. Processing of radar signals

A search radar is to detect a target with the probability of detection $P_D = 0.9$ by using one received pulse to take a decision on the presence of the target. The radar is designed to provide the range resolution $\delta R = 75\,\mathrm{m}$. The radar wavelength is $\lambda = 0.03\,\mathrm{m}$ and the transmitted pulse width is $T = 50 \cdot 10^{-6}\,\mathrm{s}$.

a) Compute the minimum signal-to-noise ratio (SNR, dB) required to ensure the specified probability of detection for a Swerling I target if the probability of false alarm $P_{FA} = 10^{-6}$. (1 p)

b) Assume the transmitted pulse is an LFM (linear-frequency modulated) signal. Write an analytical expression for the complex envelope of this signal and compute its bandwidth B and compression ratio CR. Explain the physical meaning of the compression ratio. (2 p)

c) The radar observes two zero-Doppler point targets having the same angular coordinates but separated in range by a distance of D m. Draw the envelopes of corresponding echo signals at the output of the LFM matched filter for the two cases: (i) D = 150 m and (ii) D = 50 m. Ignore the receiver noise. How could you characterize the possibility to distinguish the targets in case (i) and (ii) taking into account the radar resolution ability? (3 p)

d) Estimate the maximum range measurement error ΔR due to the range-Doppler coupling effect for LFM signals if the target radial velocity $|V_r| \le V_{r\max}$, where $V_{r\max}$ = 150 m/s. Illustrate this effect by using a contour plot of the LFM ambiguity function. (4 p)

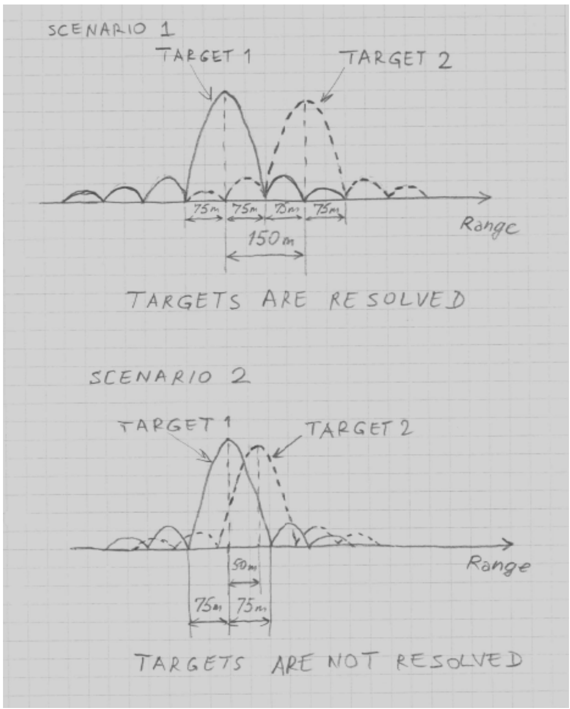
Solution 4

a) $$SNR=\frac{\ln P_{FA}}{\ln P_D}-1=\frac{\ln 10^{-6}}{\ln 0.9}-1=108.272;\quad SNR[\mathrm{dB}]=10\lg SNR=10\lg 108.272=20.35\,\mathrm{dB}$$

b) $$u(t)=\frac{1}{\sqrt{T}}\exp\left[j\pi kt^2\right],\ 0\le t\le T,\ \text{zero elsewhere}$$

$$B=\frac{c}{2\delta R}=\frac{3\cdot10^8}{2\cdot75}=2\cdot10^6\,\mathrm{Hz};\quad CR=B\cdot T=2\cdot10^6\times50\cdot10^{-6}=100;\quad CR=\frac{\text{Uncompressed pulsewidth}}{\text{Compressed pulsewidth}}$$

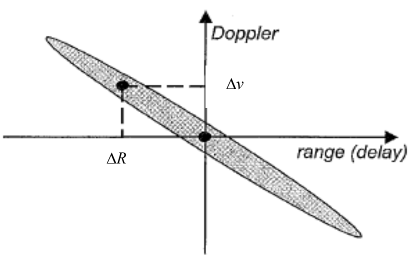
c)



d)

$$\Delta\nu=\frac{2|V_{r\max}|}{\lambda}=\frac{2\cdot150}{0.03}=10^4\,\mathrm{Hz};\quad \Delta\tau=\Delta\nu\frac{T}{B}=10^4\frac{50\cdot10^{-6}}{2\cdot10^6}=25\cdot10^{-8}\,\mathrm{s}$$

$$\Delta R=\frac{c\Delta\tau}{2}=\frac{3\cdot10^8\cdot25\cdot10^{-8}}{2}=37.5\,\mathrm{m}=0.5\,\delta\mathrm{R}$$



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b) Real measurements show that Venus radar echoes are time-varying and have a Doppler bandwidth of about 0.5 Hz. Assume a Swerling case 1, i.e. coherent integration of a steady target during a dwell time of 2 s (reciprocal of the Doppler bandwidth) followed by incoherent integration of a target with randomly varying radar cross section σ according to the probability density function $p(\sigma) = (1/\bar{\sigma})\exp(-\sigma/\bar{\sigma})$. Determine the required total integration time for detection. (4p)

c) The NASA JPL Goldstone radar also made measurements during the same time period. It operated using a higher frequency of 2388 MHz, system noise temperature of 64 K, average transmit power of 13 kW and total system losses of 1 dB. Did the Goldstone radar need more or less integration time to detect Venus compared to the Millstone radar? Motivate your answer. Assume the same effective aperture area as the Millstone radar antenna. The expected attenuation through the entire atmosphere as a function of frequency for different elevation angles is given in the graph below. (2p)

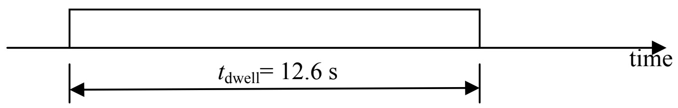
Solution 5

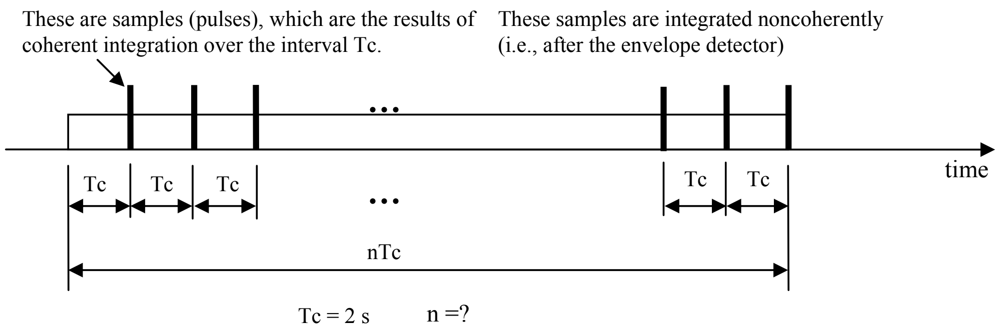
a) $SNR = D_0(1) = 12.5\ \text{dB}$ □ $G = \dfrac{4\pi A_e}{\lambda^2} = 5600$

$$t_{dwell} = SNR\,\frac{(4\pi)^3 R^4 k_B T_s L}{P_{avg} G^2 \lambda^2 \sigma} = 10^{1.25}\,\frac{(4\pi)^3 \left(4.24\cdot 10^{10}\right)^4 \left(1.38\cdot 10^{-23}\cdot 240\right) 10^{0.1}}{\left(1.5\cdot 10^5\right)\cdot 5600^2\cdot 0.68^2\cdot 1.7\cdot 10^{13}} = 12.6\ s$$

b) Since the coherent processing interval (CPI) is limited to be $T_c = 2\ s$, the SNR that can be achieved after the coherent integration over this interval is

$SNR_c = \dfrac{D_0(1)}{n_0} = \dfrac{10^{12.5/10}}{12.6/2} = 2.82 \Rightarrow 4.5\ dB$. Here $n_0 = t_{dwell}/T_c$ is the number of CPIs that can be placed within the dwell time computed assuming envelope detection of a steady target.





To determine the required total integration time nTc we need to determine the number of pulses n taking into account the integration loss (due to noncoherent integration of n pulses) and the fluctuation loss (Swerling-1 target model). The number of pulses can be determined by using equation for the required signal-to-noise ratio per one pulse

$$SNR_{req} = \frac{D_0(1) L_i(n) L_f}{n}.$$

This required SNR must be equal to that after coherent processing over the CPI, i.e. $SNR_{req} = SNR_c = \dfrac{D_0(1)}{n_0}$. Thus, we have $\dfrac{D_0(1)}{n_0} = \dfrac{D_0(1) L_i(n) L_f}{n}$ or

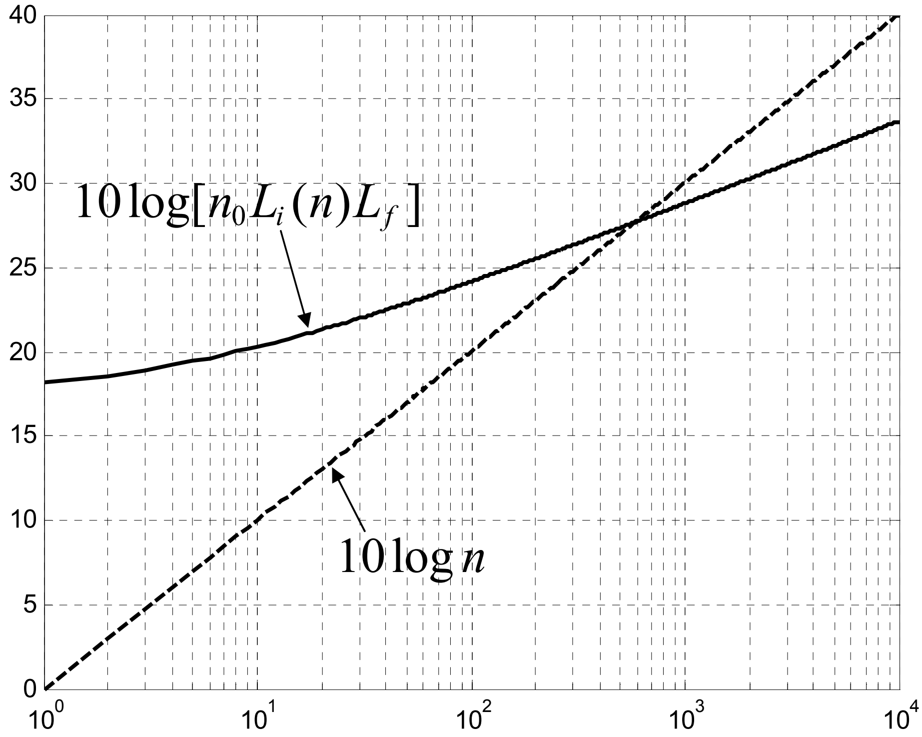
$$n_0 L_i(n) L_f = n \ (*)$$

where the integration loss $L_i(n) = \dfrac{1+\sqrt{1+\dfrac{9.2n}{D_c(1)}}}{1+\sqrt{1+\dfrac{9.2}{D_c(1)}}}$, $D_c(1) = [erfc^{-1}(2P_{FA}) - erfc^{-1}(2P_D)]^2$

and the fluctuation loss

$$L_f = \frac{D_1(1)}{D_0(1)},\ D_1(1) = \frac{\ln P_{FA}}{\ln P_D} - 1,\ D_0(1) = \left[\sqrt{\ln(1/P_{FA})}\text{-erfc}^{-1}(2P_D)\right]^2 - 1/2\,.$$

Equation (*) can be solved graphically



From figure $n \approx 588$. Thus, the total integration time is $nT_c = 588 \cdot 2 = 1176 s$ (19.6 min).

c) The factor $\frac{P_{avg}G}{T_s}$ is equal to 3.5 x 10^6 W/K for the Millstone radar compared with 3.3 x 10^7 W/K for the Goldstone radar. Atmospheric losses are greater for the Goldstone radar but the difference compared with the Millstone radar is less than 2 dB according to the graph. We therefore expect less integration time for the Goldstone radar.