Exam in Course Radar Systems and Applications (RRY080)

20th October 2008. 1400-1800.

Additional materials: Beta, Physics Handbook, Approved electronic calculator, Dictionary.

Anatoliy Kononov (tel: 1844) will visit around 1500 and 1700.

Read through all questions before this time to check that you have understood the questions.

Answers may only be given in **English**.

Each question carries a total of ten points. Where a question consists of different parts, the number of points for each part of the question is indicated in parentheses. **In most cases it should be possible to answer later parts of the question even if you cannot answer one part.**

Grades will be awarded approximately as <20 = Fail, 20-29 = 3, 30-39 = 4, ≥ 40 = 5
(exact boundaries may be adjusted later).

Review of corrected exam papers will take place on the two following occasions:

Thursday the 13th of November, 10:00-11:45
Monday the 17th of November, 13:15-15:00

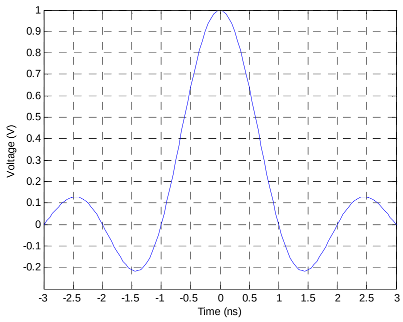
Clearly show answers (numbers and dimensions!) to each part of question just after the corresponding solution.

Begin each question on a new sheet of paper!

1. Basic terms & principles

a) What is the RCS of a large metallic sphere with radius a >> wavelength. Assume that the sphere is located in free space. (2p)

b) What is the 3 dB pulse width of the pulse in the figure. (2p)



c) What is the radial velocity resolution which can be obtained using the signal (f_0 = 15 GHz, τ = 10 ms):

$$s(t) = \begin{cases} \cos(2\pi f_0 t) & \text{for } |t| \le \tau/2 \\ 0 & \text{otherwise} \end{cases}$$ (3p)

d) What is the pulse compression ratio, i.e. the ratio of pulse width before and after compression, which can be obtained using the signal (f_0 = 10 GHz, K = 10^{13} Hz/s, τ = 10 μs):

$$s(t) = \begin{cases} \cos(2\pi f_0 t + \pi K t^2) & \text{for } |t| \le \tau/2 \\ 0 & \text{otherwise} \end{cases}$$ (3p)

2. Maximum range and unambiguous range

A mid-X band (10 GHz) radar has the average power 1 kW, 30 dB antenna gain, dwell time 30 ms, system noise temperature 600 K and system loss 6 dB.
Boltzmann's constant $= 1.38 \cdot 10^{-23} \frac{W}{K \cdot Hz}$

a) Calculate the maximum range at which the radar can detect a target of radar cross section -20 dBm^2. Assume the signal to noise ratio required for detection is 18 dB and the filter mismatch factor is 0 dB. (3 p)

b) Assume the maximum unambiguous velocity interval is 900 m/s. Calculate the maximum unambiguous range. Assume pulse width << pulse repetition interval. (3 p)

c) If we have a target at the range 8 km, what is the apparent range (range assuming that the received echo comes from the last transmitted pulse) for a PRF of 60 kHz. (2 p)

d) One way to resolve some of the range ambiguity is to use PRF switching, i.e. the same target is observed with more than one PRF. If we assume that the unambiguous range for PRF_i is $R_{ui} = m_i \Delta r$, where i = 1, 2, …., N and that no two m_i have a common factor, the overall unambiguous range is

$$R_u = \Delta r \cdot \prod_{i=1}^{N} m_i$$

Select two PRFs that give an overall unambiguous range R_u = 20 km. Assume Δr = 1 km. (2 p)

3 Target detection

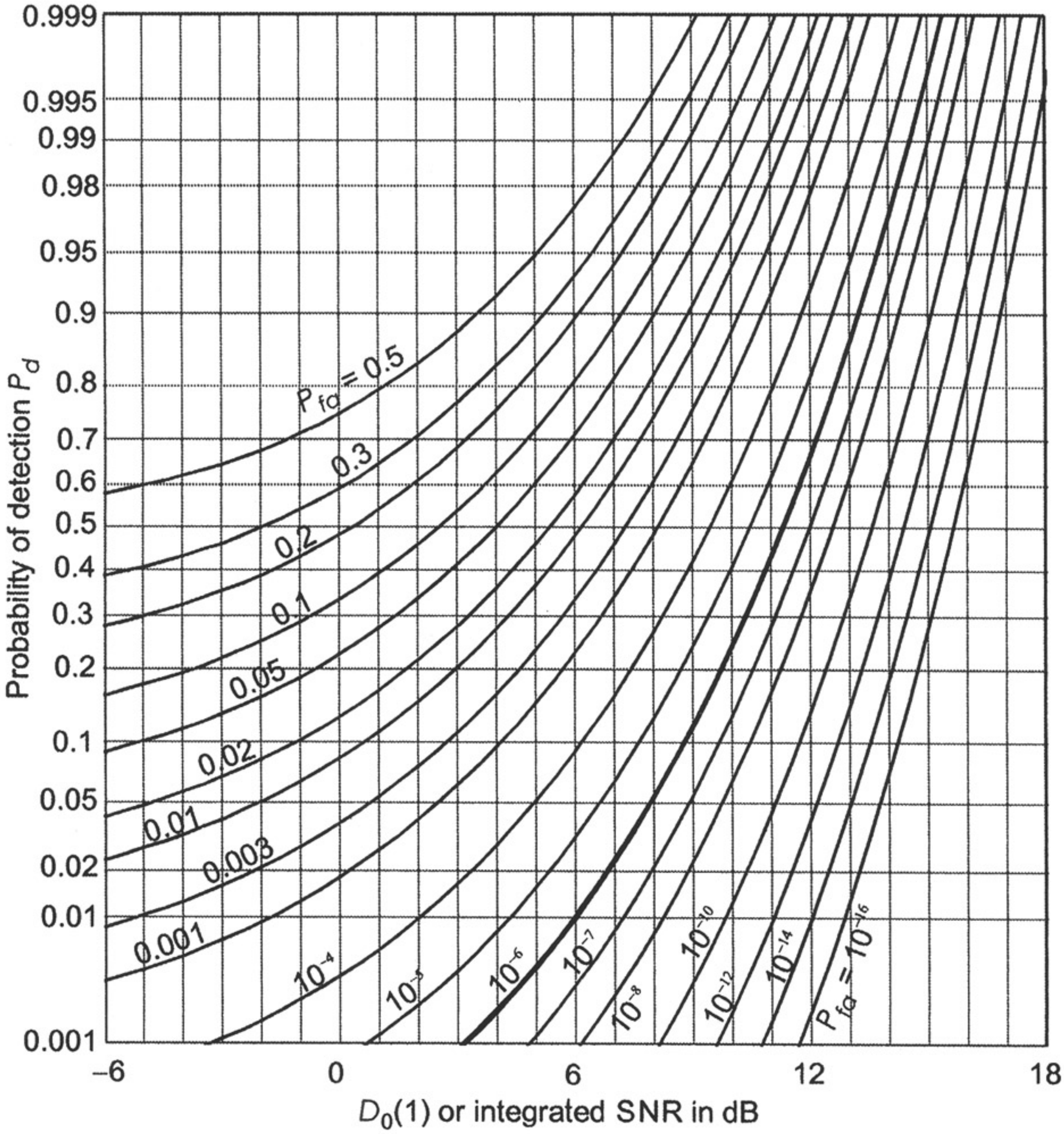
Consider an air surveillance radar with an antenna that rotates at a constant angular velocity $\Omega = 0.2\pi$ radians per second with an azimuth beamwidth of $\theta = 0.576$ degree and a pulse repetition frequency of PRF = 1 kHz. The radar employs noncoherent pulse integration of *n* pulses received on each scan and a binary integration technique by combining the results of detections from N consecutive scans within an observation interval of $T_o = 40$ seconds (it is assumed that the range between the radar and target remains constant during the observation interval). The target fluctuation model obeys Swerling-2 case.

a) Calculate the number of pulses *n* received by the radar from the target on a single scan (one scan means that the radar antenna completes 360^o sweep) (1p)

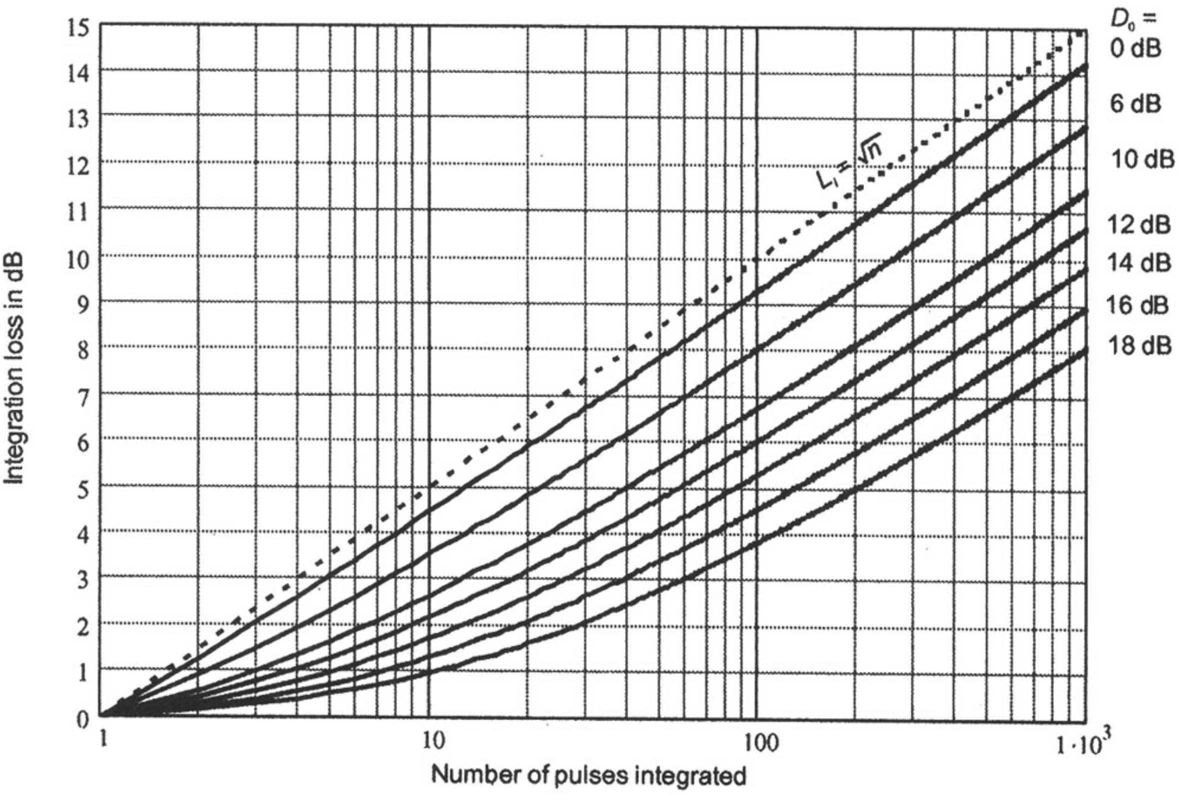
b) Calculate the number of consecutive scans N available to the radar to detect the target within the specified observation interval T_o. (1p)

c) Assume the first threshold (i.e. the detection threshold on each scan) was set to yield $P_{FA} = 10^{-4}$. What is the single-pulse signal-to-noise ratio SNR_{req} required to ensure the detection probability $P_D = 0.7$ on each scan? (4p)

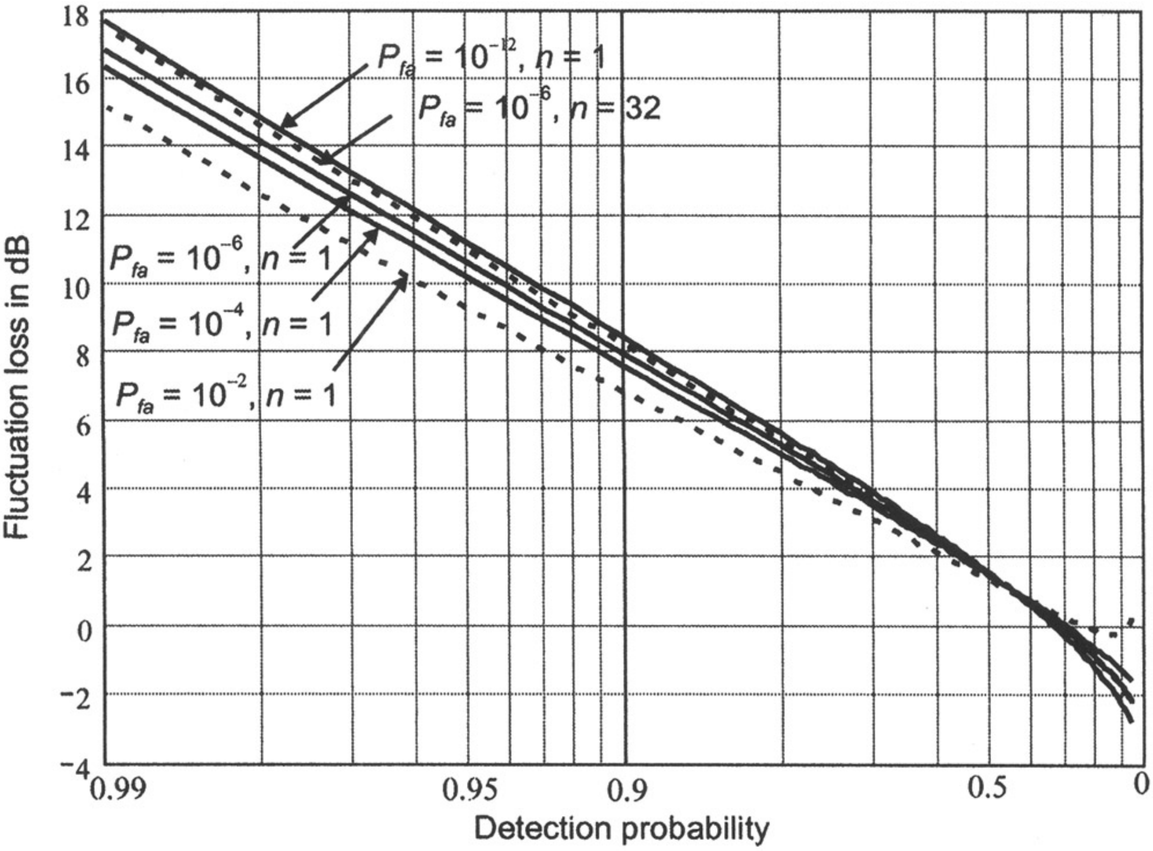
d) With $P_{FA} = 10^{-4}$ and $P_D = 0.7$, compute the cumulative probabilities of detection $P_{CD} = P_D(2, N)$ and false alarm $P_{CFA} = P_{FA}(2, N)$ for binary detection procedure based on the "2 of N" decision rule. Compare $P_D(2, N)$ with P_D and $P_{FA}(2, N)$ with P_{FA} and explain the result. (4p)



Detectability factor for a steady target
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 45]



Integration loss versus number of pulses integrated after envelope detection
for different values of output detectability factor $D_0(1)$
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 52]



Fluctuation losses for case 1 target (Swerling-1 model)
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 61]

Auxiliary Formulas

$$L_f(n_e) = \left[L_f(1)\right]^{1/n_e} \qquad (4.15)$$

$$L_f(n_e)[\text{dB}] = \frac{1}{n_e}\cdot\left(10\cdot\lg L_f(1)\right) = \frac{L_f(1)[\text{dB}]}{n_e}$$

$$n_e = \begin{cases} 1 & \text{for Swerling-1} \\ n & \text{for Swerling-2} \\ 2 & \text{for Swerling-3} \\ 2n & \text{for Swerling-4} \end{cases}$$

$n_e \to \infty$ for nonfluctuating model (Swerling - 0 or 5) and $L_f(1) = 1$ or $L_f(1)[\text{dB}] = 0$

$$SNR_{\text{req}} = D_e(n, n_e) = \frac{D_0(1)L_i(n)L_f(n_e)}{n} \qquad (4.16)$$

Binomial Distribution

For a random experiment that consists of *N* trials and satisfies the following conditions:

(1) Each trial results in only two possible outcomes ("1" or "0")
(2) The trials are independent (the outcome of one trial has no effect on outcomes from any other trial)
(3) The probability of success in each trial, denoted as *p*, remains constant.

For such a random experiment, the random variable *K* that equals the number of trials that result in a success (the number of trials with outcomes that are "1") is a so called *binomial random variable* or random variable that obeys a *binomial distribution*. The probability mass function of *K* is

$$f(k) = P'(k, N) = \binom{N}{k} p^k (1-p)^{N-k},\ k = 0,1,2,\ldots,N$$

$$\text{where } \binom{N}{k} = \frac{N!}{k!(N-k)!},\ \text{ factorial } m! = 1\cdot 2\cdot 3\cdot\ldots\cdot(m-1)\cdot m \qquad (*)$$

Equation (1) defines the probability of exactly *k* "successes" (number of trials with outcomes "1") in *N* independent trials, when the probability of success on each trial is *p*. The probability of at least *M* successes (that is the probability that *k* will be equal to *M*, or to *M*+1, …, or to *N*) for a binomial random variable is given by

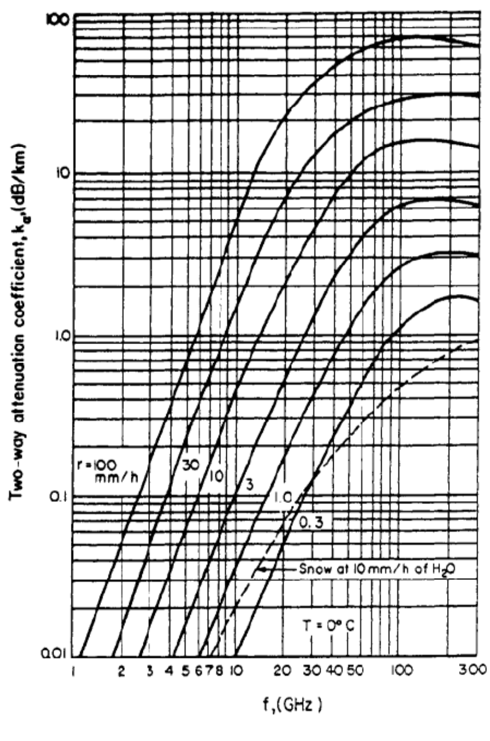
$$P(M \le k \le N, N) = P(M, N) = \sum_{k=M}^{N} \binom{N}{k} p^k (1-p)^{N-k} \qquad (**)$$

4. Radar system design

a) Maritime navigation radars are critical safety systems on board ships and must meet performance requirements also in the advent of adverse weather conditions. These radars are normally non-coherent radars with magnetron transmitters. Describe improvements in system performance which can be expected when moving to a modern coherent radar design instead. Discuss also possible disadvantages with a coherent radar design. (5p)

b) Your task as radar engineer is to establish the centre frequency to be selected for a new maritime navigation radar that maximizes the signal-to-noise ratio from small boats in extreme rainfall. Assume that all radar system parameters, except the centre frequency, are constant in the radar equation (see 1.23 in the list of formulas). The graph below shows the two-way attenuation for different rain fall rates. Assume the worst-case condition, i.e. 100 mm/hour. Assume also that the small boat is located 5 km from the antenna and has a free-space RCS which is independent of frequency. Your evaluation should include S-band (3 GHz), C-band (5 GHz), X-band (10 GHz), and Ku-band (15 GHz). Which radar band would you select if you only take attenuation loss into account? (3p)

c) Repeat the task in b), but include multi-path from reflection off the sea surface in your computations. The radar antenna is mounted 20 m above sea level and uses horizontal polarization. Assume perfect reflection with 180 deg phase shift, i.e. reflection coefficient = -1. Assume that the boat is a point target at 1 m above the sea surface. Which radar band would you select? (2p)



5. Synthetic Aperture Radar

This question concerns the design of a SAR mode for an airborne radar system for detecting boats and ships. The radar properties are given in the table below. Note that two different pulse lengths are possible to choose, with different range resolution. The idea of this question is to determine which choice to make by balancing image quality in terms of resolution and detectability of point scatterers in the presence of noise and clutter (i.e. backscatter from ocean).

To compare the different choices of radar pulse length, consider a test target of a trihedral with RCS 10 m^2, compared to the ocean with nominal backscattering coefficient of the background assumed to be σ^o= -10 dB. Assume the imaging is done at a slant range distance of 50 km, and a depression angle of 30^o below the horizontal. The aircraft speed is v=300 m/s.

Consider SAR modes using the longer uncompressed pulse length, i.e. range resolution 0.2 m after compression. We will call this Mode 1.

a) If the azimuth resolution δ_{cr} is the same as that in range (i.e. 0.2 m), show that the minimum PRF allowed for Mode 1 is 1500 Hz to avoid azimuth ambiguities? Hint: $\delta_{cr} = v/B_a$, where B_d is the Doppler bandwidth. (2p)

b) The PRF required in part a) is not possible to use in Mode 1. Explain why this is not possible, and show that the finest azimuth resolution possible without ambiguities is about 0.3 m. (2p)

The signal-to-clutter ratio (SCR) is defined as the ratio of the signal strength (in this case from the trihedral with 10 m^2), compared to the average radar cross-section of the clutter.

c) For the given resolution in range and azimuth (0.2 m x 0.3 m) calculate the SCR. (2 p)

d) Calculate the clutter-to-noise-ratio (CNR) in Mode 1, and hence give the signal-to-noise ratio (SNR) for the trihedral. Is the detectability of the trihedral limited by thermal noise or by clutter? Hint: $T_{sys}=FT_0$ (2p)

Using the shorter pulse length (with range resolution 0.4 m), a suggestion is to use a mode with an azimuth resolution of 0.1 m. We will call this Mode 2.

e) What PRF should be used for Mode 2? Calculate SCR and SNR for Mode 2. Using these value discuss which Mode (1 or 2) would you recommend for detecting ships? Motivate your answer clearly. (2p)

Radar Properties

Peak transmit power, P_{peak}	1 kW
Maximum duty cycle, f_D	5 %
Wavelength, λ	1 cm
Antenna gain (one-way, boresight), G	36 dB
Noise figure, F	8 dB
System losses, L	2 dB
Boltzmann constant, k_B	1.38×10^{-23} J/K
Reference temperature, T_0	290 K
Maximum allowed PRF to avoid range ambiguities	5 kHZ

Properties for different transmit pulse modes (linear FM)

	Mode 1	Mode 2
Pulse length (uncompressed)	50 μs	1 μs
Range resolution (compressed pulse)	0.2 m	0.4 m

Formulas available at examination of the course Radar Systems and Applications (equation and page numbers refer to the course book by Sullivan)

Radar equation

$$SNR = \frac{P_{peak} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.23)$$

$$SNR = \frac{P_{avg} G^2 \lambda^2 \sigma t_{dwell}}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.24)$$

Antennas

$$A_e = \frac{G\lambda^2}{4\pi} \qquad (1.18)$$

$$A_e = A\eta \qquad (p.\ 15)$$

$$R_{far} = \frac{2L^2}{\lambda} \qquad (p.\ 43)$$

System noise

$$k_B T_{sys} = k_B \left[T_{ant}/L_{radar} + T_{radar}(1 - 1/L_{radar}) + T_{rcvr} \right] \qquad (2.10)$$

$$T_{rcvr} = (F-1)T_0 \quad ,F > 1 \qquad (2.11)$$

Radar Cross Section

$$\sigma = \lim_{R \to \infty} 4\pi R^2 \frac{|\mathbf{E_s}|^2}{|\mathbf{E_i}|^2} \qquad (3.1)$$

$$\sigma = 4\pi \frac{A^2}{\lambda^2}$$

(broadside RCS of large metallic flat plate, p. 69-70)

Radar clutter

$$\sigma_c = \sigma^0 A \qquad (p.\ 87)$$

$$\sigma_c = \eta V \qquad (p.\ 88)$$

Envelope detection of a single pulse

$$P_D \approx \frac{1}{2}\left[erfc\left(\sqrt{\ln\left(\frac{1}{P_{FA}}\right)} - \sqrt{SNR + \frac{1}{2}} \right) \right]$$

(4.9) corrected

Matched filter

$$h(t) = Ku^*(t_m - t) \quad \Leftrightarrow \quad H(\omega) = KU^*(\omega)\exp(-j\omega t_m) \qquad (4.32)$$

Ambiguity function

$$|\chi(\tau,\upsilon)| = \left| \int_{-\infty}^{+\infty} u(t)u^*(t-\tau)\exp(j2\pi\upsilon t)dt \right|$$ (p. 116)

Doppler frequency

$$f_{doppler} = -\frac{2v}{\lambda}$$ (1.32)

Maximum unambiguous velocity interval

$$\Delta v_u = \frac{f_R \lambda}{2}$$ (p. 22)

Inverse SAR

$$\delta_{rpn} = \frac{c}{2B}$$ (6.4)

$$\delta_{crpn} = \frac{\lambda}{2\Delta\phi}$$ (6.5)

SAR

$$\delta_r \approx \frac{c}{2B}$$ (p. 216)

$$\delta_{cr} \approx \frac{\lambda}{2\Delta\theta}$$ (7.1)

$$CNR = \frac{P_{avg} G^2 \lambda^3 \sigma^0 \delta_r}{2(4\pi)^3 R^3 k_B T_{sys} LV \cos\psi}$$ (7.46)

Exam in Course Radar Systems and Applications (RRY080)

20th October 2008.

Solutions:

1. Short definitions/explanations of basic terms & principles

a): πa^2

b): 0.9 ns

c): $\lambda/(2\tau) = 1$ m/s

d): $K\tau^2 = 1000$ or 30 dB

2. Maximum range and maximum unambiguous range

a) $\lambda = 0.03$ m; G = 1000; L = 4; $\sigma = 0.01$; SNR = 63.10

$$SNR = \frac{P_{avg} G^2 \lambda^2 t_{dwell} \sigma}{(4\pi)^3 R^4 k T_s L C_B}$$

$$R^4 = \frac{1000 \cdot 1000^2 \cdot 0.03^2 \cdot 0.03 \cdot 0.01}{(4 \cdot \pi)^3 \cdot 1.38 \cdot 10^{-23} \cdot 600 \cdot 4 \cdot 1 \cdot 63.1} = 6.521 \cdot 10^{16} \quad m^4$$

R = 15980 m

b) $\Delta v_u = \frac{f_R \lambda}{2}$

$$R_u \approx \frac{c}{2 f_R} = \frac{c\lambda}{4 \Delta v_u} = \frac{3 \cdot 10^8 \cdot 0.03}{4 \cdot 900} = 2500\, m$$

c) $R_u \approx \frac{c}{2 f_R} = \frac{3 \cdot 10^8}{2 \cdot 60000} = 2500\, m$

$$8000 - 3 \cdot 2500 = 500\, m$$

d) $R_u = \Delta r\, m_1\, m_2 = 20000$ m

$$m_1 \cdot m_2 = \frac{20000}{1000} = 20$$

Choose $m_1=4$ and $m_2=5$

$$f_{R_1} \approx \frac{c}{2 \cdot R_{u_1}} = \frac{3 \cdot 10^8}{2 \cdot 1000 \cdot 4} = 37500\, Hz$$

$$f_{R_2} \approx \frac{c}{2 \cdot R_{u_2}} = \frac{3 \cdot 10^8}{2 \cdot 1000 \cdot 5} = 30000\, Hz$$

3 Target detection

a) Compute the number of pulses n received by the radar from the target on a single scan

$$n = \frac{\theta}{\Omega} PRF$$

$$\theta = 0.576^{\circ} \quad = 0.576 \frac{\pi}{180} \text{radian} \qquad \Rightarrow \quad n = \frac{0.576 \frac{\pi}{180}}{0.2\pi} 1000 = \frac{0.576 \cdot 1000}{0.2 \cdot 180}$$

$$\Omega = 0.2\pi, \quad PRF = 1\,\text{kHz} = 1000\,\text{Hz} \qquad = 16$$

$n = 16$

b) Calculate the number of consecutive scans N available to the radar to detect the target within the specified observation interval T_o.

The rotation period of the radar antenna (i.e. time needed to complete 360^{o} sweep)

$$T_{rot} = \frac{2\pi}{\Omega} = \frac{2\pi}{0.2\pi} = 10 \text{ sec}$$

The number of successive scans available to the radar to detect the target is

$$N = \frac{T_o}{T_{rot}} = \frac{40}{10} = 4$$

$N = 4$

c)

Step 1: Determine the detectability factor $D_0(1)$ [this corresponds to envelope detection of a single steady (i.e. nonfluctuating) pulse, see Roger J. Sullivan, Radar Foundations, p. 106] from Figure 2.5 [David K. Barton, 2004, Radar System Analysis and Modeling, p. 45] : $D_0(1) = 10.45$ dB (see dashed blue lines in Fig. 1 below)

Step 2: Determine the integration loss for noncoherent integration of n = 16 pulses $L_i(\text{n}) = L_i(16)$ from Figure 2.8 [David K. Barton, 2004, Radar System Analysis and Modeling, p. 52]: for $D_0(1) = 10.45$ dB we obtain $L_i(16) \approx 3.3$ dB (see dashed blue lines in Fig. 2)

Step 3: Determine the fluctuation loss $L_f(\text{n}_e)$ for Swerling-2 model and number of pulses $n = 16$.

4.1 From Figure 2.10 [David K. Barton, 2004, Radar System Analysis and Modeling, p. 61] (see dashed blue lines in Fig. 3 below) we get $L_f(1)$ for Swerling-1: $L_f(1) = 3.5$ dB

4.2. Then, using equation (4.15) [Sullivan, Radar Foundations, p. 109] yields:

$$L_f(n_e) = \left[L_f(1)\right]^{1/n_e}, \; L_f(n_e)[\text{dB}] = 10\lg L_f(n_e), \; L_f(n_e)[\text{dB}] = \frac{1}{n_e} \cdot \left(10 \cdot \lg L_f(1)\right) = \frac{L_f(1)[\text{dB}]}{n_e}$$

Since given target fluctuation model is Swerling-2 case we have $n_e = n = 16$ and

$$L_f(16) = 3.5/16 = 0.22 \text{ dB}$$

Step 4. Compute the required signal-to-noise ration SNR_{req}.

To compute this value equation (4.16) [Roger J. Sullivan, Radar Foundations, p. 109] is used. It is handy to use a dB-form of the equation (4.16). Using eq. (4.16) yields:

$$SNR_{\text{req}} = D_e(n, n_e) = \frac{D_0(1) L_i(n) L_f(n_e)}{n}$$

$$SNR_{\text{req}}[\text{dB}] = D_0(1)[\text{dB}] + L_i(n)[\text{dB}] + L_f(n_e)[\text{dB}] - n[\text{dB}]$$

$$n[\text{dB}] = 10\lg n \;\rightarrow\; 10\lg 16 = 12.04\text{dB}$$

Thus, $\underline{SNR_{\text{req}}[\text{dB}] = 10.45 + 3.3 + 0.22 - 12.04 = 1.93 \text{ dB}}$

$\underline{SNR_{\text{req}} = 10^{0.1 SNRreq[\text{dB}]} = 10^{0.193} = 1.56}$

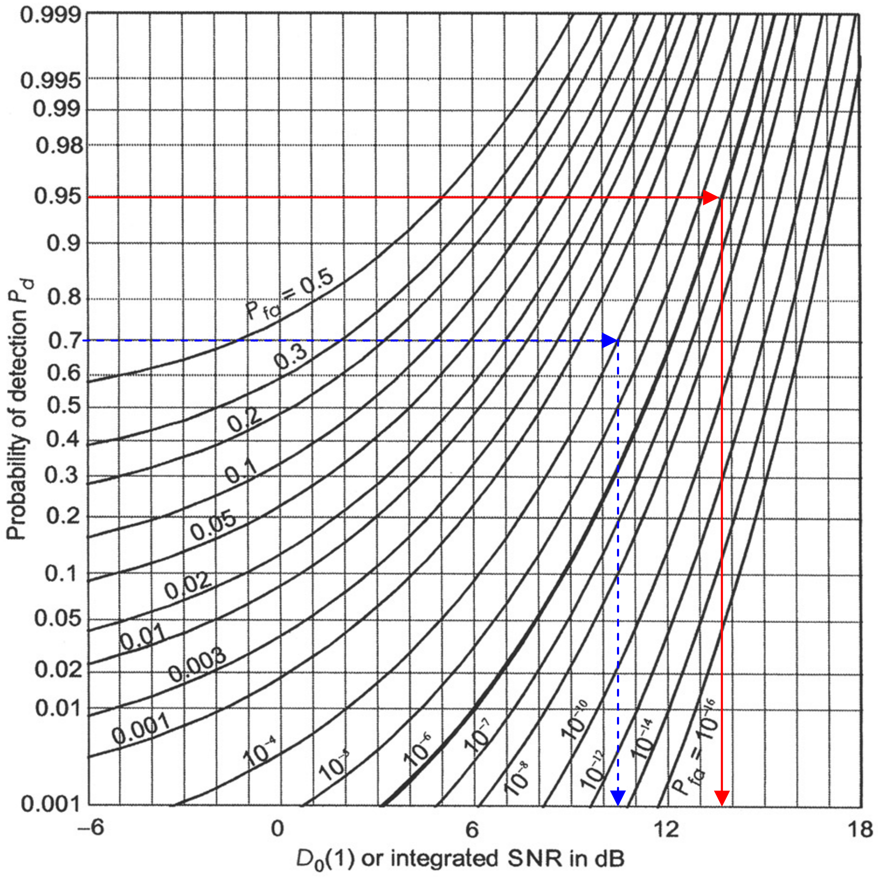


Fig. 1 Finding the detectability factor $D_0(1)$

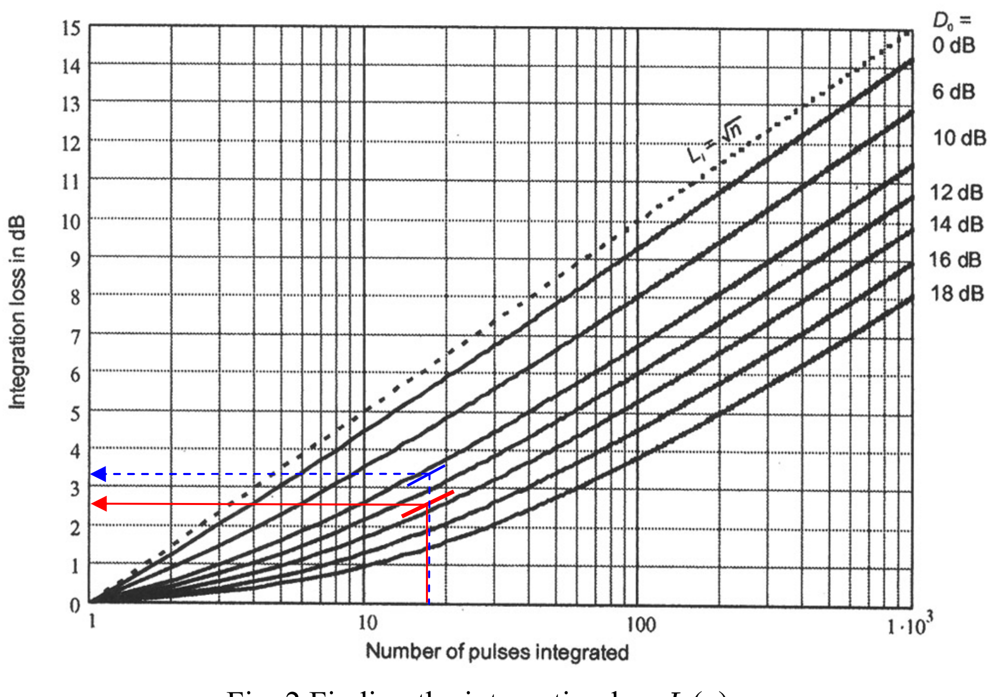


Fig. 2 Finding the integration loss L_i(n)

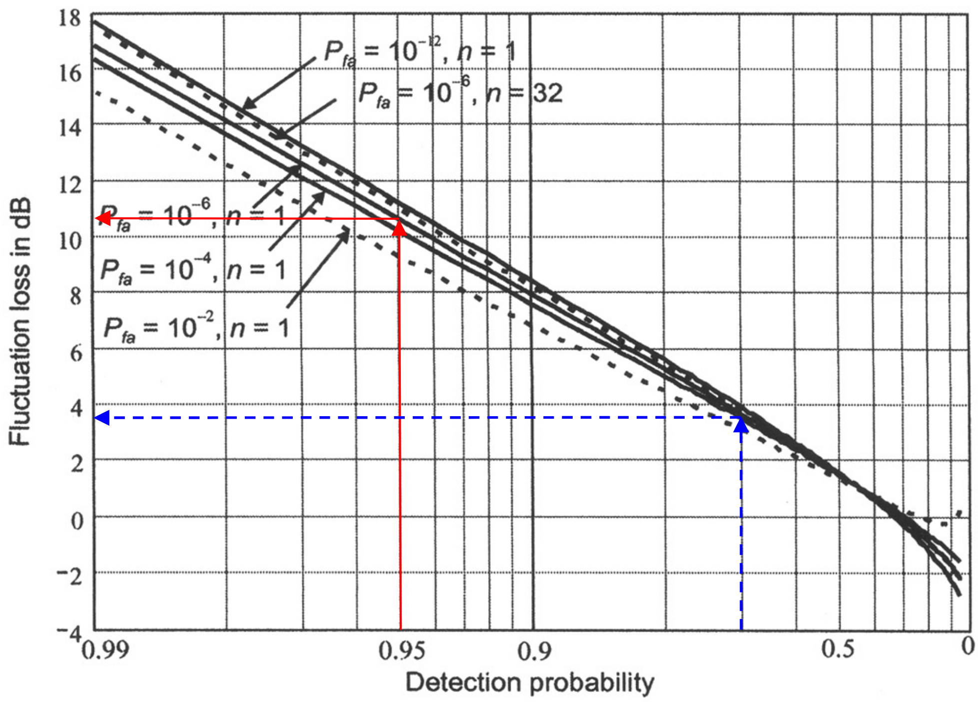


Fig. 3 Finding the fluctuation loss $L_f(1)$ for case 1 target (Swerling-1 model)

d) Using equations $P_D(M,N) = \sum_{k=M}^{N} \binom{N}{k} P_D^k (1-P_D)^{N-k}$ and $P_{FA}(M,N) = \sum_{k=M}^{N} \binom{N}{k} P_{FA}^k (1-P_{FA})^{N-k}$

we have for the "2 of 4" decision rule

$$
\begin{aligned}
P_D(2,4) &= \sum_{k=2}^{4} \binom{4}{k} 0.7^k (1-0.7)^{4-k} \\
&= \binom{4}{2} 0.7^2\, 0.3^2 + \binom{4}{3} 0.7^3\, 0.3^1 + \binom{4}{4} 0.7^4\, 0.3^0 \\
&= 6\cdot 0.7^2 \cdot 0.3^2 + 4\cdot 0.7^3 \cdot 0.3^1 + 1\cdot 0.7^4 \cdot 0.3^0 \\
&= 0.2646 + 0.4116 + 0.2401 \\
&= 0.9163
\end{aligned}
$$

$$
\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!\,2!} = \frac{1\cdot 2\cdot 3\cdot 4}{(1\cdot 2)\cdot(1\cdot 2)} = 3\cdot 2 = 6
$$

$$
\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4!}{3!\,1!} = \frac{1\cdot 2\cdot 3\cdot 4}{(1\cdot 2\cdot 3)\cdot(1)} = 4
$$

$$
\binom{4}{4} = \frac{4!}{4!(4-4)!} = \frac{4!}{4!\,0!} = \frac{4!}{4!\cdot 1} = 1
$$

$$
\begin{aligned}
P_{FA}(2,4) &= \sum_{k=2}^{4} \binom{4}{k} 0.0001^k (1-0.0001)^{4-k} \\
&= \binom{4}{2} 0.0001^2\, 0.9999^2 + \binom{4}{3} 0.0001^3\, 0.9999^1 + \binom{4}{4} 0.0001^4\, 0.9999^0 \\
&= 6\cdot 0.0001^2\, 0.9999^2 + 4\cdot 0.0001^3\, 0.9999^1 + 1\cdot 0.0001^4\, 0.9999^0 \\
&= 5.9992\cdot 10^{-8} \approx 6\cdot 10^{-8}
\end{aligned}
$$

Thus, we have $\underline{P_D(2,4) = 0.9163 > P_D = 0.7}$

$\underline{P_{FA}(2,4) = 6\cdot 10^{-8} << P_{FA} = 10^{-4}}$

Explanation: It is known from the theory of binary integration that the "2 of 4" rule provides acceptable false alarm reduction for small p (in particular for $P_{FA}=10^{-4}$) and detection improvement for large values of p (in particular for $P_D = 0.7$). Generally speaking, the "2 of 4" rule is the best choice for N = 4.

4. Radar system design

a) Improved range resolution can be obtained by increasing bandwidth while retaining pulse energy. Velocity resolution can be obtained by Doppler filtering using a train of coherent pulses. That is, a coherent radar design is expected to obtain improved sensitivity for small and moving targets (boats, floating obstacles etc) and enable improved measurement accuracy. Disadvantages include stringent phase noise requirements and more complex signal processing implying a more expensive radar system. Another disadvantage is increased complexity of radar control which will require a more skilled operator.

b) The two-way attenuation L_{rain} for 100 mm/h at 5 km range is 0.8 dB (S-band, 3 GHz), 3.5 dB (C-band, 5 GHz), 25 dB (X-band, 10 GHz) and 60 dB (Ku-band, 15 GHz).

c) Multi-path modifies RCS by a factor $F^4 = \left[2\sin\left(\frac{2\pi h_{ant} h_{boat}}{\lambda R}\right)\right]^4$, i.e. $SNR \propto \frac{\lambda^2 F^4}{L_{rain}}$.

	λ^2F^4 [dBm2]	L [dB]	λ^2F^4/L_{rain} [dBm2]
S-band, 3 GHz	-32.1	0.8	-32.9
C-band, 5 GHz	-28.0	3.5	-31.5
X-band, 10 GHz	-23.6	25	-48.6
Ku-band, 15 GHz	-22.8	60	-82.8

Select C-band which gives highest SNR. S-band gives similar SNR but has the disadvantage of larger physical size, e.g. antenna. Lower frequencies than S-band give even lower values for λ^2F^4 whereas the rain loss tends to 0 dB. Higher frequencies than Ku-band give extreme values of the rain loss (> 200 dB).

5. Synthetic Aperture Radar

a) Minimum PRF is determined by requirement to sample the full Doppler bandwidth. Considering SAR as a chirp, the Doppler bandwidth, B_d, determines the resolution, δ, according to $\delta = v/B_d$. Rearranging and putting in values for v=300 m/s and δ=0.2 give B_d=Min. PRF = 1500 Hz.

b) The PRF is below the 5 kHz for range ambiguities so this is not a problem. But, with the min. PRF =1500 Hz, and the pulse length 50 µs, the duty cycle is 1500 x 50×10^{-6} = 0.075. This exceeds the maximum value allowed of 0.05 (5%). Considering the duty cycle the maximum PRF is $0.05/(50\times10^{-6})$ = 1000 Hz. To avoid azimuth ambiguities we cannot allow a Doppler bandwidth greater than this PRF, so the resolution is v/1000 Hz = 0.3 m.

c) By definition $SCR=\sigma_{trihedral}/\sigma_{clutter}$
$\sigma_{trihedral}$ is given as 10 m^2.
Clutter RCS is given by the size of the resolution cells (range x azimuth resolution) multiplied by the backscattering coefficient. Note that the size of the range resolution used should be projected to the ground plane, i.e.
$\sigma_{clutter}=\sigma^o\,(\delta_{range}/\cos(\psi))\,\delta_{azimuth}$
where ψ is the depression angle, 30°. δ_{range}=0.2 m and $\delta_{azimuth}$=0.3 m, and σ^o=-10 dB = 0.01
i.e. $\sigma_{clutter}$=0.01 x 0.2 x 0.3 /(0.866) = 6.9×10^{-4}
Hence SCR=1400 = 32 dB.

d) Using the formula sheet we have

$$CNR = \frac{P_{avg} G^2 \lambda^3 \sigma^0 \delta_r}{2(4\pi)^3 R^3 k T_{sys} L V \cos\psi}$$

G, λ, L and k are given directly in the table. R=50 km, and ψ=30^{o,} ,v= 300 m/s and σ^o=-10 dB are given in the question. Tsys = FT_0= $10^{(8/10)}$x290= 1800 K.
To calculate Pavg we need to use peak power and the duty cycle. In this case the PRF was chosen to maximize the duty cycle (0.05), so Pavg=1000x 0.05 = 50 W.

Putting all the values in gives CNR=3.1 = 4.9 dB.
This means that the clutter is stronger than the noise, i.e. the detectability is limited by the clutter.
To find the SNR we simply use the fact that:

$$SNR = \frac{Signal}{Nosie} = \frac{Signal}{Clutter} x \frac{Clutter}{Noise} = SCR \cdot CNR$$

In dB this means that SNR=SCR+CNR =32 +4.9 = 37 dB.

e) In this case, the azimuth resolution is 2 x finer than 0.2m in part i., i.e. 0.1m, and the minimum PRF=2x1500 Hz= 3 kHz. This is below the maximum PRF allowed (5 kHz) so is OK from the point of view of range ambiguities. And the duty cycle limitation of 5%, with a pulse length of 1 μs, gives a maximum PRF of 0.05/1x10$^-$6 = 50 kHz (i.e. way above the 5 kHz for range ambiguities, so the max. PRF is set by range ambiguities here). To maximize SNR we should use the highest possible PRF i.e. we should use PRF= 5 kHz.

The SCR can be calculated relative to the previous case. We have a range resolution which is 2 x coarser, and an azimuth resolution which is 3 x finer (since we used 0.3 m in part iii, so the SCR is 3/2 of that in part iii, i.e. SCR= 2100= 33 dB.

The CNR can be calculated using the fact the range resolution is 2 x larger (coarser) but this is offset by a lower duty cycle (5 kHz x 1 ms= 0.005= 1/10th that used previously). Hence, altogether the CNR is decreased by a factor of 5.
CNR=3.1/5 = 0.61 = -2.1 dB.

Putting these together gives SNR=33-2.1=31 dB.

Using mode 2 then, the clutter is below the noise, hence we are dominated by noise in the detection scheme.

Overall it appears that mode 1 is slightly better, since the dominating term is SCR=32 dB, which is slightly better than the SNR for mode 1 which dominates (32 dB). Mode 2 has a slightly better overall resolution, but in both cases the resolution is so much higher than the size of the boats that this is unlikely to be a major difference. I would choose mode 1.