Exam in Remote Sensing (RRY 055)

Date:	2011-03-18.
Teacher contact:	Patrick Eriksson (0730-581306).
Language:	Answers can be given in either Swedish or English.
Extent:	Exam has 5 questions and 6 pages.
Points:	The exam carries 50 points (20 points needed to be approved).
Answers:	Begin each question on a new sheet of paper.
Allowed aid:	Calculators, basic math and physics formulae collections (such as Physics Handbook) and enclosed sheet of formulae.

1. Various short questions (10 p)

a Define the absorption length in terms of optical depth. (1 p)

b How is the NDVI measure defined? (2 p)

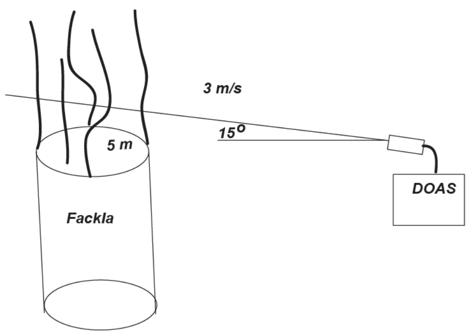
c What is the aim of "atmospheric correction" (in the field of surface imagery)? (2 p)

d Make a quick drawing of two weighting functions for a down-looking temperature sounder (eg. AMSU-A), and comment on how the corresponding channels are placed with respect to the centre of the absorption band (of O_2 or CO_2). (3 p)

e What is the basic philosophy of Bayesian inversion approaches? (2 p)

2. DOAS (10 p)

In a petrochemical factory in Houston there is a flare which burns excess gas (like a chimney). In May 2009, measurements were carried out of the gas releases from the flare during no wind conditions. A telescope was pointed at an angle of 15° above the horizon. The skylight was transmitted to a UV/visible spectrometer. Two spectra were measured, one with the telescope pointing outside the flare gas, and one with the telescope pointing through the flare gas plume, according to the schematic figure below. Figure **??** shows the ratio between the two spectra and Figure **??** shows the absorption cross section of formaldehyde (HCHO). The flare plume is circular with a diameter of 5 m and the concentration is constant across the whole plume. The plume raises with a vertical speed of 3 m/s.



a What is the gas column of formaldehyde in the measured spectrum, in the unit molecules/cm^2. Describe the methodology? (6 p)

b How many kilos of formaldehyde are released from the flare per hour? (4 p)

3. Atmospheric radiative transfer (10 p)

You plan to perform observations of the atmosphere in the mid IR range ($\lambda > 5\,\mu$m). As a preparation, you consider the physical mechanisms involved and the basic mathematical expressions to be solved.

a Discuss under which conditions you need to consider scattering. (2 p)

b Discuss under which conditions you need to consider solar radiation. (2 p)

c Write down a complete form of the radiative transfer equation. The differential form suffices, but it shall be as detailed as possible. (2 p)

d What is the unit of the quantities in the radiative transfer equation? (2 p)

e Describe the impact of refraction. For what observation geometry is refraction especially important? (2 p)

4. A satellite system (10 p)

After a disaster it is important to get fast updates of the situation at the location for the disaster. One possibility to collect information about the situation is to use satellite sensors. You will now be asked to give recommendations about some important design issues for a satellite system for detection and monitoring of a broad range of natural disasters.

a The satellite system must be placed in an orbit around the Earth. Describe which type of orbit you would choose for the satellite system. Motivate why you choose this orbit. What is the typical altitude for this orbit? (3 p)

b The budget for the satellite system is limited, so you can only use one satellite with two sensors. Select two sensor types that you want to have on a satellite for detection and monitoring of natural disasters. Motivate your answer! (3 p)

c In addition to the sensors, the satellite must have several other sub-systems, e.g. solar panels, to work properly. Mention at least four other sub-systems. (2 p)

d When a disaster occurs, information about the disaster must be distributed fast. Describe how you get the information from your satellite sensors to the people who are rescuing people at the disaster site. Your system must include two alternative way to transfer the data from the satellite. (2 p)

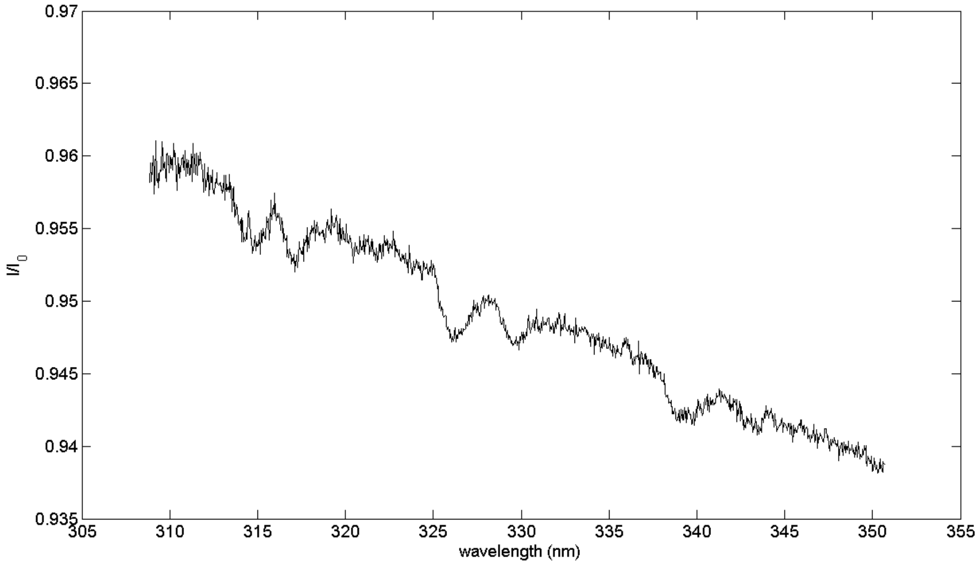


Figure 1: A transmittance spectrum corresponding to a spectrum measured in the plume divided by a reference spectrum (sky light) measured beside the plume.

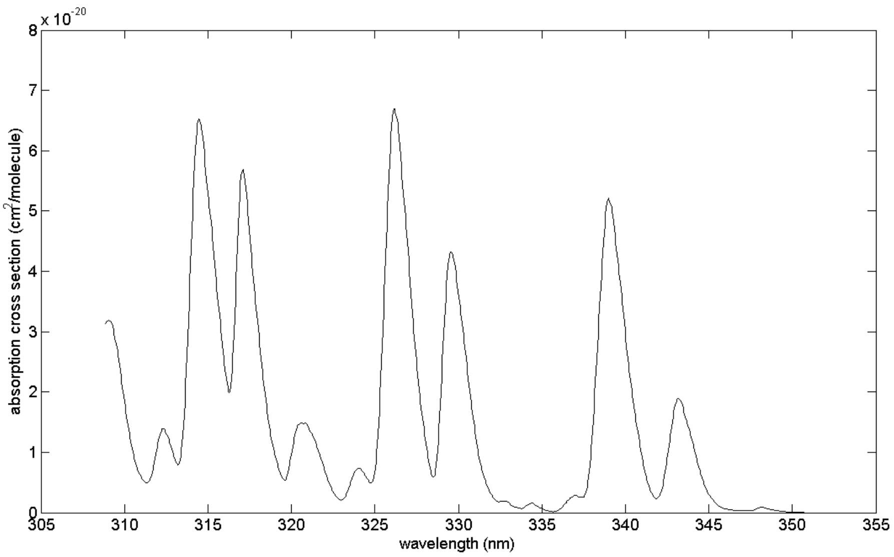


Figure 2: Absorption cross section for formaldehyde (HCHO).

5. Airborne SAR (10 p)

An airborne SAR-system with a centre frequency of 5.3 GHz travels above flat terrain (Figure ??). The antenna has an effective area of 1 m^2 and efficiency one, and the transmitted power is 15 kW. When travelling at an altitude z = 4 km, the resolution obtainable with the system is 3 m in slant range and 1 m in azimuth (along track). A field with back-scattering coefficient (σ^o) of -15 dB is located as indicated in the figure. Calculate the received power returned from the field originating from one resolution cell, when (Hint: Assume a rectangular resolution cell.):

a The flight altitude z = 4 km. (7 p)

b The flight altitude is changed to z = 6 km (all other parameters are unchanged). (3 p)

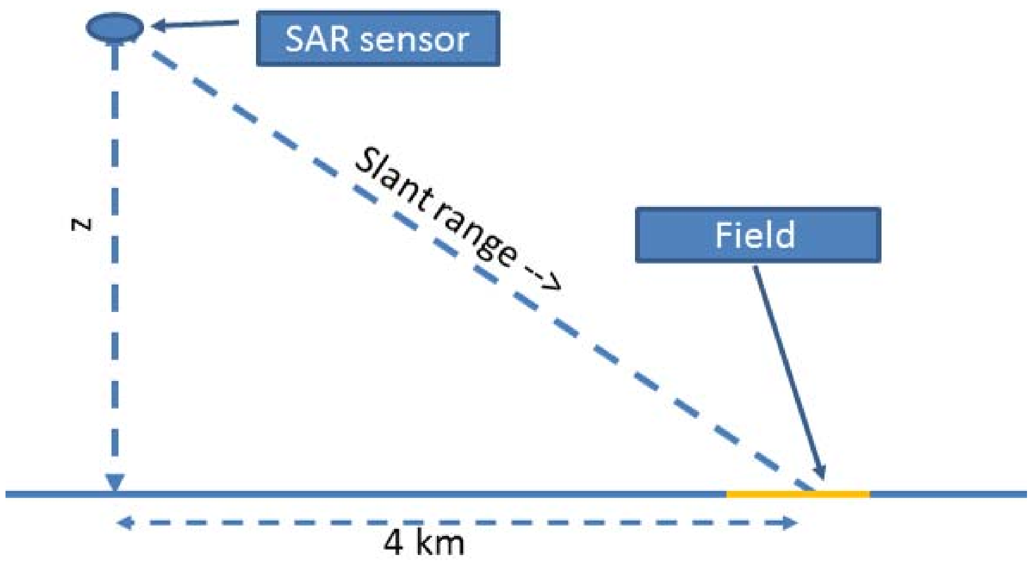


Figure 3: SAR imaging geometry. The sensor travels “out of the paper”.

Formulae

Ideal gas law

$$N = \frac{PV}{k_B T}, \qquad \frac{\rho T}{P} = \frac{M}{R}$$

Energy of a photon

$$E = hf$$

Wavenumber as spectroscopic unit

$$\tilde{\nu} = 1/\lambda$$

Stokes Vector

$$\begin{aligned}
\mathbf{s} &= [S_0\ S_1\ S_2\ S_3]^T \\
S_0 &= \langle E_{0x}^2 \rangle + \langle E_{0y}^2 \rangle \\
S_1 &= \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle \\
S_2 &= \langle 2E_{0x}^2 E_{0y}^2 \cos(\phi_y - \phi_x) \rangle \\
S_3 &= \langle 2E_{0x}^2 E_{0y}^2 \sin(\phi_y - \phi_x)) \rangle
\end{aligned}$$

Degree of polarisation

$$\frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

Refractive index

$$n = n' - in'' = \sqrt{\epsilon_r}$$

Angular frequency

$$\omega = 2\pi f$$

Complex (angular) wavenumber

$$k = \frac{\omega n}{c}$$

Absorption length

$$l_a = \frac{c}{2\omega n''} = \frac{1}{\gamma_a}$$

Snells law

$$n_1' \sin(\theta_1) = n_2' \sin(\theta_2)$$

Fresnel coefficients ($\varepsilon_1 = 1$)

$$\begin{aligned}
\Gamma_\perp &= \frac{\cos(\theta_1) - \sqrt{\epsilon_{r2} - \sin^2(\theta_1)}}{\cos(\theta_1) + \sqrt{\epsilon_{r2} - \sin^2(\theta_1)}} \\
\Gamma_\parallel &= \frac{\sqrt{\epsilon_{r2} - \sin^2(\theta_1)} - \epsilon_{r2}\cos(\theta_1)}{\sqrt{\epsilon_{r2} - \sin^2(\theta_1)} + \epsilon_{r2}\cos(\theta_1)} \\
r &= |\Gamma|^2
\end{aligned}$$

Irradiance / Exitance

$$E \text{ or } M = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} L \cos(\theta) \sin(\theta)\, \mathrm{d}\theta\, \mathrm{d}\phi$$

BRDF / surface reflectivity

$$\begin{aligned}
L(\theta_1, \phi_1) &= R(\theta, \phi, \theta_1, \phi_1) F(\theta, \phi) \cos(\theta) \\
M &= r(\theta, \phi) F(\theta, \phi) \cos(\theta) \\
r(\theta, \phi) &= \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} R \cos(\theta) \sin(\theta)\, \mathrm{d}\theta\, \mathrm{d}\phi
\end{aligned}$$

Blackbody radiation

$$B(f, T) = \frac{2hf^3}{c^2} \frac{1}{e^{hf/k_B T} - 1}$$

Rayleigh-Jeans approximation

$$\begin{aligned}
&(hf/k_B T \ll 1) \Rightarrow \\
&B(f, T) \approx 2k_B T f^2 / c^2
\end{aligned}$$

Absorption coefficient

$$\gamma_a = N\sigma_a = N \left\{ \sum [sF] + \sigma_{\text{cont}} \right\}$$

Doppler broadening

$$\begin{aligned}
F_d &= \frac{1}{\sqrt{\pi} w_d} \exp(-((f - f_0)/w_d)^2) \\
w_d &= \frac{f_0}{c} \sqrt{\frac{2RT}{M}}
\end{aligned}$$

Pressure broadening

$$\begin{aligned}
F_p &= \frac{1}{\pi} \frac{w_p}{(f - f_0)^2 + w_p^2} \\
w_p &= w_0 P \left(\frac{T}{T_0} \right)^{-n}
\end{aligned}$$

Scattering coefficient

$$\gamma_s = N\sigma_s$$

Rayleigh scattering/absorption

$$\sigma_s \sim \frac{d^6}{\lambda^4}$$

$$\sigma_a \sim \frac{d^3}{\lambda}$$

Optical thickness

$$\tau(l_1, l_2) = \int_{l_1}^{l_2} \gamma(l) \mathrm{d}l$$

Beer-Lambert's law

$$I = I_0 \exp(-\tau)$$

DOAS

$$N = \frac{ln(I_1/I_2)}{h[\sigma(\lambda_2) - \sigma(\lambda_1)]}$$

Radiative transfer without scattering

$$\begin{aligned} I(h) &= I_0 e^{-\tau(0,h)} + \int_0^h \gamma_a B e^{-\tau(l,h)} \mathrm{d}l \\ T_b &= T_b^0 e^{-\tau(0,h)} + \int_0^h \gamma_a T e^{-\tau(l,h)} \mathrm{d}l \\ I^{out} &= I^{in} e^{-\tau} + B(1 - e^{-\tau}) \end{aligned}$$

Photographic scaling factor

$$s = \frac{f}{h}$$

f/number

$$f/number = \frac{f}{D}$$

Photogrammetry

$$x' = -\frac{fx}{h-z}, \qquad y' = -\frac{fy}{h-z}$$

Michelson interferometry

$$\begin{aligned} I &= \frac{I_0}{2}\left[1 + \cos(2kl)\right] \\ V &= \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \\ I(l) &= \int_0^\infty I(f)\cos(2\pi l f/c)\mathrm{d}f \end{aligned}$$

Diffraction limited angular resolution

$$\sin(\theta) \geq \frac{\lambda}{D}$$

Antenna relationships

$$\begin{aligned} G &= \eta D_0 = \eta \frac{4\pi}{\Omega_A} \\ \Omega_A &= \frac{\lambda^2}{A_e} \end{aligned}$$

Radiometer sensitivity

$$\Delta T_b = C \frac{T_{sys}}{\sqrt{\Delta f \Delta t}}$$

Noise power

$$P_N = k_B B T_a$$

Footprint of radar altimeter

$$r = \sqrt{ch\Delta t}$$

Radar equation

$$\frac{P_r}{P_t} = \frac{\lambda^2 G^2}{(4\pi)^3 \eta h^4} \sigma^o A$$

Radar Doppler shift

$$f_D = -2v_r/\lambda$$

Weighting functions

$$\begin{aligned} y &= \int_0^h K(z)x(z)\mathrm{d}z + \varepsilon \\ \mathbf{y} &= \mathbf{Kx} + \varepsilon \end{aligned}$$

Optimal weighting

$$\begin{aligned} \hat{x} &= \frac{\sigma_2^2 x_1 + \sigma_1^2 x_2}{\sigma_1^2 + \sigma_2^2} \\ \hat{\sigma} &= \frac{\sigma_1 \sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \end{aligned}$$

Answers to exam in Remote Sensing 2011-03-18

Question 1:

a The absorption length equals an optical depth of 1.

b NDVI $= (r_i - r_r))/(r_i + r_r)$, where r_r is the optical (or more precisely red) reflectance, and r_i is the near-IR reflectance.

c To remove all atmospheric effects on the data. That is, to obtain data as measured with the atmosphere removed.

d See Figure 6.22 in the book. Channels with high optical thickness, and peaking at higher altitudes, are closer to the centre of the absorption band.

e To use statistical information as a constrain for the retrieval (required if the problem is ill-posed).

Question 2:
Using differential spectroscopy the amount of formaldehyde across the flare is $5.3{\cdot}10^{16}$ molec/cm^2. If one corrects for the slant angle then the columns is obtained according to: column = slantcolumn $\cdot \cos(15°) = 5.11{\cdot}10^{16}$ molec/cm^2. This corresponds to: $5.11{\cdot}10^{16}$ molec/cm^2 $\cdot$ MHCHO $\cdot$ 1u= $5.11{\cdot}10^{16}$ molec/cm^2 $\cdot\ 30 \cdot 1.66{\cdot}10^{-27}$ $=2.54{\cdot}10^{-9}$ kg/cm^2 = 25.4 mg/m^2. The diameter of the flare was 5 m, hence the concentration in the flare is $25.4/5 = 5.1$ mg/m^3. The flare has an area of 19.6 m^2 and the flare has in total a in cross section of formaldehyde of: $5.1 \cdot 19.6 = 100$ mg/m the flow is 3 m/s, hence an flux of $3 \cdot 100 = 300$ mg/s = 0.3 g/s = 1 kg/h is obtained.

Question 3:

a As soon as clouds are found along the line-of-sight. (Scattering due to aerosols can be in general be neglected.) For down-looking measurements, surface scattering must also be considered.

b The solar radiation dominates for observations directly towards the sun, but can otherwise be neglected.

c $$\frac{\mathrm{d}I(\theta,\phi)}{\mathrm{d}l} = -(\gamma_a + \gamma_s)I(\theta,\phi) + \gamma_a B + \frac{\gamma_s}{4\pi}\int_{4\pi} I(\theta',\phi')p(\theta',\phi',\theta,\phi)\mathrm{d}\Omega'$$

d Several units are possible. For example: I: Wm^{-2}sr^{-1}Hz^{-1}, l: m, γ_a/γ_s: m^{-1}, B: Wm^{-3}sr^{-1}Hz^{-1}, p: sr^{-1}, and Ω: sr.

e Refraction can cause that propagation path devieates from a stright line. This effect is most important for observations at high incidence angles, with limb sounding as the most extreme case.

Question 4:

Question 5:
The basic equation needed to solve this task is the radar equation, i.e.

$$\frac{P_r}{P_t} = \frac{\lambda^2 G^2}{(4\pi)^3 \eta h^4}\sigma^o A$$

The wavelength is given by $\lambda = c/f = 0.057$ m. The gain can be calculated from the wavelength, effective area A_e and efficiency η and as (see formulae collection):

$$G = \eta \frac{4\pi}{\Omega_A} = \{\Omega_A = \frac{\lambda^2}{A_e}\} = \eta \frac{4\pi}{\lambda^2} A_e = 35.9 \text{ dB}$$

The range can be readily calculated from geometry as $h = \sqrt{z^2 + x^2}$, where x is the distance to the field in the cross track direction. h is equal to 5.7 and 7.2 km in a) and b), respectively.

The only remaining part of the radar equation is the *ground area* A. This was specified to be one resolution cell, which can be assumed to be rectangular. In the azimuth direction the side length of the resolution cell is $\delta_a = 1$ m, regardless of the flight altitude. In the range direction, the slant range resolution δ_r needs to be projected to the ground to obtain the correct resolution on the ground. From figure **??** it can be seen that the resolution on the ground (or ground range resolution) is $\delta_{gr} = \delta_r / \sin(\theta)$, where $\theta = \cos^{-1}(z/h)$. The resulting resolution area on the ground ($A = \delta_{gr} \cdot \delta_a$) is 4.24 m^2 and 5.41 m^2 for a) and b), respectively. Plugging this into the radar equation gives a received power of 49 pW and 24 pW for a) and b), respectively.

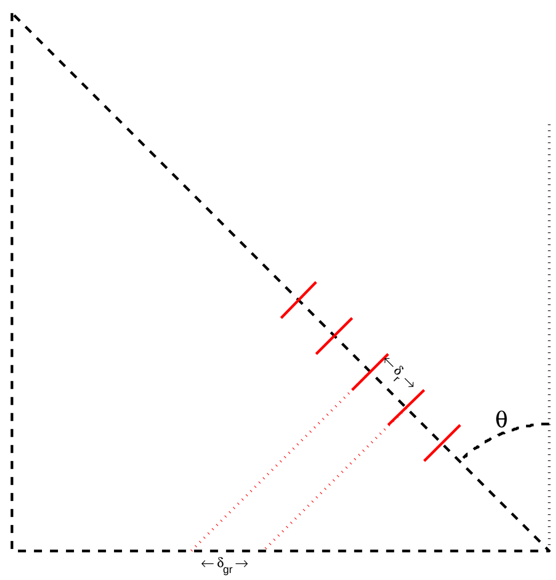


Figure 4: SAR imaging geometry. The sensor travels “out of the paper”.