MPWPS, year 1, study period 1, academic year 2011/2012

Exam
Electromagnetic Waves and Components (RRY 036), 21/10 2011
Department of Earth and Space Sciences

Teachers:
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On the exam you may use:
- Chalmers-approved calculator,
- Formulas in Electromagnetic waves (E. Palmberg 2011),
- Formulae and constants for blackbody radiation, excitation of two-level systems and radiative transfer (A.Heikkilä 2011),
- Physics Handbook, Beta, etc,
- Dictionary (not electronic).

Grade limits:
Grade 3(=pass): 20 points
Grade 4: 30 points
Grade 5: 40 points
A maximum of 50 points can be achieved on the exam.

Remember: Give full solutions to the problems you hand in, i.e. explain and motivate your answers carefully! Be careful with units! When drawing graphs, indicate clearly the quantity on each axis, and give the scale.

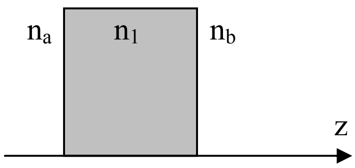
1. Laboratories use cavities maintained at specific temperatures as reference sources of blackbody radiation. Consider one such cavity at a temperature 1337.33 K (freezing point of gold). The radiation is emitted through a hole with a diameter of 6 mm.

a) Calculate the power (in watt) of the emitted radiation. (1p)
b) How many photons escape through the hole per second? (2p)
c) Imagine that you have a graph showing the spectrum of this blackbody radiator. If the temperature of the cavity is lowered, how would the spectrum of the emitted radiation change? Describe in words, no detailed calculations are needed. (1p)

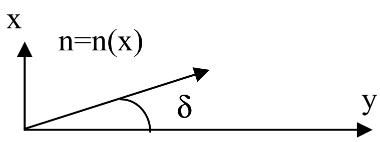
2. Consider a medium containing two-level systems (resonance frequency 24 GHz, statistical weights $g_l = g_u = 1$) which are "pumped", resulting in $N_u/N_l=5$ If the length of the medium corresponds to an optical depth of -10, and the background radiation entering the medium is negligible, what is the *brightness temperature* of the radiation emitted by the medium? (2p)

3. An unpolarized electromagnetic wave encounters a bound electron. Describe what happens using the harmonic oscillator model and explain how it leads to scattering. In which frequency regions do you expect strong and weak ω-dependence of the scattered light? Explain why the sky is blue and sunset red, and why the scattered light is partially polarized? (5p)

4. A dielectric slab with refractive index n_1 is separating two dielectrics with refractive index n_a and n_b. For a wave propagating in the z direction, which of the field quantities $\underline{E}$, $\underline{H}$, $\underline{E}_+$, $\underline{E}_-$, Z and Γ are matched at the interface (z=const)? Which of the quantities are easily propagated? What is so special with quarter-wavelength and half-wavelength thickness of the slab? Discuss the propagating properties of Z and Γ for the two cases and illustrate with some applications. (5p)



5. A wave propagates mainly in the y-direction in a dielectric medium with a refractive index n that varies perpendicular to the direction of propagation with $n(x)=n_0(1-0.5\alpha^2x^2)$ and $|\alpha x|<<1$. Use the ray equation and the paraxial approximation to derive an equation for the ray in the x-y plane and solve the equation to get x=x(y). The ray passes through x=y=0 and makes the angle δ with the y-axis at x=0. (8p)

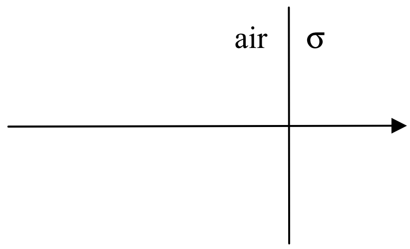


6. A ground penetrating radar is used to detect underground objects. The earth conductivity is $\sigma = 2 \times 10^{-3}$ S/m, permittivity $\varepsilon=4\varepsilon_0$ and $\mu=\mu_0$. The radar operates at 850 MHz. a) Is the earth a good or bad conductor at this frequency? b) Calculate the earth refractive index. c) Determine the maximum depth of detecting an object if detectability requires that the roundtrip power attenuation (from the surface of the object and back to the surface, neglecting reflection at the earth surface) is not greater than 35 dB. (8p)

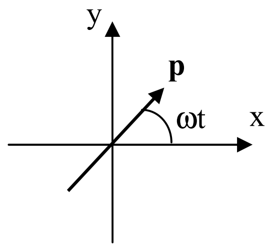
7. A left-hand polarized plane wave with frequency f=1 THz is normally incident from air (n=1) on a metal wall at z=0. The metal wall has $\sigma = 0.2\cdot10^3$ S/m, permittivity $\varepsilon=\varepsilon_0$ and $\mu=\mu_0$. The incident complex field can be written $\underline{\mathbf{E}}_i(z)=E_0(\hat{x}+j\hat{y})e^{-jkz}$.

a) Determine the reflected electric and magnetic fields $\mathbf{E}_r(z,t)$ and $\mathbf{H}_r(z,t)$.
b) Determine the polarization of the reflected wave.

(8p)

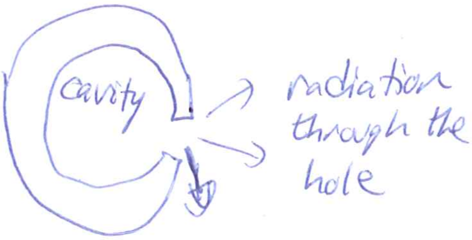


8. An electric dipole with $|\mathbf{p}|=p_0$ is rotating in the x-y plane ($\mathbf{p}=p_0\hat{x}$ at t=0) at z=0 with angular frequency ω. Find the electric field $\mathbf{E}(r,t)$ and magnetic field $\mathbf{H}(r,t)$ on the y and z axes in the radiation zone of the rotating dipole. Discuss the polarization properties of the wave in the two cases. Find the total radiated power of the rotating dipole. (10p)



EMWC exam 21/10-2011

(1)



a) $P = A \cdot I \simeq \frac{\pi d^2}{4} \cdot 5{,}67 \cdot 10^{-8} T^4 \simeq$

area of the hole

$\simeq \underline{\underline{5{,}1 \text{ W}}}$

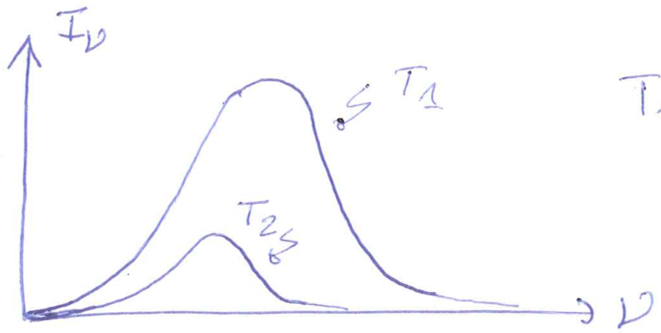
b) Photon flux $= \frac{P}{\text{energy/photon}}$

average energy per photon $= \frac{\rho}{n} \simeq \frac{7{,}56 \cdot 10^{-16} T^4}{2{,}03 \cdot 10^{7} T^3}$

$\simeq 5{,}055 \cdot 10^{-20}$ J/photon

$\Rightarrow$ Flux $\simeq \underline{\underline{1{,}0 \cdot 10^{20} \text{ photons/second}}}$

c)



$T_1 > T_2$

Max. intensity decreases & freq. at which max I_ν occurs is lowered when the temp. decreases.

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(2) $T_b \simeq T_{ex} \cdot (1 - e^{-\tau_{\nu_{ul}}})$ if background radiation is neglected.

$\frac{N_u}{N_l} = e^{-h\nu_{ul}/kT_{ex}} \Rightarrow T_{ex} = \frac{-h\nu_{ul}/k}{\ln\left(\frac{N_u}{N_l}\right)} \simeq -0{,}716$ kelvin

$\Rightarrow T_b \simeq \underline{\underline{1{,}6 \cdot 10^4 \text{ kelvin}}}$

Negative T_{ex} & $\tau_{\nu_{ul}} \Rightarrow$ amplifying medium, i.e. the upper level is over-populated and more stim. em. than absorption of photons.

3+4 - see lecture notes

5.

Ray eq. $\frac{d}{dl} n \frac{d\vec{r}}{dl} = \nabla n$

$n = n_0 (1 - \frac{1}{2}\alpha^2 x^2)$, $|\alpha x| << 1$

Paraxial approx: $d/dl \simeq d/dy$

$\nabla n = -\hat{x} n_0 \alpha^2 x$

$$\Rightarrow \frac{d}{dy} n(x) \frac{dx}{dy} \hat{x} = -\hat{x} n_0 \alpha^2 x$$

$$\Rightarrow \frac{d^2 x}{dy^2} = -\frac{n_0 \alpha^2 x}{n(x)} \simeq -\alpha^2 x \quad (|\alpha x| << 1)$$

$$\Rightarrow x = A \sin(\alpha y + \theta)$$

$x = 0$ at $y = 0 \Rightarrow \theta = 0$

$$\left.\frac{dx}{dy}\right|_{y=0} = A \alpha \cos 0 = A\alpha = \tan\delta \simeq \delta \quad \left(\text{paraxial approx}\right)$$

$$\Rightarrow A = \delta/\alpha$$

$$\Rightarrow \underline{x(y) = \frac{\delta}{\alpha} \sin \alpha y}$$

6. $\varepsilon_c = \varepsilon_d - j\,\sigma/\omega = 4\varepsilon_0 \left(1 - j\frac{\sigma}{4\varepsilon_0}\right)$

$\frac{\sigma}{\omega \varepsilon_0 \cdot 4} \simeq 0.01$ weakly lossy

$$n = \sqrt{\frac{\varepsilon_c}{\varepsilon_0}} = 2\sqrt{1 - j0.01} \simeq 2\left(1 - j\frac{0.01}{2}\right)$$

$$k = n k_0 = \beta - j\alpha \simeq k_0 (2 - j0.01)$$

$$\Rightarrow \alpha = \frac{\omega}{c_0} n_I = \frac{2\pi \cdot 850 \cdot 10^6}{3 \cdot 10^8} \cdot 0.01 = 0.178$$

$\frac{P(z)}{P(0)} = e^{-2\alpha z_{tot}}$, $-10\log_{10}\frac{P(z)}{P(0)} = 35\text{ dB}$

$\Rightarrow \frac{P(z)}{P(0)} = 3.16 \cdot 10^{-4}$, $e^{-2\alpha z_{tot}} = 3.16 \cdot 10^{-4} \Rightarrow z_{tot} = 22.6\text{ m}$

$$\Rightarrow \underline{z = \frac{z_{tot}}{2} = 11.3\text{ m}}$$

7. $f = 10^{12}$ Hz , $\sigma = 0.2 \cdot 10^{3}$ S/m

$n_{air} = 1$, $n_c = \sqrt{\varepsilon_c/\varepsilon_0}$

$\varepsilon_c = \varepsilon_0 \left(1 - j\, \sigma/\omega\varepsilon_0\right)$

$$\frac{\sigma}{\omega\varepsilon_0} = 3.6$$

$$n_c = \sqrt{1-j3.6} = 1.93\, e^{-j\,37.2^\circ} = 1.54 - j\,1.17$$

$$\rho = \frac{1-n_c}{1+n_c} = 0.46\, e^{j\,139.5^\circ}$$

a) $E_r = \rho E_i$

$$\vec{E}_r(z) = 0.46\, e^{j\,139.5^\circ} E_0 (\hat{x} + j\hat{y})\, e^{+jkz}$$
$$= 0.46\, E_0 (\hat{x} + j\hat{y})\, e^{j(kz+139.5^\circ)}$$
$$= 0.46\, E_0 \left\{ \hat{x}\, e^{j(kz+139.5^\circ)} + \hat{y}\, e^{j(kz-130.5^\circ)} \right\}$$

$$\vec{E}_r(z,t) = Re\left\{ \vec{E}_r\, e^{j\omega t} \right\}$$
$$= 0.46\, E_0 \cos(\omega t + kz + 139.5^\circ)\, \hat{x}$$
$$+ 0.46\, E_0 \cos(\omega t + kz - 130.5^\circ)\, \hat{y}$$

$$\vec{H}_r(z) = \frac{1}{\eta_0}(-\hat{z}) \times \vec{E}_r = \frac{0.46\, E_0}{\eta_0}(-\hat{y} + j\hat{x})\, e^{jkz}\, e^{j\,139.5^\circ}$$

$$\vec{H}(z,t) = \frac{0.46\, E_0}{\eta_0} \cos(\omega t + kz - 40.5^\circ)\, \hat{y}$$
$$+ \frac{0.46\, E_0}{\eta_0} \cos(\omega t + kz - 130.5^\circ)\, \hat{x}$$

b) Same relation between $\hat{x}$ and $\hat{y}$ components for reflected wave as for incident wave $\Rightarrow$ circular polarization.
Different direction of propagation ($-\hat{z}$ vs $\hat{z}$) $\Rightarrow$ right-hand circular polarized (same amplitude of $\hat{x}$ and $\hat{y}$ components, 90° phase difference).

8 $\vec{p} = p_0 \cos\omega t\,\hat{x} + p_0 \sin\omega t\,\hat{y}$

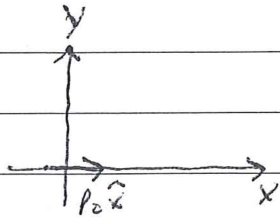
Assume short dipole, $\lambda >> \ell$

$\vec{p} = p_0\,\hat{x} - j p_0\,\hat{y}$

Fields can be superposed from the two dipoles.

Radiation zone on y-axis (e.g. $y >> 1$)

No contribution from $\hat{y}$ component of $\vec{p}$.



Formulas:

$$\vec{E} = -\hat{\theta}\,\frac{k^2 p_0 \sin\theta}{4\pi\epsilon_0 r}\,e^{-jkr}$$

Here: $r = y$, $\theta = \pi/2$, $-\hat{\theta} = \hat{x}$

$$\Rightarrow \vec{E} = \hat{x}\,\frac{k^2 p_0}{4\pi\epsilon_0 y}\,e^{-jky}$$

$$\vec{E}(r,t) = \hat{x}\,\frac{k^2 p_0}{4\pi\epsilon_0 y}\cos(\omega t - ky)$$

$$\vec{H}(r,t) = -\hat{z}\,\frac{k^2 p_0}{4\pi\epsilon_0 y\,\eta_0}\cos(\omega t - ky)$$

Polarization: linearly polarized on $\hat{x}$ direction

Radiation zone on z-axis (e.g. $z >> 1$)

Superposition of contributions from $p_0\hat{x}$ and $-jp_0\hat{y}$

Here: $r = z$, $\theta = \pi/2$, $-\hat{\theta} = \hat{x}$ or $-\hat{\theta} = \hat{y}$

$$\vec{E}(\vec{r}) = \hat{x}\,\frac{k^2 p_0}{4\pi\epsilon_0 z}\,e^{-jkz} + \hat{y}\,\frac{k^2 p_0\,e^{-j90^\circ}}{4\pi\epsilon_0 z}\,e^{-jkz}$$

$$\vec{E}(r,t) = \hat{x}\,\frac{k^2 p_0}{4\pi\epsilon_0 z}\cos(\omega t - kz) + \hat{y}\,\frac{k^2 p_0}{4\pi\epsilon_0 z}\underbrace{\cos(\omega t - kz - 90^\circ)}_{= \sin(\omega t - kz)}$$

$$\vec{H}(r,t) = \hat{y}\,\frac{k^2 p_0}{4\pi\epsilon_0 \eta_0 z}\cos(\omega t - kz) - \hat{x}\,\frac{k^2 p_0}{4\pi\epsilon_0 \eta_0 z}\sin(\omega t - kz)$$

b) ~~Same~~ Polarization: ~~Left-hand~~ Circular polarization (for large z)

8 continued

Power can usually not be superposed. Here, the fields due to $p_0 \hat{x}$ and $i p_0 \hat{y}$ are 90° out of phase. Then the cross-terms are zero and the ~~fields~~ power from each dipole can be added.

$$P_{tot} = P_1 + P_2 = \frac{\omega^4 p_0}{12\pi \varepsilon_0 c_0^3} + \frac{\omega^4 p_0}{12\pi \varepsilon_0 c_0^3} = \frac{\omega^4 p_0}{6\pi \varepsilon_0 c_0^3}$$