IMAGE PROCESSING (RRY025)

One of the Exams in 2011/2012

1 IMAGE ENHANCEMENT/RESTORATION [15 points]

(a) [2p] Describe histogram equalization as a tool for image enhancement.

(b) [2p] Same question as in **(a)**, but for histogram specification (matching).

(c) [2p] Describe the ideal low-pass filter as a tool for noise reduction, discuss its advantages and disadvantages, and explain for which type of noise it can be used.

(d) [2p] Same questions as in **(c)**, but for the Gaussian low-pass filter.

(e) [7p] You have a digital photo of 1024×1024 pixels. The quality of the photo is high only within the central 896×896 pixels. The surrounding 'frame', which has a thickness of 64 pixels [$64 = (1024 - 896)/2$], is polluted by additive Gaussian white noise of unknown standard deviation. This is unfortunate because such a 'frame' encloses important information, which you absolutely want to restore. Indeed, you took this photo on a very special occasion, using a camera that was just about to break :-(So explain in detail, and with illustrations, how you would de-noise the photo!

2 MISCELLANEA [15 points]

(a) [3p] Suppose that you have an image of 1010×1020 pixels, and that you want to transform it using Fourier or wavelet methods. What should you do first? And why? And if your image is of size 1020×1030 or 1030×1040? Are you sure? Think again! For example, if the image is of size 1025×1025?? Explain why!!

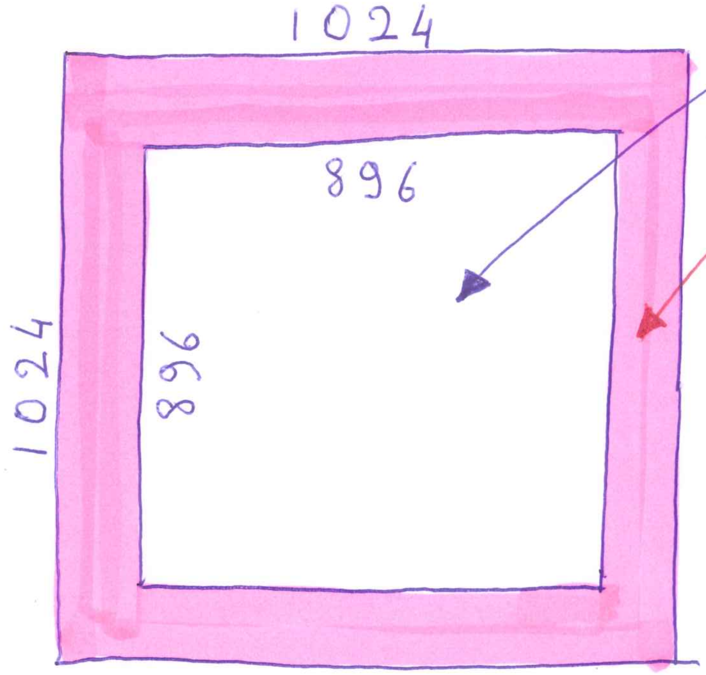
(b) [4p] Suppose that you have a low-contrast image, and that you enhance it by equalizing its histogram. Suppose also that you want to compress both images (the original one, and the enhanced one) without loss, coding each pixel separately. Which one of these two images can be compressed more? And why?

(c) [5p] Consider a signal $S(X)$ that is almost perfectly regular: it is continuous, together with its 1st and 2nd derivatives. But its 3rd derivative has a discontinuity at a certain point X_0. The signal is also very simple: it is a polynomial of degree 3 both for $X < X_0$ and for $X > X_0$. How would you detect the 'breakdown' point X_0? What would be the uncertainty ΔX of your detection? Explain in detail!

(d) [3p] In your opinion, what is the most interesting topic of the course? Explain how important this topic is in the context of image processing, and how important it is for your studies/job.

1 Image Enhancement/Restoration

(e)

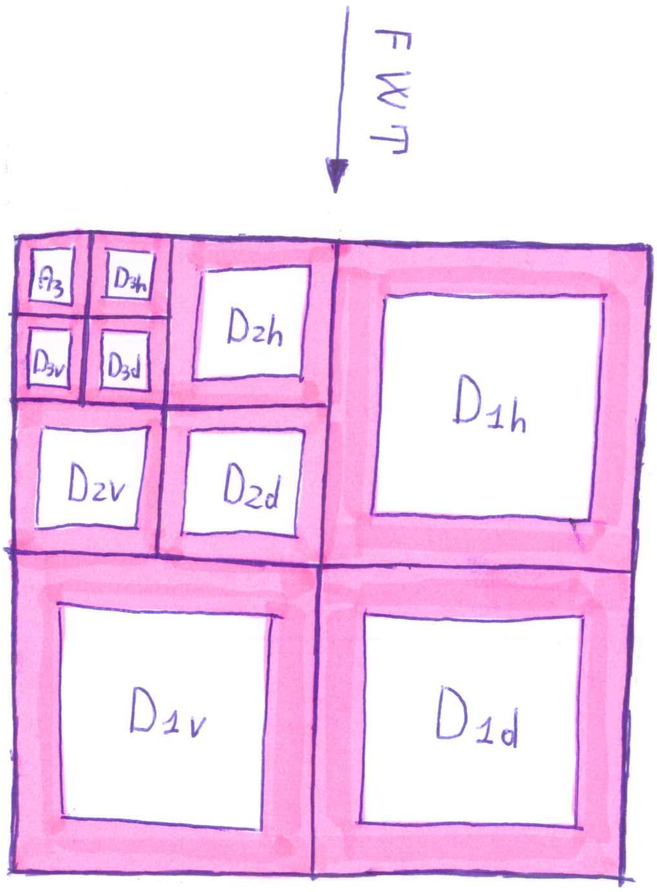


high-quality

polluted by noise:

- additive
- Gaussian
- white
- standard deviation?

→ ... and LOCALIZED!



De-noise the image using the Fast Wavelet Transform

- Choose the wavelet:

  bi-orthogonal & quasi-orthogonal

  → bior4.4 or rbio6.8

- Choose the level:

  $$2^{l-1} \times \underbrace{\text{wavelet size}}_{\substack{*\,12 \text{ pixels for bior4.4} \\ *\,20 \text{ pixels for rbio6.8}}} \approx \underbrace{\text{'frame' thickness}}_{64 \text{ pixels}}$$

  → $l = 3$ in both cases

- FWT the original image at level 3

- Compute the standard deviation of noise:

  $$\sigma = \frac{1}{0.6745} \times \text{Median Absolute Deviation} \{D_1\}_{frame}$$

- Compute the threshold:

  $$T = \sqrt{2 \ln \underbrace{N_{frame}}_{1024^2 - 896^2}}\ \sigma$$

- Threshold $\{D_1\}_{frame}$, $\{D_2\}_{frame}$ and $\{D_3\}_{frame}$

- IFWT 

2 Miscellanea

(a) We know that:

- Given an image of $M \times N$ pixels, the FFT and the FWT are <u>best</u> computed if $M = 2^m$ and $N = 2^n$, where m and n are positive integers.

- If the size of the image is not a power of two, then the usual recipe is to (zero-) pad:

  * $1010 \times 1020 \rightarrow 1024 \times 1024$
  * $1020 \times 1030 \rightarrow 1024 \times 2048$
  * $1030 \times 1040 \rightarrow 2048 \times 2048$
  * $1025 \times 1025 \rightarrow 2048 \times 2048$

<u>BUT</u>:

- When you want to transform (not to convolve) an image, whatever type of padding you use, it will always produce artifacts.
- Padding 'slows down' the transform.
- Usually, the information contained near the boundaries of an image is irrelevant.

→ 1025 x 1025 crop to 1024 x 1024.

Why? This is like cutting off the outer ≈ 0.1 millimeters from a square image of ≈ 10 centimeters!

→ 1030 x 1040 crop to 1024 x 1024.

Do you think that the information contained in the outer 1–1.5 mm of a 10 cm image is significant?!

... 1020 x 1030 → 1024 x 1024

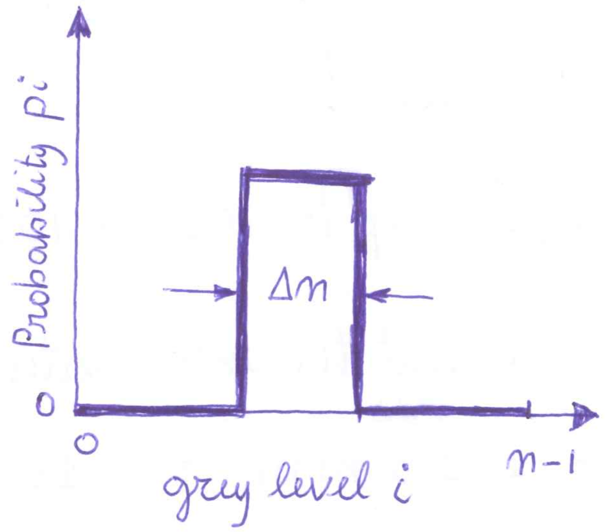
1010 x 1020 → 1024 x 1024

(b)

Low-contrast image → narrow histogram → low single-pixel entropy
Histogram-equalised image → flat histogram → high single-pixel entropy

→ The original image can be compressed more than the enhanced one.

For example, consider the following two 'toy models':

- 

Low-contrast image

* Single-pixel entropy

$$H_1 = - \sum_{i=0}^{n-1} p_i \log_2 p_i$$

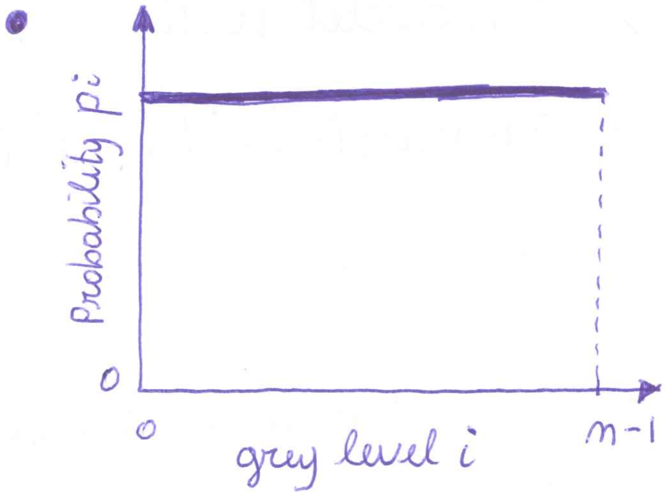
$$= - \Delta n \left(\frac{1}{\Delta n} \log_2 \frac{1}{\Delta n} \right)$$

$$= \log_2 \Delta n$$

* Theoretical maximum compression =

$$\frac{\text{\# bits/pixel in the image}}{\text{single-pixel entropy}} = \frac{\log_2 n}{\log_2 \Delta n} > 1$$

→ The smaller Δn, the more the image can be compressed!



histogram-equalized image

* Single-pixel entropy

$$H_1 = -\sum_{i=0}^{n-1} p_i \log_2 p_i$$

$$= -n\left(\frac{1}{n}\log_2\frac{1}{n}\right)$$

$$= \log_2 n$$

* Theoretical maximum compression ... =

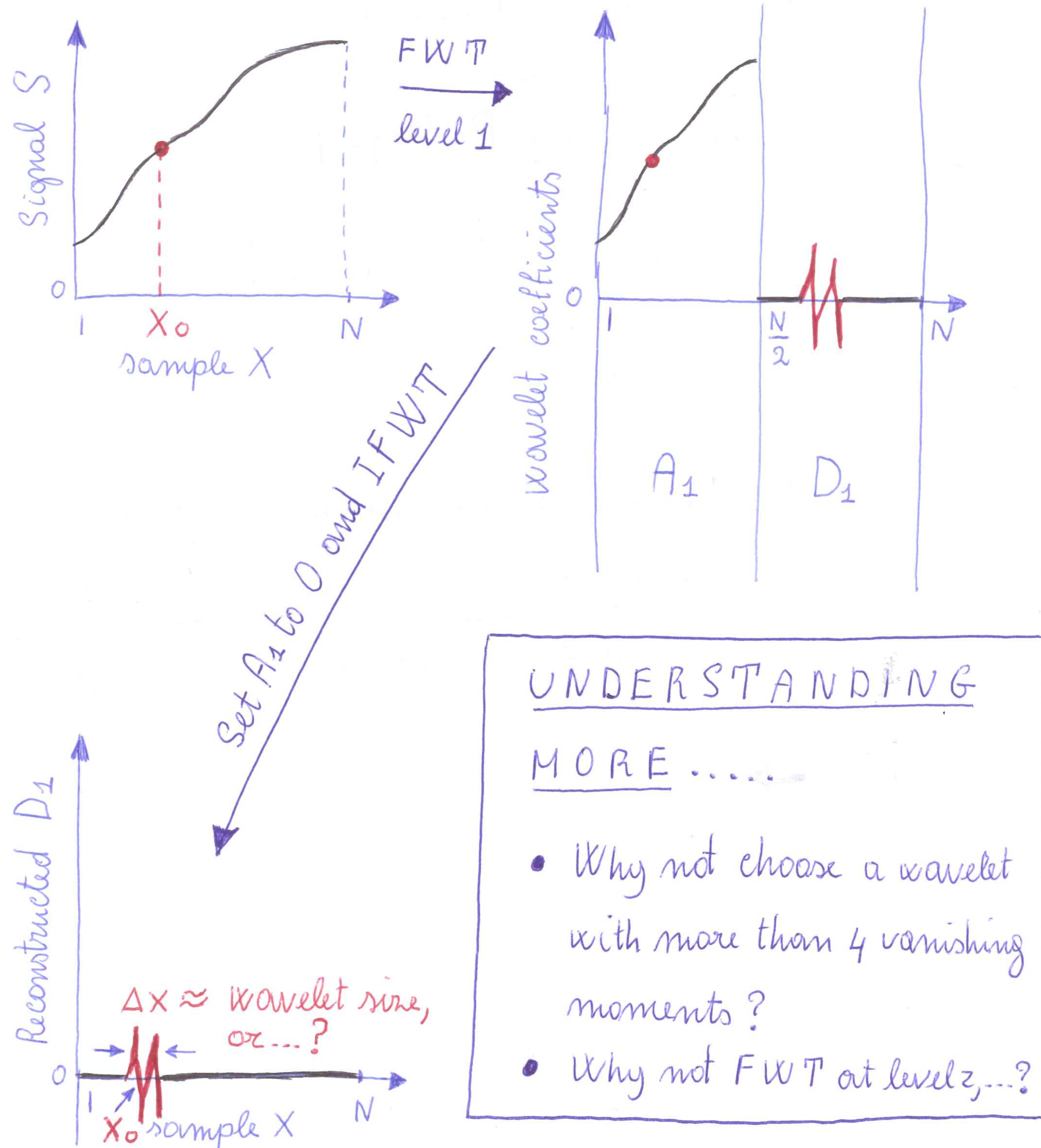
$$\frac{\text{\# bits/pixel in the image}}{\text{single-pixel entropy}} = \frac{\log_2 n}{\log_2 n} = 1$$

→ <u>No</u> compression!

(c) SMART solution

A wavelet with $n+1$ vanishing moments is 'blind' to polynomials of degree n.

→ Choose a wavelet with 4 vanishing moments, such as the 'FBI' wavelet bior 4.4, and then:



UNDERSTANDING MORE....

- Why not choose a wavelet with more than 4 vanishing moments?
- Why not FWT at level 2,...?

STANDARD solution

- The 3rd derivative of the signal shows an <u>edge</u> at $x \approx x_0$!

  (Why not a discontinuous jump at $x = x_0$?)

➡ Compute its 4th derivative and detect the edge!

<u>But how can we compute those derivatives?</u>

- $\underbrace{d_2(x)}_{\text{2nd derivative}} = S(x+1) - 2S(x) + S(x-1)$ ... we know that.
- $\underbrace{d_3(x)}_{\text{3rd derivative}} = d_2(x+1) - d_2(x-1)$ ... centred at x.

  $= S(x+2) - 2S(x+1) + 2S(x-1) - S(x-2)$
- $\underbrace{d_4(x)}_{\text{4th derivative}} = d_2(x+1) - 2d_2(x) + d_2(x-1)$ ... centred at x.

  $= S(x+2) - 4S(x+1) + 6S(x) - 4S(x-1) + S(x-2)$

<u>So what are the corresponding filters?</u>

- $d_2 = [1 \;\; -2 \;\; 1]$
- $d_3 = [-1 \;\; 2 \;\; 0 \;\; -2 \;\; 1]$
- $d_4 = [1 \;\; -4 \;\; 6 \;\; -4 \;\; 1]$

➡ WHAT DO WE LEARN?! ⬅