IMAGE PROCESSING (RRY025)

One of the Exams in 2008/2009

1 IMAGE ENHANCEMENT

Two students of this course, X and Y, want to test their understanding of image enhancement.

(a) X asks Y to enhance a 4-bit image of size 4×4 by histogram equalization. X sends the image by horizontal raster scanning:

15, 1, 2, 12, 4, 10, 9, 7, 8, 6, 5, 11, 3, 13, 14, 0.

Y receives this image and breaks into a laugh! Can you explain why? [2 points]

(b) X then sends a 2-bit image of the same size:

1, 1, 2, 2, 2, 1, 3, 1, 1, 1, 0, 1, 1, 2, 2, 1.

How does Y equalize the histogram of this image? What is the output image? Any comment on the output histogram? [3 points]

(c) Now Y asks X to equalize the histogram of an image in which the gray level is equal everywhere. How do the output and input images differ in this case? [2 points]

(d) Finally, X and Y want to do some experiments involving exponential noise, for which the probability distribution function is $p(x) = a\,e^{-ax}$ if $x \geq 0$, and $p(x) = 0$ otherwise. The problem is that their toolbox has a command to generate only uniform noise, for which the probability distribution function is $q(y) = 1$ if $0 \leq y \leq 1$, and $q(y) = 0$ otherwise. How can they generate exponential noise? [3 points]

2 WAVELETS

Explain the following points, AND sketch simple diagrams to illustrate them:

(a) the fundamental property of wavelets; [2 points]

(b) the fast wavelet transform; [2 points]

(c) the idea behind data compression using wavelets; [3 points]

(d) the idea behind data de-noising using wavelets; [3 points]

3 IMAGE COMPRESSION

A gang of criminals threaten to paralyse the World Wide Web by transmitting huge images. They will do so unless someone finds a smart way to compress their images efficiently and without loss. You accept the challenge. Warm up your brain [**(a)** and **(b)**], and save the world [**(c)** and **(d)**]!

(a) Consider the same image as in **1(a)** (image enhancement). What is the theoretical maximum compression without loss, if each pixel is coded separately? Note now that this image is a magic square: the sum of the elements in each row, or in each column, or in each diagonal is the same. Using such information, how much can you surely compress this image without loss? And why can you compress more than the theoretical maximum evaluated above? [2 points]

(b) Consider the same image as in **1(b)** (image enhancement). How much can you compress this image by Huffman coding each pixel separately? [2 points]

(c) In order to give you a chance to understand, the criminals send a 6-bit miniature image of size 4×4 by horizontal raster scanning:

18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63.

You deduce that the criminals like mathematics and are predictable. So find a smart way to compress this image efficiently and without loss! If the image had a larger size and obeyed the same mathematical rule, would your compression scheme be more efficient or not? Explain! [3 points]

(d) Finally, the criminals send a 7-bit miniature image of size 4×4:

0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120.

Same questions as in **(c)**. [3 points]

BRIEF ANSWERS TO ALESSANDRO'S QUESTIONS

1 (a)

15	1	2	12
4	10	9	7
8	6	5	11
3	13	14	0

4 bits/pixel

4 × 4 pixels

→ The histogram is already flat!

1 (b)

1	1	2	2
2	1	3	1
1	1	0	1
1	2	2	1

2 bits/pixel

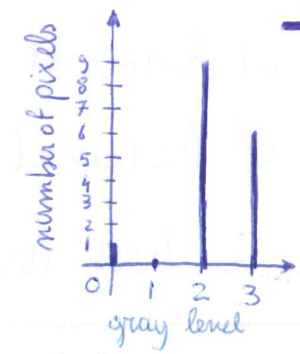
4 × 4 pixels

→

gray level	probability	cumulative probability	round to nearest multiple of 1/3	output gray level
0	1/16	1/16	0/3	0
1	9/16	10/16	2/3	2
2	5/16	15/16	3/3	3
3	1/16	16/16	3/3	3

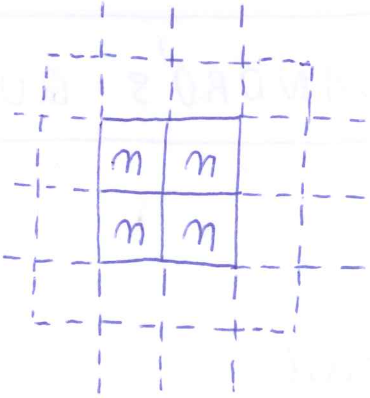
→ output:

2	2	3	3
3	2	3	2
2	2	0	2
2	3	3	2



→ The histogram is badly equalized because of discretization and the small number of bits and pixels.

1 (c)



each pixel has the same gray level n

N gray levels are possible $(2^{\# \text{bits}})$.

$0 \leqslant n \leqslant N-1$

n occurs with probability $= 1$

cumulative probability $= 1$

output gray level $= N-1$

➔ the output image is white.

1 (d)

$$\int_0^y q(y')\,dy' = \int_0^x p(x')\,dx'$$

as in histogram matching (equalization)

$$y = 1 - e^{-ax}$$

$$\Rightarrow x = -\frac{1}{a}\ln(1-y)$$

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2 (a) See Sect. 2.1 of Romeo et al. (2004) and Fig. 1 of Romeo et al. (2003).

2 (b) See Sect. 2.2 of Romeo et al. (2004) and Fig. 2 of Romeo et al. (2003) [see also Fig. 6 of Romeo et al. (2004)].

2 (c) See Sect. 3.1 of Romeo et al. (2004).

2 (d) See Sect. 3.2 of Romeo et al. (2004).

NOTE: The references above are linked from the lecture notes.

3 (a) Single-pixel entropy $H_1 = -\sum_{i=0}^{15} p_i \log_2 p_i = 4$ bits/pixel. (3)

Theoretical maximum compression ..... =

$$\frac{\#\text{ bits/pixel in the image}}{\text{single-pixel entropy}} = 1 \rightarrow \underline{\text{no}} \text{ compression!}$$

Magic square ☺

How many pixels do we really need to transmit?

Yes	Yes	Yes	Yes
Yes	Yes	Yes	No
Yes	Yes	Yes	No
No	No	No	No

10 pixels → compression = 1.6;

or, e.g.

Y	Y	Y	Y
Y	Y	Y	N
N	Y	Y	N
N	N	N	N

9 pixels → higher compression $\simeq 1.8$, but more difficult to transmit.

→ compression > theoretical maximum...

because pixels are correlated and we are reducing inter-pixel redundancy.

3 (b)

gray level	probability	Huffman code	length
1	9/16	0	1
2	5/16	1 0	2
0	1/16	1 1 0	3
3	1/16	1 1 1	3

Mean Huffman-code length $N_{mean} = \sum_{i=0}^{3} p_i N_i \simeq 1.6$ bits/pixel.

→ compression $\simeq 1.3$ (the original image has 2 bits/pixel).

3 (c)

18	21	24	27
30	33	36	39
42	45	48	51
54	57	60	63

6 bits/pixel

4 × 4 pixels

→ Lossless predictive coding: $\bar{f}_n = f_{n-1}$

$e_n = f_n - \bar{f}_n = f_n - f_{n-1}$

18	3	3	3
3	3	3	3
3	3	3	3
3	3	3	3

→ Run length coding: 18 (3, 15)

6 bits + 7 bits + 4 bits = 17 bits

original # bits = 96 bits

→ compression $\simeq 5.6$.

NOTE: minimum number of assumptions!

3 (e)
continued

Larger-size image, e.g.

18	21	24	27	30
33	36	39	42	45
48	51	54	57	60
63	66	69	72	75
78	81	84	87	90

7 bits/pixel
5 × 5 pixels

→ same coding as before : 18 (3, 24)

7 bits + 8 bits + 5 bits = 20 bits
original # bits = 175 bits

→ compression ≃ 8.8,
higher efficiency!

NOTE: a rigorous generalization is non-trivial!
But the trend is clear !!

3 (d)

0	1	3	6
10	15	21	28
36	45	55	66
78	91	105	120

7 bits/pixel

4 × 4 pixels

→ Lossless predictive coding: $\bar{f}_n = 2 f_{n-1} - f_{n-2}$ (gradient-based)

$e_n = f_n - \bar{f}_n = f_n - 2 f_{n-1} + f_{n-2}$

0	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

→ Run length coding: 0 1 (1, 14)

7 bits + 7 bits + 9 bits + 4 bits = 27 bits

original # bits = 112 bits

→ compression ≃ 4.1. (same note as in 3c).

Larger-size image, e.g.

0	1	3	6	10
15	21	28	36	45
55	66	78	91	105
120	136	153	171	190
210	231	253	276	300

9 bits/pixel

5 × 5 pixels

→ same coding as before: 0 1 (1, 23)

9 bits + 9 bits + 11 bits + 5 bits = 34 bits

original # bits = 225 bits

→ compression ≃ 6.6,

higher efficiency! (same note as in 3c).