Sannolikhet, statistik och risk MVE300
Written examination - 28th May 2013, 14 - 18.

Allowed aids: Mathematical or statistical tables. The tables of formulæ of the course. Any calculator (not PC). Dictionaries for translation. Jour: Igor Rychlik 0707405575.

1 Suppose that one has observed n independent lifetimes $t_1, \ldots, t_n$ of exponentially distributed random variable T, it means that $F_T(t) = 1 - e^{-t/a}$. Derive maximum likelihood estimate of the unknown paramter a. (10 p)

2 At a determination of the yield Z (unit: %) when oxidating ammonia in a converter, the following quantities are measured: X = the amount NH_3 in the reactant gas; Y = the amount NO in the product gas. The yield Z was calculated as

$$Z = Y(100/X - 1.25).$$

The following standard deviations have been found:

$$\sigma_X = 7,0 \cdot 10^{-2}, \quad \sigma_Y = 10 \cdot 10^{-2},$$

while expected values are $E[X] = 11.0$, $E[Y] = 13.5$. Suppose that X and Y are independent then calculate approximately the standard deviation of the yield. (10 p)

3 A new heating system has been installed. It is a modern system which can give a warning when reliability/efficiency is decreasing and then a major service is recommended. Suppose that service times form a stationary Poisson stream of events with an unknown intensity λ [year^{-1}]. Depending on quality of fuel and amount of needed energy, the intensity λ may vary. The dealer claims that on average, service is needed once in two years, i.e. $\lambda = 0.5$ year^{-1}.

(a) When ordering the system there is an option for a constant price of service, $c = 4000$ SEK. Since λ is intensity (frequency) of services needed, the average cost per year is $c\lambda$. What is the predicted yearly cost based on the information given by the dealer? (3 p)

(b) Suppose that you choose the option of constant service price and that service was needed once in the first 6 months of use. How does this information affect your predicted yearly service cost? (Hint: Use a suitable exponential prior, update and then compute $\mathsf{E}[c\Lambda]$.) (7 p).

4 In years 1987-2005 severe storms (significant wave height exceeding 8 meters) has been observed at some location in the North Atlantic. The number of severe storms years 1987 - 2005 are:

0 1 2 2 1 0 3 1 1 3 5 3 1 4 4 3 3 0 5

A person claims that the number of severe storms observed during a year is Poisson distributed with the same expected value. Use a suitable test to check this claim (hypothesis). (20 p)

5 Suppose that a number of small specimens of a material have been tested. The strength X of the specimens is well described by Weibull distribution with mean 100 MPa and coefficient of variation $R_X = 0.281$. Suppose that a component is composed of 100 of such specimens in series and has strength Y, i.e. $Y = \min(X_1, \dots, X_{100})$. Give the value of characteristic strength of components, i.e. the 0.9 quantile of Y. (20 p)

c	2.00	2.10	2.70	3.00	3.68	4.00	5.00	5.73	8.00
R_X	0.523	0.500	0.400	0.363	0.302	0.281	0.220	0.200	0.118
$\Gamma(1+1/c)$	0.866	0.886	0.889	0.893	0.902	0.906	0.918	0.926	0.942

6 Ekofisk is an oil field in the Norwegian sector of the North Sea discovered in 1969 and its time horizon for the operation is 2028. A challenging problem at Ekofisk has been the subsidence of the seabed. In 1986, deck structures of a number of platforms had to be elevated by 6 meters. When measured against the sea level at that time, it was 27 meters above the sea level. However, the subsidence today is 8.5 meters so the deck structure is now 18.5 meters above sea level. It is assumed that the annual maximum wave height, H (meters above sea level), is Gumbel distributed, with the estimated parameters $a^* = 1.95$ and $b^* = 5.2$.

(a) Are the estimates of yearly probability of wet deck, i.e. of waves reaching the platform deck structures, computed year 1986 the same as the one computed no, i.e. 2013)? (3 p)

(b) Estimate present yearly probability of wet deck, i.e. of waves reaching the platform deck structures in the next 12 months? (7 p)

(c) Assume that future subsidence of the seabed can be neglected. Estimate the probability of wet deck in years 2013-2028. (10 p)

Good luck!

Solutions: Written examination - 28 May 2013
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1 The probability density is $f(t) = a^{-1}\exp(-t/a)$ hence likelihood

$$L(a) = f(t_1)\cdot ... \cdot f(t_n) = a^{-n}\exp(-\sum t_i/a).$$

The loglikelihood function $l(a) = -n\ln(a) - \sum t_i/a$, $l'(a) = -n/a + \sum t_i/a^2$. ML estimate a^* solves $l'(a^*) = 0$ and hence $a^* = \sum t_i/n$ that is mean of the observations.

2 We will use Gauss' approximation. Introduce $g(x, y) = y(100/x - 1.25)$. Then

$$\frac{\partial g}{\partial x} = -\frac{100y}{x^2}, \quad \frac{\partial g}{\partial y} = \frac{100}{x} - 1.25$$

and Gauss' approximation gives

$$\mathsf{V}(Z) = \sigma_X^2\left(-\frac{100\mathsf{E}(Y)}{(\mathsf{E}(X))^2}\right)^2 + \sigma_Y^2\left(\frac{100}{\mathsf{E}(X)} - 1.25\right)^2 = 1.225.$$

It follows that $\mathsf{D}(Z) = 1.11$.

3 **(a)** The average cost is $\mathsf{E}[c\cdot X] = c\cdot\mathsf{E}[X] = c\cdot\lambda = 2000$ SEK/year.

(b) Since our information is that intensity of failure is in average 0.5 year^{-1} the conjugate gamma prior for Λ could be Gamma(α,β) with $\alpha = 1$ and $\beta = 2$. Because it is conjugate to the Poisson distribution, the posterior distribution of Λ is given by Gamma$(\alpha + x, \beta + t)$ =Gamma$(2, 2.5)$ hence using the mean value of Gamma distribution gives $\mathsf{E}[c\Lambda] = 4000\frac{2}{2.5} = 16000/5 = 3200$ SEK.

4 χ^2-test, 4-classes $N \le 1$, $N = 2$, $N = 3$, $N = 4$, $N \ge 4$, $\lambda^* = 2.211$, $p_i^* = (0.352,$ $0.2679, 0.1974, 0.1827)$, $n_i = (8, 2, 5, 4)$, $n = 19 \Rightarrow Q = 2.63 < \chi^2_{0.05}(4-1-1) = 5.99$, Do not reject.

5 Using the table we find that the shape parameter $c = 4$ and $\Gamma(1+1/c) = 0.906$. Since $E[X] = a\Gamma(1 + 1/c)$ we get that $a = 90.5$ MPa.

Let $y_{0.9}$ be the characteristic strengh of Y, then $P(Y > y_{0.9}) = 0.9$. Now $P(X > x) = e^{-(x/a)^c}$ hence $P(Y > y) = [e^{-(y/a)^c}]^n$. Hence $e^{-n(y_{0.9}/a)^c} = 0.9$ giving $y_{0.9} = 90.5 * [-\ln(0.9)/100]^{1/4} = 16.3$ MPa.

6 **(a)** No.

(b) Present yearly probability of wet deck is

$$P(H > 18.5) = 1 - e^{-e^{-(18.5-5.2)/1.95}} = 0.0011 \approx 1/1000.$$

(c) Probability of wet deck in years 2013-2028 is

$$P(H > 18.5) = 1 - e^{-e^{-(18.5-5.2-1.95*\ln(2028-2013+1))/1.95}} = 0.0174,$$

or $1 - (1 - P(H > 18.5))^{16} = 0.0175$ (ups numerical errors).