MATEMATIK
GU, Chalmers
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Datum: 2014-01-14 Tid: 8:30
Inga hjälpmedel. Mobiltelefoner är förbjudna.
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FOR STUDENTS WHO STUDIED IN YEAR 2013.

Tenta i ODE och matematisk modellering, MMG511, MVE160.

Answer first those questions that look simpler, then take more complicated ones etc.
Good luck!

1. Define what is a generalized eigenspace of a matrix. Formulate and proof the theorem on the stability of solutions to a linear system of ODEs $\frac{d\overrightarrow{r}(t)}{dt} = A\overrightarrow{r}(t)$ with constant matrix A using generalized egenspaces of the matrix A. (4p)

2. Formulate and proof the theorem on the extensibility of solutions for ODEs $\frac{d\overrightarrow{r}(t)}{dt} = \overrightarrow{F}(\overrightarrow{r}, t)$ with at most linear growth of the right hand side $\overrightarrow{F}(\overrightarrow{r}, t)$ with respect to $\overrightarrow{r}$. (4p)

3. Consider the following system of ODE:

$$\frac{d\overrightarrow{r}(t)}{dt} = A\overrightarrow{r}(t), \text{ with a constant matrix } A = \begin{bmatrix} -2 & 4 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

   Find all those initial vectors $\overrightarrow{r_0} = \overrightarrow{r}(0)$ that give unbounded solutions to the system. (4p)

4. Consider the following linear ODE with periodic coefficients. Find its monodromy matrix and Floquet exponents and investigate stability of its solutions.

$$\frac{d\overrightarrow{r}(t)}{dt} = A(t)\,\overrightarrow{r}(t), \text{ with matrix } A(t) = \left(\sin^2(t) - \tfrac{3}{4}\right)\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}. \quad (4p)$$

5. Consider the following system of ODE

$$\begin{cases} x' = y \\ y' = -y - 6x - 3x^2 \end{cases}$$

   Find a Lyapunov function, show that the origin is asymptoticaly stable and find its attracting region. (4p)

6. Show that the following system of ODE has no periodic solutions.

$$\begin{cases} x' = \frac{1}{7} + x^2 - yx + y^2 \\ y' = -\frac{1}{5} - y^2 \end{cases} \quad (4p)$$

   Max. 24 points;

   Thresholding for marks: for GU: VG: 19 points; G: 12 points. For Chalmers: **5**: 21 points; **4**: 17 points; **3**: 12 points;

One must pass both home assignments and the exam to pass the course.
Total points for the course will be the average of the points for the home assignments (30%) and for this exam (70%).
The same thresholding is valid for the exam, for the home assignments, and for the total points.