

### Tränings tenta i matematisk modellering, MMG511, MVE160

The examination includes two theorems with proofs and four problems of both theoretical and calculational character.

1. Formulate what is a generalised eigenspace of a matrix. Formulate and prove the theorem describing conditions implying that a solution to a linear autonomous system of ODE will tend to zero with time going to infinity or will be bounded for all times. (4p)
2. Formulate and give a proof for Liapunovs theorem on stability of a fixed point and give definitions of the notions used in the formulation. (4p)

#### 3. Linear systems

Consider the following ODE:

$$\frac{d\vec{r}}{dt} = A\vec{r}(t), \quad \vec{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \end{bmatrix} \text{ with a constant matrix } A \text{ defined as } A = 2I + C =$$
$$-2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Calculate  $\exp(At)$  and the general solution for the ODE. (4p)

4. Show that a solutions to the following system of equations exist on an infinite time interval.

$$\begin{aligned} x' &= \sin(xy) + 2x/(1+x^2) \\ y' &= \ln(1+x^2) + 3y \end{aligned} \quad (4p)$$

5. Consider the following system of ODE and investigate stability of the fixed point in the origin. (4p)

$$\begin{cases} x' = x^3 + 2xy^2 \\ y' = x^2y \end{cases}$$

6. Show that the following system of ODE has a periodic solution. (4p)

$$\begin{cases} x' = y \\ y' = -x + y(1 - 3x^2 - 2y^2) \end{cases}$$

Max. 24 points;

For GU: **VG**: 19 points; **G**: 12 points. For Chalmers: **5**: 21 points; **4**: 17 points; **3**: 12 points;

One must pass both the home assignments and the exam to pass the course.

Total points for the course will be the average of points for the home assignments (30%) and for this exam (70%)