INGA LÖSNINGAR ERHÅLLNA!

MATEMATIK
GU, Chalmers
A.Heintz

Datum: 2012-01-10
Hjälpmedel: Inga
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Tenta i matematisk modellering, MMG510, MVE160

Problems from the topics for which a student has got bonus points should not be solved at the examination.

1. **Lyapunov functions and stability of stationary points.**

   a) Formulate a criteria for asymptotically stable stationary point using a Lyapunov function that is not a stong Lyapunov function. (2p)

   b) Consider the system of ODE: $\begin{aligned} x' &= y \\ y' &= -y + y^3 - x^5 \end{aligned}$

   Use Lyapunovs theory for the given equation and investigate if the stationary point in the origin is asymptotically stable. (2p)

2. **Periodical solutions to ODE.**

   Formulate Bendixsons theorem on nonexistence of periodic solutions in the plane. Proof this theorem using Greens formula.

3. **Limit cycles**

   Formulate the Poincare - Bendixsons sats and use it to show that the following system of ODE has a limit cycle in a "ring shaped" domain around the origin.

   $x' = x^3 + y - x^3(x^2 + 4y^2)$;

   $y' = -x + y^3 - y^3(x^2 + 4y^2)$ (4p)

   Hint. Combine equations to get an expression for the derivative $\left(x^2 + y^2\right)'$ and use its properties.

4. **Chemical reactions and the Gillespie method**

   Consider the following reactions: $X + X \underset{c_2}{\overset{c_1}{\rightleftarrows}} W$, $W + X \underset{c_4}{\overset{c_3}{\rightleftarrows}} P$ where $c_i\,dt$ is the probability that during time dt the reaction with index i will take place. $i = 1, 2, 3, 4$.

   a) Write differential equations for the number of particles for these reactions. (2p)

   b) Give formulas for Gillespies algorithm that would model stochastically these reactions by Gillespies method. (2p)

Max. 16 points;
For GU: VG: 13 points; G: 8 points;
For Chalmers: 5: 14 points; 4: 11 points; 3: 8 points;