MATEMATIK
GU, Chalmers
A.Heintz

Datum: 2011-05-25
Hjälpmedel: Beta
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Tenta i matematisk modellering, MMG510, MVE160

1. **Linear systems.**

   Consider the following ODE:

   $\frac{d\vec{r}(t)}{dt} = A\vec{r}(t)$, $\vec{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix}$ with $A = \begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix}$,

   Find the evolution operator for this system. **(2p)**

   Find which type has the stationary point at the origin and give a possibly exact sketch of the phase portrait. **(2p)**

2. **Ljapunovs functions and stability of fixed points.**

   Consider the system of equations: $\begin{cases} x' = -x + 2xy^2 \\ y' = -(1-x^2)y^3 \end{cases}$

   Investigate stability of the fixed point in the origin.

3. **Periodic solutions to ODE.**

   Show that the system of equations

   $\begin{cases} x' = \sin(y) + x(y^2+1) \\ y' = (x-1)^2 + x^2 y \end{cases}$

   does not have periodic solutions. **(4p)**

4. **Hopf bifurcation.**

   Explain the notion Hopf bifurcation.

   Show that the system $\begin{cases} x' = \mu x + y - x^3\cos(x) \\ y' = -x + \mu^2 y \end{cases}$

   has a Hopf bifurcation at $\mu = 0$. **(4p)**

5. **Chemical reactions by Gillespies method**

   Consider the following reactions: $X + Z \underset{c_2}{\overset{c_1}{\rightleftarrows}} W$, $W + Z \underset{c_4}{\overset{c_3}{\rightleftarrows}} P$ where $c_i\,dt$ is the probability that during time dt the reaction with index i will take place $i = 1, 2, 3, 4$.

   a) Write down differential equations for the number of particles for these reactions. **(2p)**

   b) Give formulas for the algorithm that models these reactions stochastically by Gillespies method. **(2p)**

   Max. 20 points;

For GU: VG: 15 points; G: 10 points. For Chalmers: **5:** 17 points; **4:** 14 points; **3:** 10 points;
Total points for the course will be an average of points for the project (60%) and for this exam together with bonus points for home assignments(40%).