MATEMATIK
GU, Chalmers
A.Heintz

Datum: 2010-08-18
Hjälpmedel: Inga
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Tid: 8:30 - 13.30
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Tenta i matematisk modellering, MMG510, MVE160

1. **Linear systems.**

   Consider the following ODE:

   $$\frac{d\vec{r}(t)}{dt} = A\vec{r}(t),\ \vec{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} \text{ with } A = \begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix},$$

   Find the evolution operator for this system. **(2p)**

   Find which type has the stationary point at the origin and give a possibly exact sketch of the phase portrait. **(2p)**

2. **Lyapunovs functions and stability of stationary points.**

   Consider the system of equations: $\begin{cases} x' = -x + 2xy^2 \\ y' = -x^2y^3 \end{cases}$

   Show that $V(x,y) = x^2 + y^2$ is a weak Lyapunov function at the origin. **(2p)**

   Find if the origin is an asymptotically stable stationary point. **(2p)**

3. **Periodical solutions to ODE.**

   Use Poincare - Bendixsons theorem to show that the system of equations.

   $$\begin{cases} x' = 1 - xy \\ y' = x \end{cases}$$

   does not have periodical solutions. **(4p)**

4. **Hopf bifurcation.**

   Show that the system $\begin{cases} x' = \mu x + y - x^3\cos(x) \\ y' = -x + \mu y \end{cases}$

   has a Hopf bifurcation for $\mu = 0$. Explain what means Hopf bifurcation. **(4p)**

5. **Chemical reactions by Gillespies method**

   Consider the following reactions: $X + Z \underset{c_2}{\overset{c_1}{\rightleftarrows}} W$, $\quad W + Z \underset{c_4}{\overset{c_3}{\rightleftarrows}} P$ where $c_i\,dt$ is the probability that during time dt the reaction with index i will take place $i = 1,2,3,4$.

   a) Write down differential equations for the number of particles for these reactions. **(2p)**

   b) Give formulas for the algorithm that shell model these reactions stochastically by Gillespies method. **(2p)**

   Max. 20 points;

For GU: **VG:** 15 points; **G:** 10 points. For Chalmers: **5:** 17 points; **4:** 14 points; **3:** 10 points;
Total points for the course will be an average of points for the project (60%) and for this exam (40%).