1. Linear systems.
    Consider the following ODE:
    \[ \frac{d\vec{v}(t)}{dt} = A\vec{v}(t), \quad \vec{v}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} \] with \( A = \begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix} \).
    Find the evolution operator for this system.
    Find which type has the stationary point at the origin and give a possibly exact sketch of the phase portrait. (2p)

2. Lyapunov’s functions and stability of stationary points.
    Consider the system of equations:
    \[ \begin{cases} x' = -x + 2xy^2 \\ y' = -x^2y^3 \end{cases} \]
    Show that \( V(x, y) = x^2 + y^2 \) is a weak Lyapunov function at the origin.
    Find if the origin is an asymptotically stable stationary point. (2p)

3. Periodical solutions to ODE.
    Use Poincaré - Bendixson’s theorem to show that the system of equations:
    \[ \begin{cases} x' = 1 - xy \\ y' = x \end{cases} \]
    does not have periodical solutions. (4p)

    Show that the system
    \[ \begin{cases} x' = \mu x + y - x^3 \cos(x) \\ y' = -x + \mu y \end{cases} \]
    has a Hopf bifurcation for \( \mu = 0 \). Explain what means Hopf bifurcation. (4p)

5. Chemical reactions by Gillespies method
    Consider the following reactions:
    \[ X + Z \rightarrow W, \quad W + Z \rightarrow P \] where \( c_i dt \) is the probability that during time \( dt \) the reaction with index \( i \) will take place \( i = 1, 2, 3, 4 \).
    a) Write down differential equations for the number of particles for these reactions. (2p)
    b) Give formulas for the algorithm that shell model these reactions stochastically by Gillespies method. (2p)

Max. 20 points;

For GU: VG: 15 points; G: 10 points. For Chalmers: 5: 17 points; 4: 14 points; 3: 10 points; Total points for the course will be an average of points for the project (60%) and for this exam (40%).