

Tenta i matematisk modellering, MMG510, MVE160

You do not need to solve the last problem if you have got bonus points for the Gillespie method.

1. Linear systems

Consider the following ODE:

$$\frac{d\vec{r}(t)}{dt} = A\vec{r}(t), \vec{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} \text{ with } A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}, \text{ eigenvectors:}$$

Find the evolution operator for this system. (2p)

Find which type has the stationary point at the origin and give a possibly exact sketch of the phase portrait. (2p)

2. Lyapunovs functions and stability of stationary points.

Formulate the criterion for asymptotic stability of a stationary point of an ODE using only a weak Lyapunov function.

Consider the system of equations:
$$\begin{cases} x' = -x + y^2 \\ y' = -xy - x^2 \end{cases}$$

Show that $V(x, y) = x^2 + y^2$ is a weak Lyapunov function and decide if the stationary point at the origin is asymptotically stable. (4p)

3. Periodical solutions to ODE.

Formulate the Poincare - Bendixson theorem. Use Poincare - Bendixsons theorem to show that the system of equations

$$\begin{cases} x' = -y + x(1 - x^2 - y^4) \\ y' = x + y(1 - x^2 - y^4) \end{cases}$$

has at least one periodical solution.

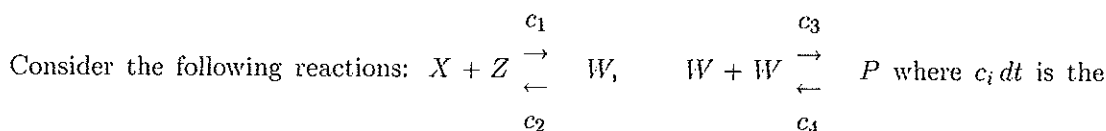
Hint. Use polar coordinates and write down an equation for r . (4p)

4. Hopf bifurcation.

Show that the system
$$\begin{cases} x' = y - x^3 \\ y' = -x + \mu y - x^2 y \end{cases}$$

has a Hopf bifurcation for $\mu = 0$ and explain what does it mean. (4p)

5. Chemical reactions by Gillespies method



probability that during time dt the reaction with index i will take place $i = 1, 2, 3, 4$.

a) Write down differential equations for the number of particles for these reactions. (2p)

b) Give formulas for the algorithm that shall model these reactions stochastically by Gillespies method. (2p)

Max. 20 points;

For GU: VG: 15 points; G: 10 points. For Chalmers: 5: 17 points; 4: 14 points; 3: 10 points; Total points for the course will be an average of points for the project (60%) and for this exam (40%).