MATEMATIK Datum: 2010-05-26 Tid: 8:30–13.30
GU, Chalmers Hjälpmedel: Inga
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Tenta i matematisk modellering, MMG510, MVE160

You do not need to solve the last problem if you have got bonus points for the Gillespie method.

1. **Linear systems**

   Consider the following ODE:

   $$\frac{d\overrightarrow{r}(t)}{dt} = A\overrightarrow{r}(t),\ \overrightarrow{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} \text{ with } A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}, \text{ eigenvectors:}$$

   Find the evolution operator for this system. **(2p)**

   Find which type has the stationary point at the origin and give a possibly exact sketch of the phase portrait. **(2p)**

2. **Lyapunovs functions and stability of stationary points.**

   Formulate the criterion for asymptotic stability of a stationary point of an ODE using only a weak Lyapunov function.

   Consider the system of equations: $\begin{cases} x' = -x + y^2 \\ y' = -xy - x^2 \end{cases}$

   Show that $V(x,y) = x^2 + y^2$ is a weak Lyapunov function and decide if the stationary point at the origin is asymptotically stable. **(4p)**

3. **Periodical solutions to ODE.**

   Formulate the Poincare - Bendixson theorem. Use Poincare - Bendixsons theorem to show that the system of equations

   $$\begin{cases} x' = -y + x(1 - x^2 - y^4) \\ y' = x + y(1 - x^2 - y^4) \end{cases}$$

   has at least one periodical solution.

   Hint. Use polar coordinates and write down an equation for r. **(4p)**

4. **Hopf bifurcation.**

   Show that the system $\begin{cases} x' = y - x^3 \\ y' = -x + \mu y - x^2 y \end{cases}$

   has a Hopf bifurcation for $\mu = 0$ and explain what does it mean. **(4p)**

5. **Chemical reactions by Gillespies method**

   Consider the following reactions: $X + Z \underset{c_2}{\overset{c_1}{\rightleftarrows}} W,\quad W + W \underset{c_4}{\overset{c_3}{\rightleftarrows}} P$ where $c_i\,dt$ is the probability that during time dt the reaction with index i will take place $i = 1, 2, 3, 4$.

   a) Write down differential equations for the number of particles for these reactions. **(2p)**

   b) Give formulas for the algorithm that shell model these reactions stochastically by Gillespies method. **(2p)**

   Max. 20 points;

For GU: **VG:** 15 points; **G:** 10 points. For Chalmers: **5:** 17 points; **4:** 14 points; **3:** 10 points; Total points for the course will be an average of points for the project (60%) and for this exam (40%).