MATEMATIK
GU, Chalmers
A.Heintz

Datum: 2009-05-25
Hjälpmedel: Inga
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Tid: 8:30 – 13.30

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Tenta i matematisk modellering, MMG510, MVE160

OBS! You do not need to solve problems for which you have got bonus points.

1. **Lyapunovs functions and stability of stationary points.**

   a) Formulate the criterium for asymptotic stability of a stationary point of an ODE using only a weak Lyapunovs function. **(2p)**

   b) Consider the system of equations: $$\begin{aligned} x' &= -y/3 - x(3x^2 + y^2) \\ y' &= x - y(3x^2 + y^2) \end{aligned}$$

   Find a strong Lyapunovs function $V(x, y)$ and show that the stationary point in the origin is asymptotically stable. **(2p)**

   Hint. Use $V(x, y)$ in the form $V(x, y) = ax^2 + y^2$ and choose parameter a so that $V(x, y)$ will be a strong Lyapunovs function.

2. **Periodical solutions to ODE.**

   Formulate the Poincare - Bendixson theorem. Use Poincare - Bendixsons theorem to show that the system of equations $x' = -4y + x(1 - x^2 - y^2)$; $y' = 4x + y(1 - x^2 - y^2)$ has at least one periodical solution in some ring : $\rho^2 < x^2 + y^2 < R^2$.

   Hint. Use polar coordinates and write down an equation for r. **(4p)**

3. **Asymptotic methods for ODE.**

   a) consider the idea of the direct asymptotic expansion in ODE. Which terms are called by secular terms in an expansion? Consider the equation: $u'' + u = \epsilon(1 - u^2)u'$. Find terms of order zero and one in a direct asymptotic expansion of solutions to the equation for small ε. **(2p)**

   b) Describe the idea of the averaging method to get an expansion without secular terms and make first steps of the method for the given equation in part a). **(2p)**

   You do not need to give a complete solution , just describe the ideas and formulate the equations for main steps of the method.

   Useful trigonometric formulas are at the backside.

4. **Chemical reactions by Gillespies method**

   Consider the following reactions: $X + Z \underset{c_2}{\overset{c_1}{\rightleftarrows}} W, \quad W \overset{c_3}{\longrightarrow} P$ where $c_i\,dt$ is the probability that during time dt the reaction with index i will take place $i = 1, 2, 3$.

   a) Write down differential equations for the number of particles for these reactions. **(2p)**

   b) Give formulas for the algorithm that shell model these reactions stochastically by Gillespies method. **(2p)**

   Max. 16 points;

For GU: **VG:** 13 points; **G:** 8 points;
For Chalmers: **5:** 14 points; **4:** 11 points; **3:** 8 points;

Turn !!