Foundations of Probability Theory (MVE140 – MSA150)

Friday 6th of April 2018 examination questions

You are allowed to use a dictionary (to and from English) and up to a maximum of 2 double-sided pages of your own written notes. This examination has five problems with a maximum of 20 credit points for a fully satisfactory solution, so the maximal total is 100 credit points. To pass the course, you need to score at least 40 points.

Examination Questions

1. m men and w women seat themselves at random in $m+w$ seats around an oval table. Find the probability that all the women (and hence all the men) will be adjacent.

2. Two points p and q are chosen randomly and uniformly from $[0, 1]$. What is probability that both roots of the equation $x^2 + px + q = 0$ are

   a. real;

   b. positive.

3. Proportion p in a large population suffers from a disease, a person can be identified as a carrier by the presence of a particular pathogen in the blood. To identify the carriers, the following two-stage procedure is applied. The blood of k people is mixed and the mixture is then analysed for presence of the pathogen. If the result is negative, it means that all k people are free from the disease. Contrarily, if the test is positive, the blood of each of the k people is then tested. So it then takes $k + 1$ tests to identify all infected people.

   a. Find the expectation of the required number of tests.

   b. Knowing p, for which value of k the minimum of the expectation is attained?

4. Random variables $\xi_1, \dots, x_n$ are independent and normally distributed with mean a and variance σ^2. Find the joint (two-dimensional) distribution of the vector

$$S_m = \sum_{k=1}^{m} \xi_k, \; S_n = \sum_{k=1}^{n} \xi_k, \; m < n.$$

5. Let $\xi_1, \xi_2, \dots$ are independent exponentially distributed random variables with parameter 1 and $\zeta_n = \max\{\xi_1, \dots, \xi_n\}$.

   a. Find the limiting distribution of $n^{-\gamma}\zeta_n$ for $\gamma > 0$.

   b. For which sequence $\{a_n\}$, the limit of $\zeta_n - a_n$ has a non-degenerate limiting distribution?