Foundations of Probability Theory (MVE140 – MSA150)

Thursday 16th of April 2015 resit examination questions

You are allowed to use a dictionary (to and from English) and up to a maximum of 2 doublesided pages of your own written notes. This examination has five problems with a maximum of 20 credit points for a fully satisfactory solution, so the maximal total is 100 credit points. To pass the course, you need to score at least 40 points. The examiner, Prof. Sergei Zuyev, is available at the examination site around 14:30 and 16:00. Telephone: 031 772 3020.

Examination Questions

1. Let A and B be certain events and C be an impossible event.

   - Show that event 'A or B holds' is also certain.
   - Show that the probability that A and B are observed at the same time is 1.
   - Show that the probability that C happens at least once in the infinite series of independent repititions of the probability experiment is 0.

2. m men and w women seat themselves at random in $m+w$ seats arranged in a row. Find the probability that all the women will be adjacent.

3. Jack cuts off a piece from 1m length wire at an arbitrary place and takes it with him. The piece left is picked up by John who cuts off a piece from it at arbitrary place and throws away the remainder.

   (a) Find the distribution of the length of wire John has.

   (b) Find the covariance between the lengths of wires Jack and John have.

   (c) Find the joint distribution of these lengths.

   (d) Are these lengths independent?

4. Let ξ be Exponentially Exp(1) distributed random variable and m be its expectation. Compute for an $a > m$ the probability $\mathbf{P}\{|\xi - m| > a\}$ and compare this to the Chernoff bound.

5. A random variable ζ is said to have a centred *Cauchy* distribution with parameter $\theta > 0$, is its p.d.f. is

$$f_\zeta(x) = \frac{\theta}{\pi(\theta^2 + x^2)}, \quad x \in \mathbb{R}.$$

   (a) Show that the corresponding characteristic function $\chi_\zeta(t)$ s real.

   (b) By differentiatin twice the corresponding integral or otherwise, show that $\chi_\zeta(t) = e^{-\theta|t|}$, $t \in \mathbb{R}$.

   (c) Deduce from here the *stability property* of the Cauchy distribution: the sum of two indepedenent Cauchy distributed r.v.'s with parameters θ_1 and θ_2 is again Cauchy-distributed with parameter $\theta_1 + \theta_2$.