Exam for the course Options and Mathematics (CTH[MVE095], GU[MMG810]) 2022/23

For questions call the examiner at +46 (0)31 772 35 70

August the 28d, 2024 (8.30-12.30)

REMARKS: (1) NO aids permitted (2) Write as clear as possible: if some step is not clearly readable it will be assumed to be wrong

Part I

1. Show that in binomial 1-period model a self-financing arbitrage portfolio exists if and only if $r \notin (d, u)$. (3 p.)

2. Derive the Black-Scholes price $C(t, S(t), K, T)$ at time t of the European call option with strike price $K > 0$ and maturity $T > 0$.(3 p.)

3. Define and explain use in the course of geometric Brownian motion. (3 p.)

4. Name at least two reasons to buy options on a stock?

   (3 p.)

Part II

1. Let $S(t) > 0$ be the price at time t of a non-dividend paying stock.

$$f(S) = \begin{cases} 100, & \text{for } x < 100; \\ 0, & \text{for } 100 \le x \le 200; \\ x - 200, & \text{for } x > 200; \end{cases}$$

(a) Find a constant portfolio $\mathcal{A}$ of European calls on a stock and, if applicable, the stock itself and cash, which at time T has the value $V(\mathcal{A}) = f(S(T))$. (3 p.)

(b) Find a constant portfolio $\mathcal{A}$ of European puts on a stock and, if applicable, the stock itself and cash, which at time T has the value $V(\mathcal{A}) = f(S(T))$. (2 p.)

(c) If annual rate of return on zero coupon 2 years maturity bond issued now, is 12%. The face of the bond is 14400kr, what is the current price of the bond. (1 p.)

2. Given is a stock of current value 900kr, we use to price it a binomial model in which at every time interval the stock has log-return $\log \frac{4}{3}$ with probability 1/3 and log-return $\log \frac{2}{3}$ with probability 2/3. We assume that a risk-free zero coupon bond used in the model has a constant price.

   (a) Find the price at time $t = 0$ of a European call with strike 600, which matures in 2 time intervals. (2 p.)

   (b) Find the price at time $t = 0$ of a American put with strike 700, which matures in 2 time intervals. (2 p.)

   (c) Find the price at time $t = 0$ of an Asian put which matures in 2 time intervals and can be executed only at time 2 with the pay-off $Y = (900 - \frac{S(1)+S(2)}{2})_+$. (2 p.)

3. Given a stock of current value 1000kr and constant log-volatility $\sigma = 9$, which is supposed to pay 3% of is market value as dividends in 3 years time (at $t_d = 3$) use Black-Scholes model to set prices on following derivatives based on the stock. The interest rate on the risk-free asset is 0.5. (Observe that you are allowed to give an answer as an expression combining elementary functions, Φ-function and numbers without evaluating it. If the answer is an integral, you should evaluate the integral in order to obtain full points.)

   (a) A European call with strike 800kr and maturity in 1 year. (2 p.)

   (b) A standard Europian derivative with pay-off $Y = (S(T))^3$ and maturity in 2 years. (2 p.)

   (c) A European put with strike 1300kr and maturity in 5 years. (2 p.)