Exam for the course Options and Mathematics (CTH[MVE095], GU[MMG810]) 2023/24

For questions call the examiner at +46 (0)31 772 35 70

April the 4th, 2024 (8.30-12.30)

REMARKS: (1) NO aids permitted (2) Write as clear as possible: if some step is not clearly readable it will be assumed to be wrong

Part I

1. Formulate and prove put-call parity under the arbitrage-free principle assumption (Theorem 2.3). (3 p.)

2. Under which condition on parameters of the binomial model a self-financing arbitrage portfolio can exist? Formulate and prove (for one-period model) (Theorem 3.23).(3 p.)

3. Define self-financing portfolio process in binomial model. (Definition 3.13). (3 p.)

4. Consider small investor acting through a stock broker vs big investor trading directly on a stock exchange buying/selling a frequently traded stock which doesn't pay dividends. Which conditions of the frictionless market are likely to be satisfied for one and for the other? Motivate your answer. (3 p.)

Part II

1. Let $S(t) > 0$ be the price at time t of a non-dividend paying stock.

$$f(S) = \begin{cases} 200, & \text{for } x < 100; \\ 100 + x, & \text{for } 100 \leq x \leq 200; \\ 300, & \text{for } x > 200; \end{cases}$$

   (a) Find a constant portfolio $\mathcal{A}$ of European calls on a stock and, if applicable, the stock itself and cash/bonds, which at time T has the value $V(\mathcal{A}) = f(S(T))$. (3 p.)

   (b) Find a constant portfolio $\mathcal{A}$ of European puts and calls on a stock and, if applicable, the stock itself (obs! no cash/bonds are allowed), which at time T has the value $V(\mathcal{A}) = f(S(T))$. (2 p.)

   (c) If annualized rate of return on zero coupon 2 years maturity bond, is 5%. The current price of the bond is 1000kr, what is the face value of the bond. (1 p.)

2. Given is a stock of current value 900kr, we use to price it a binomial model in which at every time interval the stock has log-return rate $\log \frac{6}{3}$ with probability 2/3 and log-return rate $\log \frac{2}{3}$ with probability 1/3. We assume that a risk-free zero coupon bond used in the model has a log-return rate $\log \frac{4}{3}$.

   (a) Find the price at time $t = 0$ of a European put with strike 800, which matures in 2 time intervals. (2 p.)

   (b) Find the price at time $t = 0$ of a American put with strike 800, which matures in 2 time intervals. (2 p.)

   (c) Find the price at time $t = 0$ of an Asian call which matures in 2 time intervals and can be executed only at time 2 with the pay-off $Y = (800 - \frac{S(1)+S(2)}{2})_+$. (2 p.)

3. Given a stock of current value 1200kr and constant log-volatility $\sigma = 3$, which is supposed to pay 10% of is market value as dividends in 3 years time (at $t_d = 3$) use Black-Scholes model to set prices on following derivatives based on the stock. The interest rate on the risk-free asset

is 0.02. (Observe that you are allowed to give an answer as an expression combining elementary functions, Φ-function and numbers without evaluating it.)

(a) A European put with strike 1100kr and maturity in 2 years. (2 p.)

(b) A standard Europian derivative with pay-off $Y(S(T)) = S(T)^3$ and maturity in 1 year. (2 p.)

(c) A European call with strike 1100kr and maturity in 4 years.(2 p.)