Exam for the course Options and Mathematics (CTH[MVE095], GU[MMG810]) 2023/24

For questions call the examiner at +46 (0)31 772 35 70

January the 13th, 2024 (8.30-12.30)

REMARKS: (1) NO aids permitted (2) Write as clear as possible: if some step is not clearly readable it will be assumed to be wrong

Part I

1. Find density function of the geometric Brownian motion (Theorem 8.20). (3 p.)

2. State and derive formula for Black-Scholes pricing of an European option with a pay-off before the time of maturity (Theorem 8.34).(3 p.)

3. Define and explain Black-Scholes price of European derivatives at tome $t = 0$. (Definition 8.23). (3 p.)

4. Consider following two crisis situations:

   A) World War III has started by several states attacking several other states at the same time. The stock exchanges around the world are still open, but a panic has set off and many private investors want to trade their stocks before the stock exchanges will be closed.

   B) A new pandemic was declared and everybody suppose to isolate. Temporary vacation was given to most of people and the prices on essential goods are slowly rising. Stock exchanges are open and operate on-line 24 hours a day.

In which situation the arbitrage-free condition is more likely to be satisfied? Motivate your answer. (3 p.)

Part II

1. Let $S(t) > 0$ be the price at time t of a non-dividend paying stock.

$$f(S) = \begin{cases} 200, & \text{for } x < 100; \\ 300 - x, & \text{for } 100 \leq x \leq 200; \\ 100, & \text{for } x > 200; \end{cases}$$

   (a) Find a constant portfolio $\mathcal{A}$ of European calls on a stock and, if applicable, the stock itself and cash/bonds, which at time T has the value $V(\mathcal{A}) = f(S(T))$. (3 p.)

   (b) Find a constant portfolio $\mathcal{A}$ of European puts and calls on a stock and, if applicable, the stock itself (obs! no cash/bonds are allowed), which at time T has the value $V(\mathcal{A}) = f(S(T))$. (2 p.)

   (c) If annual rate of return on zero coupon 2 years maturity bond, is 5%. The face of the bond is 2205kr, what is the current price of the bond. (1 p.)

2. Given is a stock of current value 800kr, we use to price it a binomial model in which at every time interval the stock has log-return rate $\log \frac{7}{4}$ with probability 2/3 and log-return rate $\log \frac{3}{4}$ with probability 1/3. We assume that a risk-free zero coupon bond used in the model has a log-return rate $\log \frac{5}{4}$.

   (a) Find the price at time $t = 0$ of a European call with strike 1450, which matures in 2 time intervals. (2 p.)

   (b) Find the price at time $t = 0$ of a American put with strike 800, which matures in 2 time intervals. (2 p.)

   (c) Find the price at time $t = 0$ of an Asian put which matures in 2 time intervals and can be executed only at time 2 with the pay-off $Y = (1025 - \frac{S(1)+S(2)}{2})_+$. (2 p.)

3. Given a stock of current value 1500kr and constant log-volatility $\sigma = 2$, which is supposed to pay 5% of is market value as dividends in 3 years

time (at $t_d = 3$) use Black-Scholes model to set prices on following derivatives based on the stock. The interest rate on the risk-free asset is 0.03. (Observe that you are allowed to give an answer as an expression combining elementary functions, Φ-function and numbers without evaluating it.)

(a) A European call with strike 1400kr and maturity in 2 years. (2 p.)

(b) A standard Europian derivative with pay-off $Y(S(T)) = S(T)^2 + S(T)$ and maturity in 1 year. (2 p.)

(c) A European put with strike 1400kr and maturity in 4 years.(2 p.)