Solutions to June exam 2022

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1

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1. I didn't get an anonymous code, unlucky me!
2. Favorite topic? Maybe everything I would have to say.
3. Least favorite? When I make some stupid mistake or typo that then causes students to be confused. I hate it when that happens - although - it can actually be a rewarding learning experience for you. I also struggle with conveying the definition of regular SLP, because it is just so long and involves so many ingredients. I wish there were an easier way to explain it. Lots of people dislike Bessel functions, they find them complicated and difficult to understand. Even my colleagues. I wish I had some magic way to explain them to make them less complicated (but honestly they are pretty complicated by nature - I even have a research article under review right now about them!!)
4. First thing to solve this problem: separate variables!
5. Nope, this is not a regular SLP. The problem is the function that is multiplying $\lambda f(x)$. The problem has
$$f''(x) + \lambda \sin(x) f(x) = 0, \quad x \in (0, 4).$$
The function that multiplies $f(x)$ is supposed to be positive. However, for $x \in (\pi, 4)$ the function $\sin(x) < 0$. So that makes it *not* a regular SLP.
6. We begin by separating variables because the equation is homogeneous:
$$u_{tt} = u_{xx},$$
so we start with a function of the form $T(t)X(x)$ and put it into the PDE:
$$T''X = X''T \text{ divide by } XT \implies \frac{T''}{T} = \frac{X''}{X}.$$
Since the two sides depend on different variables, they are both constant. Let's call it λ. Which side do we solve for first? *The boundary conditions*

will help us to see, what the values of λ need to be! So, we solve for X because it's the one that has boundary conditions:

$$\frac{X''}{X} = \lambda, \quad X'(0) = 0 = X'(4).$$

If $\lambda = 0$, then X is a linear function, and with the boundary conditions, it must be constant. Okay. If $\lambda \neq 0$ a basis of solutions is furnished by $e^{\pm\sqrt{\lambda}x}$. Writing our solution as an unknown linear combination of these and testing the boundary conditions:

$$a\sqrt{\lambda} - b\sqrt{\lambda} = 0 \iff a = b \text{ from the boundary condition at } x = 0,$$

$$\sqrt{\lambda}e^{4\sqrt{\lambda}} - \sqrt{\lambda}e^{-4\sqrt{\lambda}} = 0 \iff e^{4\sqrt{\lambda}} = e^{-4\sqrt{\lambda}},$$

$$\iff e^{8\sqrt{\lambda}} = 1.$$

Remember our complex numbers... there are a whole lot of solutions to $e^z = 1$, not just $z = 0$ if $z \in \mathbb{C}$. In particular the equation is satisfied for all integers $n \in \mathbb{Z}$ such that

$$8\sqrt{\lambda} = 2n\pi i \iff \lambda = \left(\frac{2n\pi i}{8}\right)^2 = -\frac{n^2\pi^2}{4^2}.$$

Then up to multiplication by a constant the corresponding $X_n(x) = \cos(n\pi x/4)$. Returning to the T we get that T_n is a linear combination of $\cos(n\pi t/4)$ and $\sin(n\pi t/4)$. We use superposition to add all these solutions together, leaving the unknown coefficients for last:

$$u(x,t) = \sum_{n\geq 0} \cos(n\pi x/4)\left(a_n \cos(n\pi t/4) + b_n \sin(n\pi t/4)\right).$$

Now I have come up with another rhyme for you: *The initial conditions will help us to find coefficients of functions that depend on time.* Since $u(x,0) = 0$, the coefficients of the $\cos(n\pi t/4)$ will all vanish. The coefficients of the $\sin(n\pi t/4)$ will be whatever, depending on that function g. So our solution will be

$$u(x,t) = \sum_{n\geq 0} \cos(n\pi x/4) b_n \sin(n\pi t/4).$$

A series with cosinus in x and sinus in t.

7. The frequencies, i.e. $|\lambda|$ values are

$$\frac{n^2\pi^2}{4^2}, \quad n \geq 0.$$

The first of these is zero, and then next is $\frac{\pi^2}{16} \approx 0.6$.

8. Can there be a nonzero function that is $\mathcal{L}^2$ orthogonal to e^{2inx} for all integers n on the interval $[0, \pi]$? In other words, do the functions e^{2inx} span $\mathcal{L}^2(0, \pi)$? This is a little tricky... It turns out they do because they are all the eigenfunctions of the regular SLP

$$X''(x) + \lambda X(x) = 0, \quad x \in (0, \pi), \quad X(0) = X(\pi), \quad X'(0) = X'(\pi).$$

9. This is an SLP type problem, but actually the same issue occurs as with the previous problem of this type. The weight function is

$$\sin(2x) \quad x \in (0, \pi).$$

For $x \in (\pi/2, \pi)$ this function is negative. The weight function is supposed to be positive.

10. This problem is asking if there is a non-zero $\mathcal{L}^2$ function on the interval $(0, \pi)$ that is orthogonal to x^n for all $n \geq 0$. Such a function would then be orthogonal to all polynomials on that interval. However, we know thanks to our orthogonal polynomial theory that every interval has an $\mathcal{L}^2$ basis consisting of orthogonal polynomials. If you're orthogonal to everything in a basis, you must be zero. So the answer here is no.

11. Okay, so here we have that

$$a_n = \int_0^\pi f(x) e^{2inx} dx = \langle f, e^{-2inx} \rangle.$$

Let's call

$$\phi_n(x) = e^{-2inx}.$$

Then the $\mathcal{L}^2$ norm of this function on $(0, \pi)$ is π, so if we define

$$\varphi_n(x) = \frac{1}{\sqrt{\pi}} e^{-2inx}$$

that function has unit $\mathcal{L}^2$ norm. By the previous problem, these are an orthonormal basis for $\mathcal{L}^2(0, \pi)$. So, by our theory, we have

$$||f||^2 = \int_0^\pi |f(x)|^2 dx = \sum_{n \in \mathbb{Z}} |\langle f, \varphi_n \rangle|^2 = \frac{1}{\pi} \sum_{n \in \mathbb{Z}} |a_n|^2.$$

The exercise says that f is bounded and positive, so its definitely in $\mathcal{L}^2$, and its norm is positive. So we have the inequality:

$$0 < ||f||^2 = \frac{1}{\pi} \sum_{n \in \mathbb{Z}} |a_n|^2 < \sum_{n \in \mathbb{Z}} |a_n|^2,$$

since at least some of the a_n must be non-zero (since the functions e^{2inx} are a basis and so are e^{-2inx} and f is not the zero function).

12. We wish to solve this:

$$\begin{cases} u_t - u_{xx} = 0, & 0 < x < 1, 0 < t, \\ u(0,t) = e^t, \\ u(1,t) = e^{-t}, \\ u(x,0) = f(x). \end{cases}$$

Those are some crazy-ass boundary conditions! There are two of them. This is kind of awful, but we have learned how to deal with it. We wish for a function that is e^t at $x = 0$ and e^{-t} at $x = 1$. So, we introduce

$$\Phi(x,t) := (1-x)e^t + xe^{-t} \implies \Phi(0,t) = e^t, \quad \Phi(1,t) = e^{-t}.$$

Then, we apply the PDE to $\Phi(x,t)$ and see what we get:

$$\Phi_t - \Phi_{xx} = (1-x)e^t - xe^{-t} =: F(x,t).$$

So, we look for a solution to a modified problem

$$\begin{cases} v_t - v_{xx} = -F(x,t), & 0 < x < 1, 0 < t, \\ u(0,t) = 0, \\ u(1,t) = 0, \\ u(x,0) = f(x) - \Phi(x,0). \end{cases}$$

Our solution to the original problem will be $v(x,t) + \Phi(x,t)$. To solve this problem, we use the series method. The boundary conditions are beautiful, so we solve the RSLP $X'' + \lambda X = 0$, on $(0,1)$ with the boundary condition $X(0) = X(1) = 0$. We obtain the functions $X_n(x) = \sin(n\pi x)$. We write

$$v(x,t) = \sum_{n \geq 0} T_n(t) X_n(x).$$

We expand

$$F(x,t) = \sum_{n \geq 0} F_n(t) X_n(x), \quad F_n(t) = \frac{\int_0^1 F(x,t) X_n(x) dx}{\int_0^1 |X_n(x)|^2 dx}.$$

Then we get ODEs for the unknown functions $T_n(t)$ and we solve them. So, the only answer that is appearing in our solution is the regular SLP that we use to build our solution. None of the other methods listed will work. It *might* be possible to solve with Laplace transform but the solution would be identical to the series solution we obtain. Consequently, when you tried to invert the Laplace transform, you would get a contour integral and would need to do some super sophisticated complex analysis involving the residue theorem, because there would be infinitely many poles (resulting in this series). Actually, it is not even clear that you would get something invertible for the Laplace transform, so I am really skeptical this could ever work... I leave it as a challenge for those of you for whom this exam was too easy.

13. This is an inhomogeneous heat equation on the real line. We learned how to solve this using the Fourier transform. The result is going to be a convolution, that is an integral over the whole real line. No weird heavyside stuff to make parts of the integral vanish like it does with Laplace transform.

14. We are supposed to calculate

$$\lim_{n\to\infty}\int_{\mathbb{R}} e^x \cos(2nx)dx.$$

Yeah. That actually is infinite for like all values of n. So this limit doesn't exist.

15. In this exercise we are evaluating the Fourier series of xe^x on the interval $(-\pi,\pi)$ with respect to the functions e^{inx}. We are evaluating it with $x=\pi$ (or equivalently $x=-\pi$). Here's a rhyme for you: *Fourier series can be summed with haste if we remember to copy and paste.* When we copy and paste this function to $\mathbb{R}$ to be 2π periodic, it has jumps at odd integer multiples of π. So, the series converges to the average of the left and right limits at these points. That is

$$\frac{\pi e^{\pi}-\pi e^{-\pi}}{2}=\pi\frac{e^{\pi}-e^{-\pi}}{2}=\pi\sinh(\pi)\approx 36.$$

16. Here we are calculating something similar, although the coefficients are not normalized. So, let's note that

$$c_n=\int_{-\pi}^{\pi} e^{x^2}e^{-inx}dx=2\pi\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{x^2}e^{-inx}dx=2\pi C_n.$$

Then, note that

$$\sum_{n\in\mathbb{Z}} C_n=\sum_{n\in\mathbb{Z}} C_n e^{in0}$$

is the series summed at $x=0$. The function e^{x^2} is continuous at this point in the interval $(-\pi,\pi)$ so *this series* converges to its value there, 1. So,

$$\sum_{n\in\mathbb{Z}} C_n=1=\frac{1}{2\pi}\sum_{n\in\mathbb{Z}} c_n \implies \sum_{n\in\mathbb{Z}} c_n=2\pi\approx 6.$$

17. Well, here we go again. This time the coefficients are normalized, and the function we are expanding is $x\cosh x$. We are supposed to evaluate at the point $x=\pi$. *Fourier series can be summed with haste if we remember to copy and paste.* When we copy and paste this function to $\mathbb{R}$ to be 2π periodic, it has jumps at odd integer multiples of π. So, the series converges to the average of the left and right limits at these points. That is

$$\frac{\pi\cosh\pi-\pi\cosh(-\pi)}{2}=\frac{\pi}{2}(\cosh\pi-\cosh\pi)=0.$$

This is because cosh is an even function.

18. We have the problem:

$$(e^x X'(x))' + \lambda e^x X(x) = 0, \quad 1 < x < 2, \quad X(1) = 0, \quad X(2) = 0.$$

It checks all the boxes to be an RSLP. So, theory says that the eigenvalues λ will tend towards plus infinity. All we need to see is if there will be any negative eigenvalues and or zero eigenvalues. So, let's work it out:

$$e^x X'(x) + e^x X''(x) + \lambda e^x X(x) = 0 \iff X'' + X' + \lambda X = 0.$$

We have a theorem from the beginning of the course that tells us a basis of solutions will be determined by solutions of the quadratic equation, so we calculate

$$-\frac{1}{2} \pm \frac{\sqrt{1-4\lambda}}{2} = -\frac{1}{2} \pm \sqrt{1/4 - \lambda}.$$

A basis of solutions is then either

$$e^{-x/2} e^{\sqrt{1/4-\lambda}x}, \quad e^{-x/2} e^{-\sqrt{1/4-\lambda}x}, \quad \lambda \neq 1/4$$

or

$$e^{-x/2}, \quad x e^{-x/2}, \quad \lambda = 1/4.$$

Let's check what happens if $\lambda \leq 1/4$. Are there solutions? Let's call $u(x) = e^{-x/2} e^{\sqrt{1/4-\lambda}x}$ and $v(x) = e^{-x/2}$ either multiplied by x (if $\lambda = 1/4$) or by $e^{-\sqrt{1/4-\lambda}x}$ if $\lambda < 1/4$. We need for the first boundary condition the coefficients to satisfy

$$au(1) + bv(1) = 0.$$

So, we need

$$b = -a\frac{u(1)}{v(1)}.$$

This is okay since $v(1) \neq 0$. Now let's look at the other boundary condition. Again $v(2) \neq 0$ so we need

$$b = -a\frac{u(2)}{v(2)}.$$

So, this means we need

$$\frac{u(1)}{v(1)} = \frac{u(2)}{v(2)}.$$

Here we look at the different cases. In case $\lambda = 1/4$ we need:

$$\frac{u(1)}{v(1)} = 1 = \frac{u(2)}{v(2)} = \frac{1}{2}.$$

Preposterous. In case $\lambda < \frac{1}{4}$ we need

$$\frac{u(1)}{v(1)} = e^{2\sqrt{1/4-\lambda}} = \frac{u(2)}{v(2)} = e^{4\sqrt{1/4-\lambda}}$$

which is also impossible. So the eigenvalues cannot be $\leq 1/4$.

19. It's time for supper! So, the boundary conditions of the problem are not homogeneous or self-adjoint, and we must deal with those first. They are time independent so we can find a SS solution.

20. YIKES! But it is not so bad. We found the SS solution, and that is just 180. So the new problem to solve is

$$\heartsuit = \begin{cases} v_t - kv_{xx} = 0, & 0 < t,\ 0 < x < 8 \\ v(0,t) = 0 = v(8,t) \\ v(x,0) = -175. \end{cases}$$

Note that this is super similar to the March exam, I just changed the numbers. So hopefully you learned how to do it and can just proceed similarly with new numbers. Similarly, I can copy and paste my solution and just change numbers, how nice. We separate variables as we did in a previous problem and arrive at the RSLP for the x dependent part

$$\frac{T'}{T} = k\frac{X''}{X} = \Lambda, \quad X'' + \lambda X = 0, \quad \Lambda = -k\lambda.$$

The BCS are $X(0) = 0 = X(8)$. We have solved this many times before, the solutions are

$$X_n(x) = \sin\left(\frac{n\pi x}{8}\right), \quad \lambda_n = \frac{n^2\pi^2}{8^2} \implies \Lambda_n = -k\frac{n^2\pi^2}{8^2}.$$

This tells our partner functions up to the constant factors are $e^{-ktn^2\pi^2/8^2}$. So, the solution

$$u(x,t) = 180 + \sum_{n\geq 1} c_n e^{-ktn^2\pi^2/8^2} X_n(x).$$

The coefficients we get by expanding the initial data in terms of the OB functions X_n, so

$$c_n = \frac{\int_0^8 -175\sin(\pi x/8)dx}{\int_0^8 |\sin(\pi x/8)|^2 dx} = -\frac{700}{\pi}.$$

This was kindly given to us. We're told to use the constant term and the next term to approximate the solution, so we are approximating with

$$u(x,t) \approx 180 - \frac{700}{\pi} e^{-kt\pi^2/8^2}\sin(\pi x/8).$$

In the middle, that is $x = 4$, we get $\sin(\pi 4/8) = 1$. We are told that the temperature is 100 after 80 minutes, meaning

$$100 = 180 - \frac{700}{\pi} e^{-k(80)\pi^2/8^2} \iff 80 = \frac{700}{\pi} e^{-k\pi^2(5/4)}.$$

So we solve this for k:

$$\ln\left(\frac{8\pi}{70}\right) = -\frac{5\pi^2 k}{4} \iff k = \frac{4}{5\pi^2}\ln\left(\frac{70}{8\pi}\right) \approx 0,08.$$

You could enter the answer rounded to two decimal places as either $0,08$ or 0.08. Both would be marked correct.

21. This problem should be a breath of fresh air after the last one. It is classic textbook: extend evenly due to the boundary condition at $x = 0$, then use Fourier transform in x.

22. Similarly, if you didn't figure it out by now, look at the preceding problem. They are *so similar.* You could almost use "exam psychology" to guess this if you didn't know how to do it. Looks almost the same as the previous one, right? Just slightly different. So this one and the previous one should have almost the same method, but maybe a little different eh? Exactly. Odd extension and Fourier transform in x here.

23. This is one way to test theory. If you learned how to *prove* the generating function for the Bessel functions, then you should recognize this stuff. Then you'd know that you should look up what that theorem says:

$$e^{\frac{x}{2}\left(z-\frac{1}{z}\right)} = \sum_{n\in\mathbb{Z}} J_n(x)z^n.$$

Okay, so we see that we must have $x = 2$, and $z = 2$. So we plug that in on the left side to obtain that the sum is

$$e^{(2-1/2)} = e^{3/2} \approx 4.5.$$

Similarly you could write $4,5$ which would be okay.

24. So, now we are inspired by the previous problem, and we look at the left side which is

$$e^{\frac{x}{2}\left(z-\frac{1}{z}\right)}.$$

For any fixed $x \neq 0$ we note that

$$\lim_{z\to 0} e^{xz/2} = 1, \quad \lim_{z\downarrow 0} e^{-\frac{x}{2z}} = 0.$$

Hence for any $x \neq 0$ and indeed for $x = 2$ we get that

$$\lim_{z\downarrow 0}\sum_{n\in\mathbb{Z}} J_n(2)z^n = 0.$$

25. If the last one showed you anything - and if you indeed learned the theory item about generating function for Hermite - then you know what to do here. That theorem says

$$e^{2xz-z^2} = \sum_{n\geq 0}\frac{H_n(x)z^n}{n!}.$$

In the sum we have $x = 1$ and $z = 2$. So, the left side is

$$e^{2*1*2-2^2} = e^0 = 1.$$

26. Here we are supposed to calculate an integral

$$\int_0^\infty e^{-x/2} \cos(x/2) dx.$$

This looks Fourierish to me. So, the first thing I do to make it more Fourierish is to make the integral over $\mathbb{R}$ using the fact that cosine is even

$$= \int_{\mathbb{R}} e^{-|x|/2} \cos(x/2) dx.$$

Note that by the evenness of cosine, evenness of $e^{-|x|/2}$ and oddness of sine, this is equal to

$$\int_{\mathbb{R}} e^{-|x|/2} e^{-ix/2} dx.$$

That is the Fourier transform of $e^{-|x|/2}$ evaluated at $\frac{1}{2}$. We look it up in the textbook, to get that the Fourier transform of $e^{-a|x|}$ at ξ is

$$\frac{2a}{\xi^2 + a^2}.$$

So putting $a = \xi = \frac{1}{2}$ this is

$$\frac{1}{\frac{1}{4} + \frac{1}{4}} = 2.$$

27. We are really crazy about summing Fourier series on this exam! Here we have the trig Fourier series of the function 2^x on the interval $(-\pi, \pi)$. Since $e^{in\pi} = (-1)^n$ we are summing the series at $x = \pi$. The trig Fourier series has jump discontinuities at odd integer multiples of π due to the copy-pasting (2π periodicity). The theorem on the convergence of trig Fourier series says that it converges to the average of the left and right limits that is

$$\frac{2^\pi + 2^{-\pi}}{2} \approx 4.5$$

It's okay to write $4, 5$ also.

28. I hope this doesn't make you hate music... At least this question is super similar to a question on the last exam. Everything is theta independent so theta will not make an appearance. We solve for the r part next, using separation of variables (at least the PDE is homogeneous right?).

$$\frac{T''}{T} = \frac{R''}{R} + r^{-1}\frac{R'}{R} = \Lambda.$$

So both sides are constant that we have called Λ. We solve for the R side because it has the nice boundary condition $R(8) = 0$. The equation is:

$$r^2 R'' + rR' - r^2 \Lambda R = 0.$$

If $\Lambda = 0$, the equation is just

$$\frac{R''}{R'} = -\frac{1}{r} \implies \ln(R')' = -\frac{1}{r} \implies \ln(R') = -\ln(r) + c$$

$$\implies R' = e^c \frac{1}{r} \implies R(r) = C \ln(r) + B,$$

for two constants C and B. This is not a viable solution because it tends to $-\infty$ when $r \to 0$. If $\Lambda > 0$, the equation we have is the modified Bessel equation of order zero. The solutions are I_0 and K_0. The problem with K_0 is that it tends to infinity when $r \to 0$. With I_0 it never vanishes, so we won't be able to get $R(8) = 0$. So we turn to $\Lambda < 0$, in which case the equation is the Bessel equation of order zero with solutions J_0 and Y_0. However, Y_0 tends to infinity at 0, so that is not a physical solution. The solution is therefore, having made the suitable variable change,

$$R(r) = J_0(\sqrt{\lambda} r), \quad \lambda = -\Lambda.$$

To satisfy the boundary condition we need $R(8) = 0$, so we define

$$\pi_k = \text{ the k-th positive zero of } J_0, \quad \sqrt{\lambda_k} = \frac{\pi_k}{8}, \quad R_k(r) = J_0(\pi_k r/8),$$

and

$$\Lambda_k = -\frac{\pi_k^2}{64}.$$

Consequently,

$$T_k(t) = a_k \cos(\pi_k t/8) + b_k \sin(\pi_k t/8),$$

and

$$u(r,t) = \sum_{k \geq 1} J_0(\pi_k r/10) T_k(t).$$

To find the coefficients in the T_k function, we use the initial data. Since both u and u_t are non-zero functions at $t = 0$, we will get that the a_k and b_k in general are nonzero. So our solution is a series with Bessel function J_0 and sines and cosines.

29. This is not as awful as it seems. We just need to consider the very first term. That is

$$a_1 \cos(\pi_1 t/8) J_0(\pi_1 r/8) + b_1 \sin(\pi_1 t/8) J_0(\pi_1 r/8).$$

We are calculating at $r = 0$, and $t = 8$, and we are given that $J_0(0) = 1$, so we just need to calculate

$$a_1 \cos(\pi_1) + b_1 \sin(\pi_1).$$

The coefficient

$$a_1 = \frac{\int_0^8 (r-8) J_0(\pi_1 r/8) r dr}{\int_0^8 J_0^2(\pi_1 r/8) r dr}.$$

We are given that this is approximately

$$-\frac{54.2}{8.7}.$$

To calculate the coefficient b_1, remember that it comes from differentiating. When we differentiate the sine, a factor of $\frac{\pi_1}{8}$ comes out. So,

$$b_1 = \frac{8}{\pi_1} a_1.$$

Therefore the answer we seek is

$$-\frac{54.2}{8.7}\left(\cos(\pi_1) + \frac{8}{\pi_1}\sin(\pi_1)\right) \approx -9.$$

We get this using a calculator!

30. Now you get a bit of relief. We do the standard thing and shift around the Legendre polynomials to fit the integral. Since we want polynomials that will be orthogonal on $(-3, 3)$, we will want

$$\int_{-3}^{3} p_n(y) p_m(y) dy = \int_{-1}^{1} \ldots = 0.$$

To get the integral on the left going from -1 to 1 we therefore want the stuff inside the Legendre polynomials to be say $x/3 = t$

$$\int_{-3}^{3} P_n(x/3) P_m(x/3) dx = \int_{-1}^{1} P_n(t) P_m(t) 3dt.$$

The factor of 3 doesn't matter because it's the orthogonality we are after.

31. Similarly, we can be a bit relieved with this exercise. It will definitely be something involving Hermite. We want to get orthogonality with respect to the modified weight function $e^{-4x^2} = e^{-(2x)^2}$. So if $y = 2x$ then we want $H_n(y)$ i.e. $H_n(2x)$.

32. This one is a freebee. It is the half line with the Laguerre weight, so yeah, we use the Laguerre polynomials.

33. This looks a little hard, but then relax and just go to the table. On the right is

$$\int_{\mathbb{R}} f'(x)dx$$

which is the Fourier transform of f' at $\xi = 0$. According to the table the Fourier transform of f' at ξ is $i\xi\hat{f}(\xi)$. So if $\xi = 0$ then this just becomes zero. One could be a bit more fancy and use some theory to say that if f and f' are both transformable then

$$\lim_{x \to \pm\infty} f(x) = 0,$$

so the integral must tend to zero.

34. This problem should SCREAM at you what to do:

$$\begin{cases} u_t - u_{xx} = 0, \quad 0 < t, x, \\ u(0,t) = e^t, \\ u(x,0) = 0. \end{cases}$$

LAPLACE TRANSFORM. You don't even have to solve it all the way because there was almost the same problem on the last exam. When you go back, with all the heavysides, the convolution integral will end up going from 0 to t.

35. We have also seen something like this before:

$$\lim_{t \searrow 0} \int_{-t/2}^{t/2} f(b-z)\frac{dz}{t}.$$

The function f is continuous. The integral above is

$$\int_{\mathbb{R}} \chi_{(-t/2,t/2)}(z) f(b-z)\frac{dz}{t}.$$

Now, note that

$$\chi_{(-t/2,t/2)}(z) = \begin{cases} 1, & -t/2 < z < t/2, \\ 0, & |z| > t/2. \end{cases}$$

This is the same as

$$\chi_{(-1/2,1/2)}(z/t) = \begin{cases} 1, & -1/2 < z/t < 1/2, \\ 0, & |z/t| > 1/2. \end{cases}$$

So the integral is $f * g_t(b)$

$$g(z) = \chi_{(-1/2,1/2)}(z), \quad g_t(z) = \frac{g(z/t)}{t}.$$

The CAT tells us that the limit is $f(b)$ times the integral over the real line of g which is one. This is because f is continuous.

36. The CAT is at it again! We consider:

$$\lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \frac{1}{1+(\pi - x)^2} \cos^2(\pi - x) e^{-|x|/\epsilon} \frac{dx}{\epsilon}.$$

So, the g function is $e^{-|x|}$. The other function we are convolving with is $f(x) = \frac{1}{1+x^2}\cos^2(x)$, and the point we are convolving at is π. So, since f is continuous everywhere we set in $x = \pi$ and multiply with the integral of g over the real line which is 2, obtaining

$$2\frac{1}{1+\pi^2}\cos^2(\pi) = \frac{2}{1+\pi^2} \approx 0.18.$$

Rounding to the nearest whole number yields 0.

37. We introduce the weird g to apply Bessel's inequality to the Fourier coefficients of g, because the stuff we are estimating turns out to be the sum of like the N^{th} and $-N^{th}$ (or maybe one of these is $N+1$ but it doesn't change the reasoning) Fourier coefficients of g. Since g is bounded, its Fourier coefficients tend to zero. That completes the proof.

38. Here one must use the theorem relating the Fourier coefficients of f and f' and the Cauchy-Schwarz inequality in ℓ^2.

39. This follows from Bessel's inequality which we use to show that the partial sums form a Cauchy sequence, together with the definition of Hilbert space, that says it is complete so every Cauchy sequence converges.

40. One *must* estimate near zero, because that sets a fixed quantity (whatever we call it, usually $y_0 > 0$ or $y_0 < 0$) and *that* is key to the estimates that follow. It won't work to do it in any other order (believe me I have tried!).

41. The proof must use the statement number one, so we define

$$g = \sum \langle f, \phi_n \rangle \phi_n,$$

and we show that the scalar product of $f - g$ with ϕ_n vanishes for all n. This allows us to use statement number one, which we get to assume is true, to conclude that $f - g = 0$. Well, that means $f = g$.

If this didn't go as well for you as you hoped, please study this exam and the previous one (March 2022) to prepare for August. August will be similar!!

i Försättsblad

Chalmers MVE030 & MVE290
Den 8:e juni 2022

MVE030/MVE290 - Fourieranalys och Fourier Metoder
Ägare: TKTFY/TKKEF – Teknisk fysik/Kemiteknik med fysik
Institution: 11 – MATEMATISKA VETENSKAPER
Examinator: Julie Rowlett
Telefon: 031-772 34 19
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Tentamen innehåller 40 frågor, 2 poäng per fråga. Bonuspoäng från duggorna läggs till (upp till 5 poäng). (Det finns 41 men #1 är för att lägga till bonuspoäng - bara skriv din anonymkod där snälla!)

För betyg 3 krävs minst 40 poäng.

För betyg 4 krävs minst 53 poäng.

För betyg 5 krävs minst 67 poäng.

Tillåtna hjälpmedel är:

1. Kursbok både i inspera som resurs (pdf) samt uttryckt (papper) version av kursboken. I kursboken tillåts anteckningar, highlights, sticky-notes/post-its, osv.

2. BETA handbook. Anteckningar, highlights, sticky-notes/post-its, osv tillåts.

3. Miniräknare (vilken som helst).

Lycka till! ^_^ You got this! May the mathematical forces be with you!

1 Bonus_Points

Skriv snälla din anonymkod här och vi ska lägga till dina bonus poäng i så fall du har några.

Skriv in ditt svar här

Teckenf...

Ord: 0

Totalpoäng: 5

2 Favorite topic?

What was your favorite topic from this course? // Vilket var ditt favoritämne från den här kursen?

Skriv in ditt svar här

Teckenf...

Ord: 0

Totalpoäng: 2

3 Least favorite/hardest part

What was your least favorite part of this course and/or what did you find most challenging? // Vilken var din minst favoritdel av den här kursen och/eller vad tyckte du var jobbigast?

Skriv in ditt svar här

Teckenf...

Ord: 0

Totalpoäng: 2

4 Wave equation 1 June 2022

Vi ska lösa problemet

$$\begin{cases} u_{tt} = u_{xx}, & t > 0, \quad 0 < x < 4, \\ u_x(0,t) = 0, \\ u_x(4,t) = 0, \\ u(x,0) = f(x), \\ u_t(x,0) = g(x). \end{cases}$$

Vad ska vi göra först?

Välj ett alternativ:

- Använd Laplace transform i x
- Använd Fourier transform i x
- Hitta en steady-state lösning
- Använd Laplace transform i x
- Variabel separation
- Lös ett Sturm-Liouville problem
- Använd Laplace transform i t
- Utveckla en serie av Bessel funktioner

Totalpoäng: 2

5 RSLP eller inte 1 June 2022

Är följande ett reguljärt Sturm-Liouville Problem?

$f''(x) + \lambda \sin(x) f(x) = 0, \quad x \in (0, 4), \quad f'(0) = 0, \quad f'(4) = 0.$

Om svaret är ja, vad kan ni säga om egenvärdena?

Välj ett alternativ

- De går mot positiv oändligt men vi kan inte räkna exakt vad de är.
- De går mot negativ oändligt men vi kan inte räkna exakt vad de är.
- De börjar med cirka -5 och blir mindre (negative).
- De börjar med cirka 0,05 och blir större.
- De börjar med 0 och blir mindre (negativ).
- De börjar med cirka 3 och blir större.
- Svaret är nej.
- De börjar med 0 och blir större (positiv).

Totalpoäng: 2

6 Wave eqn 2 June 2022

Lös problemet: $$\begin{cases} u_{tt} = u_{xx}, & t > 0, \quad 0 < x < 7, \\ u_x(0,t) = 0, \\ u_x(4,t) = 0, \\ u(x,0) = 0, \\ u_t(x,0) = g(x). \end{cases}$$

Lösningen u(x,t) är:

Välj ett alternativ:

- En serie med sinus i t variabel och cosinus i x variabel
- En serie med sinus i både x och t variabel
- En serie med cosinus i x variabel och cosh i t variabel
- En faltning (convolution)
- En serie med sinus i x variabel och cosinus i t variabel
- En integral från 0 till ∞
- En serie med sinus i x variabel och cosh i t variabel
- En serie med sinus i x variabel och sinh i t variabel
- En integral på intervallet [0, 4]
- En serie med cosinus i både x och t variabel

Totalpoäng: 2

7 Wave eqn 3 June 2022

En av följande frekvenserna (dvs $|\lambda|$) finns i lösningen till föregående uppgiften. Vilken? (frekvenserna är avrundat)

Välj ett alternativ:

- 0,2
- 1,0
- 0,6
- 0,7
- 0,9
- 0,8
- 0,4
- 0,1
- 0,5
- 0,3

Totalpoäng: 2

8 Finns det 1... June 2022

Finns det en funktion i $[0, \pi]$ som uppfyller

$\int_0^\pi |f(x)|^2 dx > 0$ och $\int_0^\pi f(x)e^{2inx} dx = 0, \quad \forall n \in \mathbb{Z}$?

Välj ett alternativ:

- Det är för lite information för att kunna bestämma
- Nej
- Ja

Totalpoäng: 2

9 Typ av problem June 2022

Vilket typ av problem är: $\begin{cases} f''(x) + \lambda \sin(2x) f(x) = 0, & 0 < x < \pi \\ f(0) + f(\pi) = 0, \\ f'(0) + f'(\pi) = 0. \end{cases}$

Välj ett alternativ

- Det är en homogen partiell differential ekvation
- Det är ett SLP typ problem men inte regulärt på grund av vikt funktionen
- Det är en Bessel ekvation
- Det är en inhomogen partiell differential ekvation
- Det är ett regulärt Sturm Liouville Problem
- Det är ett SLP typ problem men inte regulärt på grund av randvillkorerna
- Det är en modifierad Bessel ekvation

Totalpoäng: 2

10 Finns det 2 June 2022

Finns det en funktion i $[0, \pi]$ som uppfyller:
$\int_0^\pi |f(x)|^2 dx > 0, \text{ och } \int_0^\pi f(x) x^n dx = 0$
för alla $n \geq 0$? (heltal n).
Välj ett alternativ:

- Nej
- Ja
- Det är inte möjligt att bestämma med den här informationen.

Totalpoäng: 2

11 Vad vet ni om serie June 2022

Låt f(x) vara en begränsad, positiv funktion och $a_n = \int_0^\pi f(x) e^{2inx} dx$. Vad kan vi säga om serien
$\sum_{n \in \mathbb{Z}} |a_n|^2$?
Välj ett alternativ:

- $< \int_0^\pi |f(x)|^2 dx$
- $= \int_0^\pi |f(x)|^2 dx$
- Det är för lite information för att kunna svara.
- $\leq \int_0^\pi |f(x)|^2 dx$
- $> \int_0^\pi |f(x)|^2 dx$
- $\geq \int_0^\pi |f(x)|^2 dx$

Totalpoäng: 2

12 Heat eqn 1 June 2022

Vi skulle vilja lösa problemet $\begin{cases} u_t - u_{xx} = 0, & 0 < x < 1, 0 < t, \\ u(0,t) = e^t, \\ u(1,t) = e^{-t}, \\ u(x,0) = f(x). \end{cases}$

Vilken teknik kan leda till en hanterbar lösning?

Välj ett alternativ:

- Laplace transform i t
- Fourier transform i x
- Bessel funktioner
- Fourier transform i t
- Laplace transform i x
- Mellin transform i x
- Mellin transform i t
- Ett reguljärt Sturm-Liouville problem

Totalpoäng: 2

13 Heat eqn 2 June 2022

Lös problemet: $\begin{cases} u_t - u_{xx} = \frac{\sin(x)}{x}, & 0 < t, \quad -\infty < x < \infty, \\ u(x,0) = e^{-x^2}. \end{cases}$

För att lösa problemet, det funkar bra att använda: Välj alternativ
(Fouriertransform i t, Lösa ett regulärt Sturm-Liouville problem, Variabelseparation, Använd en serie med Bessel funktioner, Laplace Transform i t, Laplace Transform i x, Fouriertransform i x)
Lösningen blir
Välj alternativ (en trigonometrisk Fourierserie, en serie med Bessel funktioner, en invers Laplace transform, en integral från minus oändligt till plus oändligt).

Totalpoäng: 2

14 Calculate limit of integral 1 June 2022

Beräkna $\lim_{n\to\infty}\int_{-\infty}^{\infty} e^x \cos(2nx)dx$

Välj ett alternativ:

- 2
- $\sqrt{\pi}$
- Gränsvärdet finns inte
- 0

Totalpoäng: 2

15 Calculate series 1 June 2022

Låt $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} xe^x e^{-inx}dx$. Beräkna $\sum_{n\in\mathbb{Z}} c_n(-1)^n$ och avrunda till den närmast heltal:

[ ].

Totalpoäng: 2

16 Calculate series nr. 2 June 2022

Låt $c_n = \int_{-\pi}^{\pi} e^{x^2} e^{-inx}dx$. Beräkna $\sum_{n\in\mathbb{Z}} c_n$ och avrunda till den närmast heltal. : [ ].

Totalpoäng: 2

17 Calculate Series 3 June 2022

Låt $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\cosh xe^{-inx}dx$. Beräkna $\sum_{n\in\mathbb{Z}} c_n e^{in\pi}$ och avrunda till den närmast heltal. :

[ ].

Totalpoäng: 2

18 RSLP eller inte 2 June 2022

Är följande ett reguljärt Sturm-Liouville Problem?
$(e^x X'(x))' + \lambda e^x X(x) = 0, \quad 1 < x < 2, \quad X(1) = 0, \quad X(2) = 0.$
Om svaret är ja, vad kan ni säga om egenvärdena?

Välj ett alternativ:

- De börjar med 0 och bli mindre (negativ).
- Svaret är nej.
- De går mot negativ oändligt men vi kan inte räkna exakt vad de är.
- De börjar med cirka -2 och bli mindre (negative).
- De börjar med 0 och bli större (positiv).
- De går mot positiv oändligt men vi kan inte räkna exakt vad de är.

Totalpoäng: 2

19 Janssons del 1 June 2022

Det är dags att laga mat - Janssons frestelse. Vi ska lösa den här problemet som beskriver temperaturen i vår ungsform som vi har tagit ur kylskåpet och satt i ungnen:

$$\begin{cases} u_t - ku_{xx} = 0, & 0 < t, \quad 0 < x < 8, \\ u(0,t) = 180 = u(8,t), \\ u(x,0) = 6. \end{cases}$$

Det funkar bra att börja med att:

Välj ett alternativ:

- Hitta en steady state lösning.
- Lösa det regulärt Sturm-Liouville-Problemet:
  $f''(x) + \lambda f(x) = 0,$
  $0 < x < 8,$
  f(0) = f(8) =180.
- Använda Laplacetransform i t
- Använda Samplingssatsen
- Använda Fouriertransform i x
- Använda Laplacetransform i x
- Använda Fouriertransform i t
- Använda CAT (faltning approximering sats)

Totalpoäng: 2

20 Janssons del 2 June 2022

Lös problemet:

$$\begin{cases} u_t - k u_{xx} = 0, & 0 < t \quad 0 < x < 8, \\ u(0,t) = 180 = u(8,t) \\ u(x,0) = 5. \end{cases}$$

Maten är klart när temperaturen i mitten blir 100 (temperaturen är i C), dvs när u(4, t) = 100 grad C. Min erfarenhet ger att Janssons blir klart efter 80 minuter. Vad blir k?

Anta att du kan approximera lösningen med konstanttermen och följande termen och använd gärna: $\frac{\int_0^8 -175\sin(\pi x/8)dx}{\int_0^8 |\sin(\pi x/8)|^2 dx} = -\frac{700}{\pi}$.

Avrund svaret till två decimalplatser.

Totalpoäng: 2

21 Wave equation x>0 June 2022

Vi ska lösa problemet:

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 \quad t, x > 0, \\ u_x(0,t) = 0, \\ u(x,0) = e^{-x}, \quad u_t(x,0) = e^{-2x}. \end{cases}$$

Vilken teknik går bra att använda?

Välj alternativ (Jämn utvidning (extension) och Fourier transform i x, Laplace transform i t, Ett regulärt Sturm Liouville Problem, Laplace transform i x, Udda utvidning (extension) och Fourier transform i x, En serie med Bessel funktioner, En Laplace serie, En Fourier serie).

Totalpoäng: 2

22 Wave equation x>0 no. 2 June 2022

Vi ska lösa problemet:

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 \quad t, x > 0, \\ u(0,t) = 0, \\ u(x,0) = e^{-x^2}, \quad u_t(x,0) = 0. \end{cases}$$

Vilken teknik går bra att använda?

Välj alternativ (Laplace transform i t, Udda utvidgning (extension) och Fourier transform i x, En Fourier serie, Ett regulärt Sturm Liouville Problem, En Bessel serie, Jämn utvidgning (extension) och Fourier transform i x, En Mellin transform).

Totalpoäng: 2

23 Bessel function sum 0 June 2022

Låt J_n vara den Bessel funktion av ordning n. Beräkna och avrunda svaret till en decimal $\sum_{n\in\mathbb{Z}} J_n(2)2^n$ =

Totalpoäng: 2

24 Bessel function sum limit June 2022

Beräkna $\lim_{z\to 0+} \sum_{n\in\mathbb{Z}} J_n(2)z^n = \lim_{z\downarrow 0} \sum_{n\in\mathbb{Z}} J_n(2)z^n$.: . (Det betyder att z går mot noll och är positiv).

Totalpoäng: 2

25 Hermite polynomial sum 1 June 2022

Låt H_n vara den Hermite polynomial av grad n. Beräkna och avrunda till den närmast heltal $\sum_{n\geq 0} \frac{H_n(1)2^n}{n!}$

.

Totalpoäng: 2

26 Calculate integral half line June 2022

Beräkna och avrunda till det närmast heltal
$\int_0^\infty 2e^{-x/2}\cos(x/2)dx$

[ ].

Totalpoäng: 2

27 Calculate series 4 June 2022

Låt $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2^x e^{-inx} dx$
Beräkna $\sum_{n\in\mathbb{Z}} c_n(-1)^n$
Avrunda till en decimal.

[ ]

Totalpoäng: 2

28 Musik del 1 June 2022

Vi ska spela trumman!
Funktionen $u(r,\theta,t)$ ge höjden på trumman i tidspunkt t på punkten (r,θ) på trumman och uppfyller:

$$\begin{cases} u_{tt} - u_{rr} - r^{-1}u_r - r^{-2}u_{\theta\theta} = 0, \quad 0 < r < 8, \quad -\pi < \theta < \pi, \quad 0 < t, \\ u(8,\theta,t) = 0, \\ u(r,\theta,0) = r-8, \\ u_t(r,\theta,0) = r-8. \end{cases}$$

Lös problemet. Lösningen blir:
Välj ett alternativ:

- En serie med Bessel funktionen J_0, sinus och cosinus
- En serie med trigonometriska funktioner och exponentiella funktioner
- En faltning
- En invers Fourier transform
- En serie med Bessel funktionen J_0 och cosinus
- En serie med Bessel funktion J_0 och sinus
- En serie med Bessel funktionerna J_n för heltal n=0, 1, 2, ..., cosinus och sinus
- En invers Laplace transform

Totalpoäng: 2

29 Musik Trumma del 2 June 2022

Ni har redan löst problemet i föregående uppgiften. Vad är höjden i mitten (dvs r=0) när t=8? Om ni har fått en serie som svaret får ni använda första termen. Ni får gärna använda några formel:

Låt π_k vara den k-te positiv nollställe av J_0.
$\pi_1 \approx 2,4, \quad \int_0^8 (r-8)J_0(\pi_1 r/8) r dr \approx -54.2, \quad \int_0^8 J_0(\pi_1 r/8)^2 r dr \approx 8.7,$
och $J_0(0) = 1.$

(avrunda till närmast heltal!)

Totalpoäng: 2

30 Polynom approximering 1 June 2022

Vi vill hitta polynom p(x) av högst grad 10 som minimerar
$\int_{-3}^{3} |f(x) - p(x)|^2 dx.$
Hur kan vi hitta p(x)?
Välj ett alternativ:

- Projicera f(x) på $L_n(x/3)$ med L_n Laguerre polynomen grad n.
- Projicera f(x) på $L_n(3x)$ med L_n Laguerre polynomen grad n.
- Projicera f(x) på $H_n(3x)$ med H_n Hermite polynomen grad n.
- Projicera f(x) på $P_n(x/3)$ med P_n Legendre polynomen grad n.
- Projicera f(x) på $H_n(x/3)$ med H_n Hermite polynomen grad n.
- Projicera f(x) på $P_n(3x)$ med P_n Legendre polynomen grad n.

Totalpoäng: 2

31 Polynom approximering 2 June 2022

Vi vill hitta polynom p(x) av högst grad 8 som minimerar
$\int_{-\infty}^{\infty} |f(x) - p(x)|^2 e^{-4x^2} dx$.
Hur kan vi hitta p(x)?
Välj ett alternativ:

- Projicera f(x) på $P_n(2x)$ med P_n Legendre polynomen grad n.
- Projicera f(x) på $H_n(x/2)$ med H_n Hermite polynomen grad n.
- Projicera f(x) på $P_n(x/2)$ med P_n Legendre polynomen grad n.
- Projicera f(x) på $L_n(x/2)$ med L_n Laguerre polynomen grad n.
- Projicera f(x) på $H_n(2x)$ med H_n Hermite polynomen grad n.
- Projicera f(x) på $L_n(2x)$ med L_n Laguerre polynomen grad n.

Totalpoäng: 2

32 Polynom approximering 3 June 2022

Vi vill hitta polynom p(x) av högst grad 10 som minimerar
$\int_{0}^{\infty} |e^{-x} - p(x)|^2 e^{-x} dx$.
Hur kan vi hitta p(x)?
Välj ett alternativ:

- Projicera e^{-x} på $H_n(x)$ med H_n Hermite polynomen grad n.
- Projicera e^{-x} på $L_n(x)$ med L_n Laguerre polynomen grad n.
- Projicera e^{-x}på $P_n(x)$ med P_n Legendre polynomen grad n.

Totalpoäng: 2

33 Fouriertransform 1 June 2022

Antar att ni kan hitta Fouriertransformen av funktionen f på en tabell:
$\hat{f}(\xi) = g(\xi)$. Använd g för att beräkna $\int_{-\infty}^{\infty} f'(x)dx$

Totalpoäng: 2

34 PDE med x>0 3, June 2022

Vi ska lösa problemet:

$$\begin{cases} u_t - u_{xx} = 0, \quad 0 < t, x, \\ u(0,t) = e^t, \\ u(x,0) = 0. \end{cases}$$

Det funkar bra om vi börjar med att

Välj alternativ (använda Fourier transform i x, använda Fourier transform i t, använda Laplace transform i x, använda Laplace transform i t, separera variablerna, lösa ett regulärt SLP). När vi lösa problemet, svaret kommer att bli

Välj alternativ (en integral från minus oändligt till plus oändligt, en trigonometrisk Fourier serie, en Fourier serie med Hermite polynom, en Fourier-Bessel serie, en integral från 0 till t)

Totalpoäng: 2

35 Beräkna integral 1 June 2022

Antar att funktionen f är en begränsad och kontinuerlig funktion. Beräkna:
$\lim_{t \searrow 0} \int_{-t/2}^{t/2} f(b-z) \frac{dz}{t}$ =

Tips? MEOW! (please don't use any extra spaces!)

Totalpoäng: 2

36 Beräkna integral 2 June 2022

Antar att funktionen f är en begränsad och kontinuerlig funktion. Beräkna och avrunda svaret till det närmast heltal
$\lim_{\epsilon\to 0}\int_{-\infty}^{\infty}\frac{1}{1+(\pi-x)^2}\cos^2(\pi-x)e^{-|x|/\epsilon}\frac{dx}{\epsilon}$ =

Tips? MEOW!

Totalpoäng: 2

37 Lagom konvergenssats June 2022

I beviset av den satsen om punktvis konvergens av trigonometriska Fourier serier introducerar vi en ny funktion $g(t) := \begin{cases} \frac{f(t+x)-f(x_-)}{e^{it}-1}, & -\pi < t < 0, \\ \frac{f(t+x)-f(x_+)}{e^{it}-1}, & 0 < t < \pi. \end{cases}$

Hur använder vi funktionen g(t) i beviset?

Välj ett alternativ:

- För att sedan visa att g är lösningen till ett RSLP.
- För att sedan använda Bessels olikhet på de Fourier koefficienterna av g.
- För att sedan använda Parsevals likhet på g.
- För att sedan använda den Best Approximation satsen på g.
- För att sedan använda Plancharels sats på g.

Totalpoäng: 2

38 Triganometriska Fourier koefficienter June 2022

Antar att f är 2π periodisk och C^1 på $(-\infty, \infty)$. Hur kan vi bevisa att summan av fs Fourierkofficienterna c_n konvergar absolut?
Dvs hur kan vi bevisa att $\sum_{n\in\mathbb{Z}} |c_n|$ konvergerar?

Välj ett alternativ:

- Deriverar Fourier serien av f termvis.
- Använd Parsevals likhet.
- Använd satsen om Fourierkoefficienter av f och dess av f'.
- Det är omöjligt att bevisa med bara den här informationen om f.
- Använd Plancharels likhet.
- Använd Bessels olikhet.

Totalpoäng: 2

39 Best approximation sats June 2022

I beviset av den bästa approximeringssatsen antar vi att f ligger i ett Hilbertrum och att $\{\phi_n\}_{n\geq 0}$ ligger också i Hilbertrummet och är ortonormala. Hur vet vi att $\sum_{n\geq 0}\langle f, \phi_n\rangle\phi_n$ ligger i Hilbertrummet?

Välj ett alternativ:

- Det följer från Bessels olikhet och definitionen av Hilbertrum.
- Det följer från Pythagorus satsen och Parsevals likhet.
- Det följer från Pythagorus satsen och Plancharels likhet.
- Det följer från de 3 ekvivalenta konditioner för att vara ett ortogonal bas.

Totalpoäng: 2

40 CAT June 2022

Vilken uppskattning gör vi först i den faltnings-approximerings-satsen? (convolution approximation theorem)

Välj ett alternativ:

- Vi uppskattar integralen nära noll.
- Vi uppskattar inte utan vi börjar med variabelseparation.
- Vi uppskattar en Laplacetransform.
- Vi uppskattar integralen nära plus eller minus oändligt.
- Vi uppskattar en Fouriertransform.
- Vi uppskattar lösningen till en PDE.
- Vi uppskattar en Fourierserie.
- Vi uppskattar lösningen till ett RSLP.

Totalpoäng: 2

41 Orthogonal bases June 2022

Antar att $\{\phi_n\}_{n\geq 0}$ är ortonormala i ett Hilbertrum. Hur beviser vi att om gäller
(1) $\langle f, \phi_n \rangle = 0 \forall n \implies f = 0$
sedan gäller
(2) $f = \sum_{n\geq 0} \langle f, \phi_n \rangle \phi_n$
?

Välj ett alternativ:

- Vi definerar $g = \sum_{n\geq 0} \langle f, \phi_n \rangle$ och visar att g-f=0.
- Vi definerar $g = \sum_{n\geq 0} \langle f, \phi_n \rangle \phi_n$ och vi visar att $\langle f - g, \phi_n \rangle = 0 \forall n$
- Vi definerar $g = \sum_{n\geq 0} \langle f, \phi_n \rangle \phi_n$ och antar att $f \neq g$ och bevisa en motsägelse. (proof by contradiction)
- Vi använder triangel olikhet och Cauchy-Schwartz olikhet.
- Vi använder Plancharels likhet och Bessels olikhet.
- Vi använder Parsevals likhet och Bessels olikhet.

Totalpoäng: 2