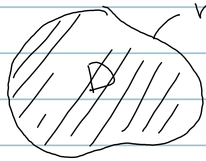
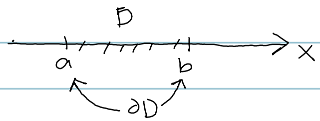
Föreläsning 17/9-13 em

Integralsatser



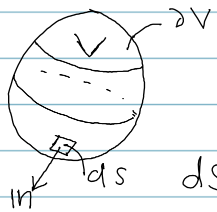
Typiskt: $\int_D (\text{derivata av fält}) = \int_{\partial D} \text{fält}$

Gauss sats i en dimension



$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a)$$

Gauss sats i tre dimensioner



$$\iiint_V \nabla \cdot \mathbb{F}\, dV = \iint_{\partial V} \mathbb{F} \cdot d\mathbb{S}$$

$d\mathbb{S} = \mathbb{n}\, ds$

Exempel

$\mathbb{F} = \mathbb{r} = (x, y, z)$, $\nabla \cdot \mathbb{F} = 3$

Exempel (forts.)

Tag V som bollen $r \leq a$

∂V: sfären $r = a$

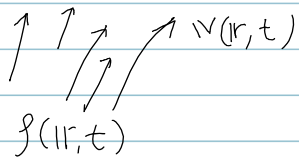
VL: $\iiint_V \nabla \cdot \mathbb{F}\, dV = 3 \iiint_V dV = \not{3} \cdot \frac{4\pi}{\not{3}} a^3 = 4\pi a^3$

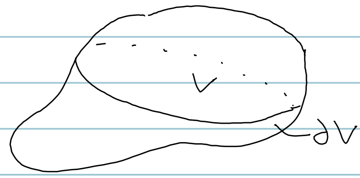
forts. →

$$H.L = \iint_{\partial V} \mathbb{F} \cdot d\mathbb{S} = a \iint_{r=a} dS = a \cdot 4\pi a^2 = 4\pi a^3$$

$HL = VL$!

Kontinuitetsekvationen





Flödestäthet $\mathbb{j} = \rho \mathbb{v}$

Massa i V: $m_V(t) = \iiint_V \rho \, dV \Rightarrow \dfrac{dm_V}{dt} = \iiint \dfrac{\partial \rho}{\partial t} dV$

Men också genom att kolla flödet genom randen: $\dfrac{dm_V}{dt} = -\iint_{\partial V} \mathbb{j} \, d\mathbb{S} = \{\text{Gauss}\} =$

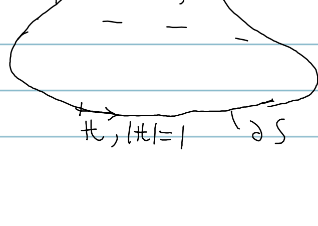
$$= -\iiint_V \nabla \cdot \mathbb{j} \, dV$$

$$\Rightarrow \iiint_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbb{j} \right) dV = 0 \Longrightarrow \boxed{\frac{\partial \rho}{\partial t} + \nabla \mathbb{j} = 0}$$

Enda antagandena:

- ρ o $\mathbb{j}$ täthet och ström för samma storhet (i exemplet: massa)
- Denna storhet är bevarad.

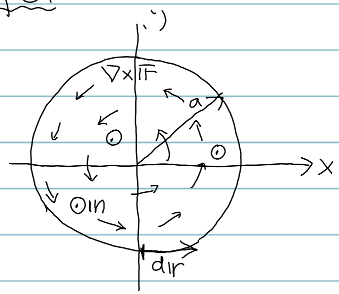
Stokes sats



$$\iint_S (\nabla \times \mathbb{F}) \cdot d\mathbb{S} = \int_{\partial S} \mathbb{F} \cdot d\mathbb{r}$$

$$d\mathbb{r} = \mathbb{t} \, ds$$

Exempel



$d\mathbb{S} = \hat{z}$, $\mathbb{F} = x\hat{y} - y\hat{x}$,

$\nabla \times \mathbb{F} = 2\hat{z}$, $\mathbb{F} = \rho\hat{\varphi} = a\hat{\varphi}$

$$\int (\nabla \times \mathbb{F})\, d\mathbb{S} = 2 \int dS = 2\pi a^2$$

$$d\mathbb{r} = \hat{\varphi}\, ds$$

$$\int_{\substack{\rho = a \\ z=0}} \mathbb{F} \circ d\mathbb{r} = a \int_{\substack{\rho = a \\ z = 0}} ds = a \cdot 2\pi a = 2\pi a^2$$

Samma, bra!