MTF053 - Fluid Mechanics

2023-10-27 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

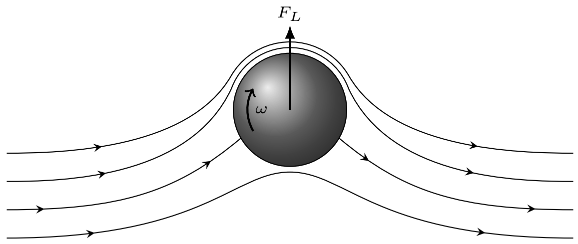
- In total 6 problems, each worth 10p

Grading:

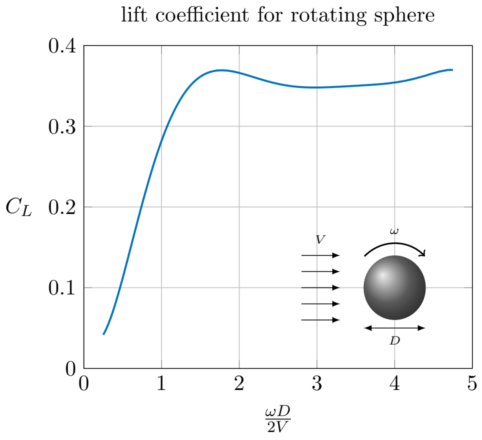
number of points on exam	24-35	36-47	48-60
grade	3	4	5

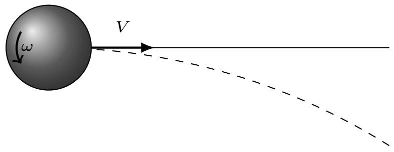
PROBLEM 1 - TABLE-TENNIS BALL (10 P.)

It is a classic result from potential flow theory that rotating cylinders and spheres will generate lift. The figure below shows a schematic illustration of the streamlines around a rotating sphere. As a consequence of the rotation, the upper part of the sphere will rotate with the flow increasing the flow velocity locally due to the no-slip condition and in the same way, the lower part will rotate against the flow direction and thus decrease the flow velocity. The result is a net turning of the flow, which will lead to generation of a lift force in the flow-normal direction (in the same way as the net turning of the flow generated by a wing is associated with a lift force). The lift force generated by rotating spheres and cylinders is often referred to as the Magnus effect and is the physical principle behind, for example, David Beckham's famous bended free kicks – in that case, there might be a certain amount of talent involved as well.



(a) Calculate the backspin (ω) required to make a table-tennis ball follow a horizontal path rather than the curved path that it would follow without adding spin to the ball. The weight and diameter of a table-tennis ball are 2.5 g and 38.0 mm, respectively. After hitting the ball, its velocity in the horizontal direction is $V = 12.0\ m/s$ (6p.)





Theory questions related to the topic:

(b) If you are going to do an experimental investigation of a problem including several important physical variables, why is it beneficial to divide the variables into non-dimensional groups? (1p.)

(c) Make a schematic representation of the pressure distribution around a cylinder for inviscid flow (potential flow), viscous flow with laminar and turbulent separation respectively. Explain why the pressure varies the way it does. Which of the three cases will give the lowest and highest pressure drag? (2p.)

(d) Why do dimpled golf balls have lower pressure drag than golf balls with smooth surfaces? (1p.)

PROBLEM 2 -WATER SKI (10 P.)

Although the flow over the bottom surface of a water ski in use is rather far from a flow over a flat plate, assuming that the flow resembles a flat plate flow will give a quite good estimate of the skin friction drag. Let's investigate the skin friction drag of a water ski that is $L = 1.5\ m$ long and $b = 0.15\ m$ wide. For the boundary layer analysis it can be assumed that transition to turbulence takes place at a local Reynolds number of $Re_x = 5.0 \times 10^5$.

(a) What is the maximum velocity for which the entire boundary layer built up under the water ski will be laminar? (2p.)

(b) Make two graphs that shows how the transition location and total drag varies with velocity (V) for velocities in the range $1.0\ m/s < V < 9.0\ m/s$ (6p.)

Theory questions related to the topic:

(c) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (1p.)

(d) Name two alternative ways to measure the boundary layer thickness than δ. How can these measures be interpreted physically? (1p.)

PROBLEM 3 - PIPE FLOW (10 P.)

An engineer works on a construction where water at 20°C flows through a 30.0 m long galvanized iron pipe (new condition) with the diameter $D = 7.5\ cm$ at a flow rate of $Q = 0.09\ m^3/s$.

(a) Based on the information given above, calculate the head loss. (3p)

(b) At a design meeting where the engineer presents his part of the construction, the calculated head loss is deemed to be too high for the pumps installed upstream. After a bit of research, the engineer finds that it would be possible to add a thin plastic liner coating to the pipe walls and make the pipe hydraulically smooth. The addition of the liner will however make the effective diameter slightly smaller. As input for the next project meeting the engineer calculates the head loss for a smooth pipe at the same Reynolds number as for the rough pipe (the maximum possible reduction of head loss) and the diameter for a smooth pipe that gives the same head loss as the rough pipe (the smallest diameter that could be allowed). Calculate these values. (5p.)

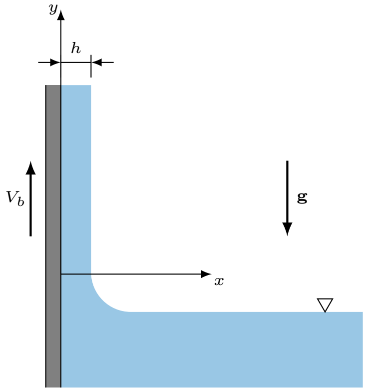
Theory questions related to the topic:

(c) What does the concept *entrance length* mean? How does the flow velocity profile change over the entrance length? (1p.)

(d) How does the turbulence viscosity μ_t compare to the fluid viscosity μ in the viscous sublayer and in the fully turbulent region, respectively? (1p.)

PROBLEM 4 - BELT-DRIVEN FLOW (10 P.)

A wide belt passes through a container filled with a viscous liquid. The belt moves vertically upward at a constant velocity V_b. Due to the viscous forces, a fluid film with the thickness h is built up over the belt surface. Since the belt moves vertically, gravity tends to make the fluid drain down the belt. The film flow can be assumed to be laminar, steady, and fully developed.



(a) Starting from the Navier-Stokes equations, derive an expression for the fluid velocity distribution in the liquid film. (5p.)

(b) For what belt velocities V_b will the average velocity in the film be positive? (3p.)

Theory questions related to the topic:

(c) Explain the physical meaning of local acceleration and convective acceleration. (1p.)

(d) How can we simplify the continuity equation on differential form under the following circumstances? (1p.)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

1: steady-state flow

2: incompressible flow

PROBLEM 5 - WATER PUMP (10 P.)

A pump delivers water at a steady flow rate of $Q = 1136.0\ L/min$. Water enters the pump through a 9.0 cm (D_{in}) pipe and leaves the pump through a 2.5 cm (D_{out}) pipe. Just upstream of the pump, the pressure is $p_{in} = 124.0\ kPa$ and the pump increases the pressure to $p_{out} = 414.0\ kPa$. There is a temperature rise over the pump of that corresponds to a rise of the internal energy of the fluid of $d\hat{u} = 278.0\ Nm/kg$. The pump can be considered to be well isolated and thus the flow is adiabatic. Under the above described conditions, the pump consumes 27.5 kW of electric power.

(a) Calculate the power consumption related to losses (the sum of viscous losses, mechanical losses, etc) (5p.)

(b) Break the nominal pump power (pump power without losses) down into its components (pressure rise, increase of kinetic energy, and increase of internal energy), i.e. calculate the fraction of the total nominal pump power that is consumed by each of these components (2p.)

Theory questions related to the topic:

(c) Explain the physical meaning of each of the terms in Reynolds transport theorem: (1p.)

$$\frac{d}{dt}\left(B_{syst}\right) = \frac{d}{dt}\left(\int_{cv} \beta \rho d\mathcal{V}\right) + \int_{cs} \beta \rho \left(\mathbf{V}_r \cdot \mathbf{n}\right) dA$$

(d) What does it mean that inlets and outlets are one-dimensional? (1p.)

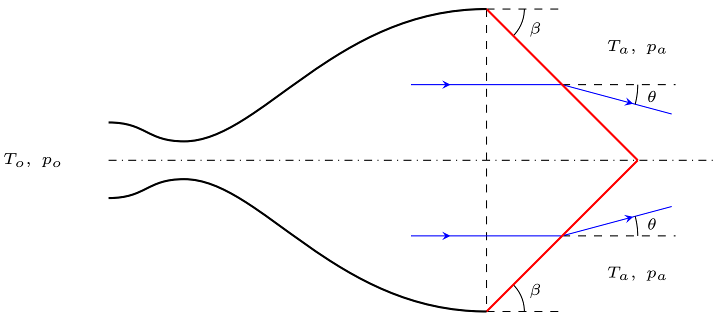
(e) The Bernoulli equation can be said to be a simplified form of the energy equation.

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const$$

In what ways are the Bernoulli equation above more limited than the energy equation? (1p.)

PROBLEM 6 - OVEREXPANDED NOZZLE FLOW (10 P.)

When a convergent-divergent nozzle operates at overexpanded conditions, oblique shocks are formed at the nozzle exit as illustrated in the figure below. The presence of the oblique shocks leads to a change of flow direction as the jet flow passes through the shock. In an experiment where air was expanded through a convergent-divergent nozzle into a room at atmospheric conditions ($p_a = 101325\ Pa$, and $T_a = 293\ K$), Schlieren imaging reviled the presence of oblique shocks downstream of the nozzle exit. From the Schlieren images, the shock angle could be estimated to be $\beta = 45°$ and the flow deflection angle (the change of flow direction over the shock) was estimated to be $\theta = 15°$.



(a) Calculate the exit-to-throat area ratio for the nozzle A/A^* (4p.)

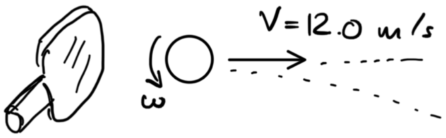
(b) Calculate the total conditions at the nozzle inlet (T_o and p_o) (4p.)

Theory questions related to the topic:

(c) Which of the properties h_o, T_o, a_o, p_o, and ρ_o are constants in a flow if the flow is adiabatic and isentropic, respectively? (1p.)

(d) Show schematically how the oblique shock formed ahead of a wedge traveling at supersonic speed if the flow deflection angle (half the wedge angle) is greater than the maximum deflection angle and less than the maximum defection angle, respectively. (1p.)

P1] TABLE-TENNIS BALL

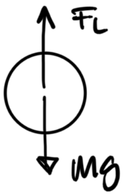


$M_{ball} = 2.5 g$

$D_{ball} = 38.0 mm$

FIND THE BACKSPIN THAT MAKES THE BALL FOLLOW A HORIZONTAL PATH..

IF THE BALL IS TO FOLLOW A HORIZONTAL PATH, THE WEIGHT OF THE BALL HAS TO BE BALANCED BY THE LIFT

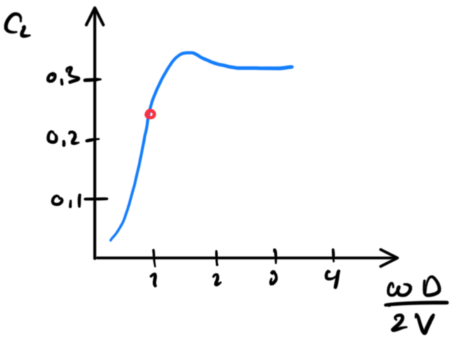


$F_L = mg$

$$F_L = C_L \frac{1}{2} \rho V^2 \frac{\pi D^2}{4} \Rightarrow$$

$$mg = C_L \frac{1}{2} \rho V^2 \frac{\pi D^2}{4} \Rightarrow$$

$$C_L = \frac{8 mg}{\rho V^2 \pi D^2} \approx 0.24$$



$$C_L = 0.24 \Rightarrow \frac{\omega D}{2V} \simeq 0.9$$

$$\Rightarrow \omega = 549.5 \text{ rad/s}$$

(5247 rpm)

P2 | WATER SKI

GIVEN:

DIMENSIONS OF WATER SKI:

LENGTH : $L = 1.5\,m$
WIDTH : $b = 0.15\,m$

TRANSITION REYNOLDS NUMBER:

5.0×10^{5}

ASSUME FRESH WATER @ 20°C

(WHO IN THEIR RIGHT MIND WOULD EVER CONSIDER SALT WATER 🙂)

$\rho = 998\ kg/m^3$
$\mu = 1.0 \times 10^{-3}\ kg/ms$

a) FIND THE MAXIMUM VELOCITY FOR WHICH THE ENTIRE BOUNDARY LAYER UNDER THE SKI IS LAMINAR.

LAMINAR BL. ⇒ TRANSITION NOT REACHED ⇒ MAX VELOCITY WHEN THE TRANSITION REYNOLDS NUMBER IS JUST REACHED AT THE TRAILING EDGE. ($@\ x = L$)

$$\Rightarrow Re_L = Re_{transition}$$

$$\frac{\rho U L}{\mu} = 5.0 \times 10^{5} \Rightarrow U = 0{,}33\ m/s$$

COMMENT: $U = 0.33\ m/s$ OR 1.2 km/h IS A FAR TO LOW VELOCITY AND THUS ONE WOULD NEVER EXPERIENCE FULLY LAMINAR BOUNDARY LAYER IN REALITY

b) MAKE GRAPHS SHOWING THE TRANSITION LOCATION AND TOTAL (VISCOUS) DRAG AS FUNCTION OF VELOCITY FOR $1.0 < U < 9.0$

$U = 1.0 > 0,33 \Rightarrow$ THERE WILL BE TRANSITION TO TURBULENCE..

$$Re_{x_{cr}} = \frac{\rho U x_{cr}}{\mu} = 5.0 \times 10^5 = Re_{tr}$$

$$\Rightarrow x_{cr} = \frac{\mu Re_{tr}}{\rho U} \qquad (1)$$

FOR BOUNDARY LAYERS WITH TRANSITION WE CAN USE EQN. 7.49

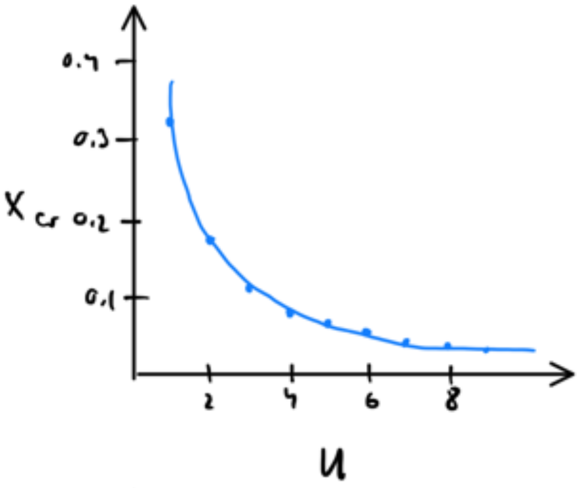
$$C_D = \frac{0,031}{Re_L^{1/7}} - \frac{1400}{Re_L} \qquad (2)$$

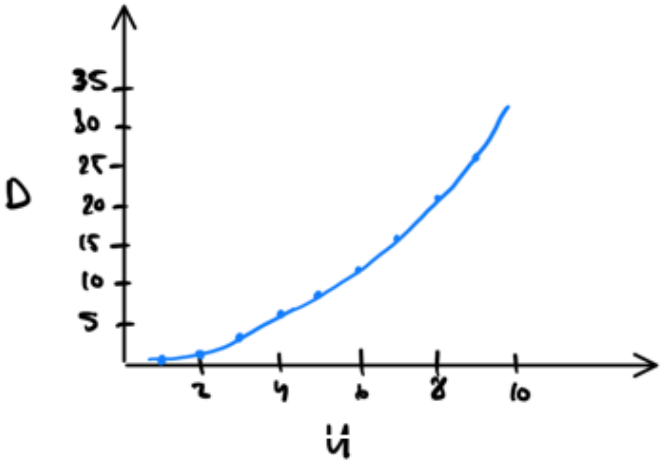
WHERE $Re_L = \frac{\rho U L}{\mu}$

THE TOTAL VISCOUS DRAG IS THEN CALCULATED AS

$$D = C_D \frac{1}{2} \rho U^2 b L \qquad (3)$$

U (m/s)	x_{cr} (m)	D (N)
1.0	0.334	0.35
1.5	0.223	0.81
2.0	0.167	1.44
2.5	0.134	2.24
3.0	0.111	3.20
3.5	0.095	4.31
4.0	0.087	5.57
4.5	0.074	6.98
5.0	0.067	8.54
5.5	0.061	10.25
6.0	0.056	12.09
6.5	0.051	14.08
7.0	0.048	16.21
7.5	0.045	18.47
8.0	0.042	20.87
8.5	0.039	23.40
9.0	0.037	26.07





P3 | PIPE FLOW

GIVEN :

WATER @ 20°C :

$$\rho = 998 \ kg/m^3$$
$$\mu = 1.0 \cdot 10^{-3} \ kg/ms$$

FLOW RATE : $Q = 0{,}09 \ m^3/s$

PIPE LENGTH : $L = 30.0 \ m$

PIPE DIAMETER : $D = 7{,}5 cm$ $(0{,}075 m)$

PIPE MATERIAL : GALVANIZED IRON (NEW)
$\Rightarrow \varepsilon = 0{,}15 \ mm$

a) CALCULATE THE HEAD LOSS

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

$$Re_D = \frac{\rho V D}{\mu} = \frac{4 \rho Q}{\mu \pi D} \approx 1.5 \cdot 10^6$$

$\Rightarrow$ TURBULENT.

$$h_f = f \frac{L V^2}{D 2g} \qquad (6.10)$$

$$h_f = \underset{\substack{\uparrow \\ \text{unknown.}}}{f} \ \frac{8 Q^2 L}{\pi^2 g D^5} \qquad (1)$$

COLEBROOK/MOODY (6.48)

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right)$$

(USE MOODY CHART WITH $\varepsilon / D = 2.0 \cdot 10^{-3}$ OR SOLVE USING ITERATIVE METHOD..)

$$\Rightarrow f = 2{,}35 \cdot 10^{-2} \qquad (2)$$

$$(2) \text{ in } (1) \Rightarrow h_f = \underline{199.2 \ m}$$

b)

THIN PLASTIC LINER COATING ⇒
HYDRAULICALLY SMOOTH PIPE

TWO SCENARIOS:

I) SAME REYNOLDS NUMBER AS IN a)
⇒ MAXIMUM REDUCTION OF HEAD LOSS

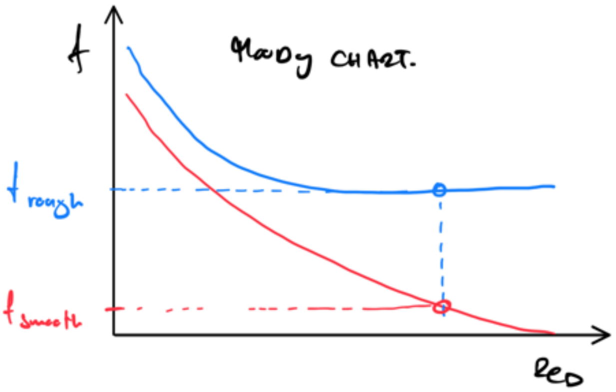
II) SAME HEAD LOSS AS IN a)
⇒ MAXIMUM DIAMETER REDUCTION ALLOWED.

I)

SAME REYNOLDS NUMBER

SAME FLOW RATE AND $Re_D = \frac{4\rho Q}{\pi D\mu}$

⇒ D IS THE SAME AS IN a)



MOODY CHART OR (6.38)

$\Rightarrow f = 1.08 \cdot 10^{-2}$ (3)

(3) in (1) $\Rightarrow h_f = 91.8$ m

II) SAME h_f AS IN a)

$$\begin{cases} h_f = f \dfrac{8Q^2 L}{\pi^2 g D^5} \\ Re_D = \dfrac{4\rho Q}{\mu \pi D} \\ \dfrac{1}{\sqrt{f}} = 2.0 \log_{10}\left(Re_D \sqrt{f}\right) - 0.8 \end{cases}$$

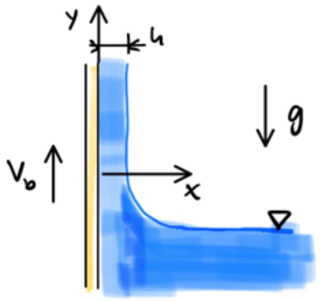
(6.38)

SOLVE ITERATIVELY:

$\Rightarrow D = 6.4$ cm ($Re_D = 1.8 \cdot 10^{6}$)

COMMENT: THE RESULT SHOW THAT THE MINIMUM HEAD LOSS THAT WE CAN GET is 91.8 m (IF THE PLASTIC COATING HAD ZERO THICKNESS..) IF THE THICKNESS OF THE COATING GIVES A NEW INNER DIAMETER SMALLER THAN 6.4 cm , THE HEAD LOSS WILL INCREASE AFTER ADDING THE COATING . THIS INFORMATION CAN BE USED TO MAKE A DECISION..

P4 BELT-DRIVEN FLOW



- WIDE BELT ⇒ 2D FLOW
- LAMINAR
- STEADY STATE
- FULLY DEVELOPED

NAVIER-STOKES EQUATIONS y-DIRECTION

$$\rho\left(\underbrace{\frac{\partial v}{\partial t}}_{=0} + \underbrace{u}_{=0}\frac{\partial v}{\partial x} + v\underbrace{\frac{\partial v}{\partial y}}_{=0} + \underbrace{w}_{=0}\frac{\partial v}{\partial z}\right) =$$

$$= -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \underbrace{\frac{\partial^2 v}{\partial y^2}}_{=0} + \underbrace{\frac{\partial^2 v}{\partial z^2}}_{=0}\right)$$

STEADY STATE ⇒ $\frac{\partial v}{\partial t} = 0$

FULLY DEVELOPED (OR CONTINUITY) ⇒ $\frac{\partial ()}{\partial y} = 0$

NO FLOW IN X OR Z DIRECTION ⇒ $u = w = 0$

2D FLOW ⇒ $\frac{\partial ()}{\partial z} = 0$

$$\Rightarrow \quad 0 = -\frac{\partial p}{\partial y} + \rho g_y + \mu\frac{\partial^2 v}{\partial x^2}$$

THE PRESSURE OUTSIDE THE FILM IS THE ATMOSPHERIC PRESSURE ⇒ $\frac{\partial p}{\partial y} = 0$

$$g_y = -g$$

⇒

$$0 = -\rho g + \mu\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\rho g}{\mu}$$

INTEGRATE:

$$\frac{\partial v}{\partial x} = \frac{\rho g}{\mu}x + C_1$$

$$v(x) = \frac{1}{2}\frac{\rho g}{\mu}x^2 + C_1x + C_2$$

BOUNDARY CONDITIONS :

1) $\tau_{xy} = 0$ @ $x = h$

$$\tau_{xy} = \mu \frac{\partial v}{\partial x} = \rho g x + C_1 \mu$$

$$\tau_{xy}\Big|_{x=h} = 0 \Rightarrow$$

$$\rho g h + C_1 \mu = 0 \Rightarrow C_1 = -\frac{\rho g h}{\mu}$$

2) $V(0) = V_b \Rightarrow$

$$V_b = \frac{1}{2}\frac{\rho g}{\mu} 0^2 + C_1 \cdot 0 + C_2$$

$$\Rightarrow C_2 = V_b$$

$$\boxed{V(x) = \frac{\rho g}{\mu} x \left(\frac{x}{2} - h\right) + V_b}$$

b) FOR WHAT BELT VELOCITIES (V_b) WILL THE AVERAGE FLOW VELOCITY BE POSITIVE?

$$V_{AV} = \frac{Q}{A} = \frac{b\,q}{b\,h} = \frac{q}{h} \quad ,(1)$$

where Q IS THE FILM FLOW RATE,
A IS THE FILM CROSS-SECTION AREA,
b IS THE BELT WIDTH , AND
q IS THE FLOW RATE PER UNIT WIDTH.

$$q = \int_0^h V(x)\,dx =$$

$$= \int_0^h \frac{\rho g}{\mu}\left(\frac{x^2}{2} - xh\right) + V_b\,dx$$

$$q = \left[\frac{\rho g}{\mu}\left(\frac{x^3}{6} - \frac{x^2 h}{2}\right) + V_b h\right]_0^h$$

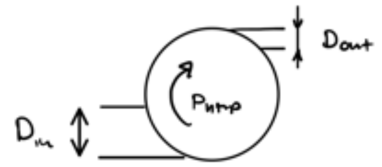
$$\Rightarrow q = V_b h - \frac{1}{3}\frac{\rho g h^3}{\mu} \quad (2)$$

(2) in (1) $\Rightarrow$

$$\boxed{V_{av} = V_b - \frac{1}{3}\frac{\rho g h^2}{\mu}}$$

$$V_{av} > 0 \Rightarrow V_b > \frac{1}{3}\frac{\rho g h^2}{\mu}$$

P5 | WATER PUMP



GIVEN:

FLOW RATE : $Q = 1136\ L/min$

$D_1 = D_{in} = 9.0\ cm$

$P_1 = 124\ kPa$

$D_2 = D_{out} = 2.5\ cm$

$P_2 = 414\ kPa$

TEMPERATURE RISE OVER PUMP

$\Rightarrow \hat{u}_2 - \hat{u}_1 = 278\ Nm/kg$

THE PUMP IS WELL ISOLATED $\Rightarrow$ ADIABATIC

CONSUMED POWER 27.5 kW

ASSUME :

STEADY STATE

ONE INLET AND ONE OUTLET

NEGLIGIBLE ELEVATION CHANGE

WATER @ 20°C $\Rightarrow \rho = 998\ kg/m^3$

a) CALCULATE THE POWER CONSUMPTION RELATED TO LOSSES..

STEADY FLOW WITH ONE INLET AND ONE OUTLET :

(3.70)

$$\hat{h}_1 + \frac{1}{2}V_1^2 + gz_1 = \hat{h}_2 + \frac{1}{2}V_2^2 + gz_2 +$$

$$- q + w_s + w_v$$

NOTE:

IT WOULD BE MORE CORRECT TO WRITE THE KINETIC ENERGY TERMS AS $\frac{1}{2}\alpha V^2$ WHERE α IS THE KINETIC ENERGY CORRECTION FACTOR.

LET'S ASSUME TURBULENT FLOW $\Rightarrow \alpha \simeq 1.0$

Now,

ADIABATIC ⇒ $q = 0$

NEGLIGIBLE CHANGE IN ELEVATION ⇒

⇒ $z_1 = z_2$

⇒

$$\hat{h}_1 + \frac{1}{2}V_1^2 = \hat{h}_2 + \frac{1}{2}V_2^2 + W_s + W_v$$

$$\hat{h} = \hat{u} + \frac{P}{\rho} \Rightarrow$$

$$\hat{u}_1 + \frac{P_1}{\rho} + \frac{1}{2}V_1^2 = \hat{u}_2 + \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + W_s + W_v$$

THE VISCOUS WORK W_v IS PART OF LOSS THAT WE ARE SUPPOSED TO CALCULATE SO WE WILL REMOVE IT HERE..

$$\hat{u}_1 + \frac{P_1}{\rho} + \frac{1}{2}V_1^2 = \hat{u}_2 + \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + W_s$$

W_s IS BY DEFINITION NEGATIVE FOR A PUMP SINCE WORK IS ADDED TO THE FLUID ⇒

$$W_s = -W_{pump}$$

$$\Rightarrow (\hat{u}_2 - \hat{u}_1) + \frac{1}{\rho}(P_2 - P_1) + \frac{1}{2}(V_2^2 - V_1^2) =$$
$$= W_{pump}$$

THE ONLY THING WE NEED NOW TO CALCULATE THE NOMINAL PUMP WORK (WORK WITH NO VISCOUS LOSSES) IS THE AVERAGE FLOW VELOCITIES..

$$V = \frac{Q}{A}$$

$$Q = 1136 \text{ L/min} = \frac{1136}{1000.}\,\frac{1}{60}\ \text{m}^3/\text{s}$$
$$= 0.019\ \text{m}^3/\text{s}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q\,4}{\pi D_1^2} = 3.0\ \text{m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{Q\,4}{\pi D_2^2} = 38.6\ \text{m/s}$$

WE ARE ASKED TO CALCULATE POWER
SO WE NEED THE MASSFLOW

$$\dot{m} = Q \cdot \rho = 18.9 \text{ kg/s}$$

$$\dot{W}_{pump} = \dot{m}\left[(\hat{u}_2 - \hat{u}_1) + \frac{1}{\rho}(p_2 - p_1) + \right.$$

$$\left. + \frac{1}{2}(V_2^2 - V_1^2)\right] = 24.8 \text{ kW}$$

CONSUMED POWER = 27.5 kW

=> POWER CONSUMED BY LOSSES:

$$\dot{W}_{loss} = 27.5 \cdot 10^3 - \dot{W}_{pump} =$$

$$= 2.7 \text{ kW}$$

COMMENT:

THE PUMP EFFICIENCY CAN NOW BE CALCULATED AS

$$\eta_{pump} = \frac{\dot{W}_{pump}}{\dot{W}_{consumed}} = \frac{24.8}{27.5} \approx 0.9$$

b) BREAK THE NOMINAL PUMP POWER DOWN INTO ITS COMPONENTS:

$$\dot{W}_{internal\ energy} = \dot{m}(\hat{u}_2 - \hat{u}_1) = 5.3 \text{ kW}$$

$$\dot{W}_{pressure} = \dot{m}\frac{1}{\rho}(p_2 - p_1) = 5.5 \text{ kW}$$

$$\dot{W}_{kinetic} = \dot{m}\frac{1}{2}(V_2^2 - V_1^2) = 14 \text{ kW}$$

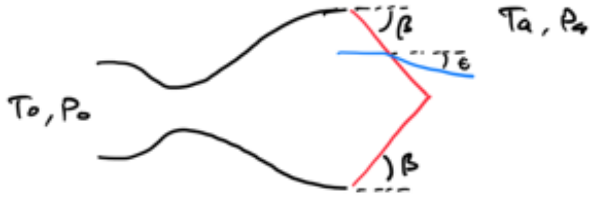
$$\frac{\dot{W}_{internal\ energy}}{\dot{W}_{pump}} = 19\%$$

$$\frac{\dot{W}_{pressure}}{\dot{W}_{pump}} = 20\%$$

$$\frac{\dot{W}_{kinetic}}{\dot{W}_{pump}} = 51\%$$

P6 | OVEREXPANDED NOZZLE FLOW

OVEREXPANDED => OBLIQUE SHOCKS
AT NOZZLE EXIT



GIVEN:

SHOCK ANGLE: $\beta = 45°$

FLOW DEFLECTION ANGLE: $\theta = 15°$

AMBIENT CONDITION:

$P_a = 101325$ Pa

$T_a = 293$ K

ASSUME:

INVISCID FLOW
AIR (CALORICALLY PERFECT)
$\gamma = 1.4$

CALCULATE:

a) A_e / A^*

b) NOZZL INLE CONDITIONS: T_0, P_0

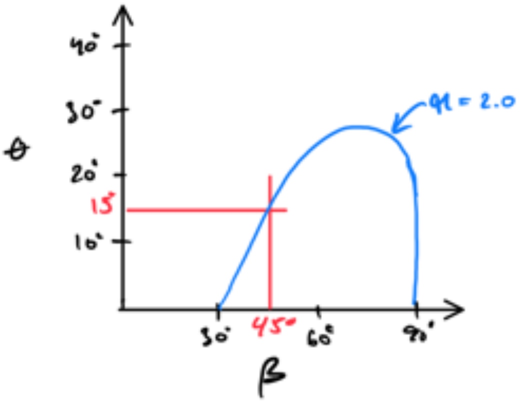
a)

WE KNOW THE SHOCK ANGLE (β) AND THE FLOW DEFFLECTION ANGLE (θ) THUS WE CAN GET THE MACH NUMBER UPSTREAM OF THE SHOCKS (WHICH IS EQUAL TO THE NOZZLE EXIT MACH NUMBER) USING THE θ-β-M -RELATION

(9.86)

$$\tan \theta = \frac{2 \cot\beta \left(M_e^2 \sin^2\beta - 1.0\right)}{M_e^2 \left(\gamma - \cos(2\beta)\right) + 2.0}$$

OR FIG. 9.23



$$\theta - \beta - M \Rightarrow M_e = 2.0$$

WITH THE EXIT MACH NUMBER KNOWN, WE CAN CALCULATE THE AREA RATIO A_e / A^* USING THE AREA-MACH-NUMBER RELATION

(9.44)

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left[\frac{2.0 + (\gamma - 1)M_e^2}{\gamma + 1}\right]^{\frac{(\gamma+1)}{(\gamma-1)}}$$

NOTE : SINCE THE FLOW IS SUPERSONIC IN THE DIVERGENT PART OF THE NOZZLE, $A^* = A_{throat}$

(THE FLOW MUST BE SUPERSONIC SINCE SHOCKS ARE FORMED DOWNSTREAM OF THE NOZZLE EXIT)

$$\frac{A_{exit}}{A_{throat}} = 1.7$$

b) FIRST WE MUST CALCULATE THE TEMPERATURE AND PRESSURE AT THE NOZZLE EXIT, WHICH WE DO USING THE OBLIQUE SHOCK RELATIONS..

1) CALCULATE THE SHOCK-NORMAL MACH NUMBER UPSTREAM OF THE OBLIQUE SHOCK:

$$M_{n_1} = M_e \sin(\beta) \qquad (9.82)$$

WHERE M_e IS THE NOZZLE EXIT MACH NUMBER CALCULATED IN a)

2) USE THE NORMAL-SHOCK RELATIONS

$$\frac{\overset{\text{AMBIENT PRESSURE}}{p_a}}{\underset{\text{NOZZLE EXIT PRESSURE}}{p_e}} = 1 + \frac{2\gamma}{\gamma+1}\left(\underset{\text{SHOCK NORMAL MACH NUMBER}}{M_{n_1}^2} - 1\right) \quad (9.55)$$

$$\frac{\overset{\text{AMBIENT TEMPERATURE}}{T_a}}{\underset{\text{NOZZLE EXIT TEMPERATURE.}}{T_e}} = \left(2.0 + (\gamma-1)M_{n_1}^2\right) \cdot \left[\frac{2\gamma M_{n_1}^2 - (\gamma-1)}{(\gamma+1)^2 \underset{\text{SHOCK-NORMAL MACH NUMBER}}{M_{n_1}^2}}\right] \quad (9.58)$$

$\Rightarrow \quad p_e = 46.0 \text{ kPa}$

$T_e = 230.6 \text{ K} \quad (-42.4\,^\circ\text{C})$

WITH THE EXIT CONDITIONS KNOWN WE CAN CALCULATE THE TOTAL PRESSURE AND TOTAL TEMPERATURE. SINCE THERE ARE NO INTERNAL SHOCKS, THE NOZZLE EXPANSION IS ISENTROPIC $\Rightarrow$ T_o AND P_o ARE CONSTANT THROUGH THE NOZZLE.

$$(9.26) \quad \frac{T_o}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 \quad \Rightarrow T_o = 417.8\,K$$

$$(9.26a) \quad \frac{P_o}{P_e} = \left(\frac{T_o}{T_e}\right)^{\gamma/(\gamma-1)} \quad \Rightarrow P_o = 368.5\,kPa$$

$\therefore$ NOZZLE INLET CONDITIONS:

$T_o = 417.8\ K$ (145°C)

$P_o = 368.5\ kPa$