MTF053 - Fluid Mechanics

2023-08-14 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Steam Turbine (10 p.)

A turbine is used to extract energy from steam. Steam flows into the turbine through a 300.0 mm duct and leaves the turbine through a 500.0 mm duct. On the inflow side of the turbine, steam enters at a velocity of 35.0 m/s, with a density of 0.6 kg/m^3 and an enthalpy of 5000.0 kJ/kg. The steam leaving the turbine at the exit has a density of 0.1 kg/m^3 and an enthalpy of 3000.0 kJ/kg. The system is sufficiently well insulated such that adiabatic conditions can be assumed. Effects of gravity can be neglected.

(a) Calculate the average flow velocity in the outlet duct (3p.)

(b) Estimate the power extracted by the turbine (5p.)

Theory questions related to the topic:

(c) What does it mean that inlets and outlets are one-dimensional? (1p.)

(d) Give examples of when it is appropriate to use fixed control volume, moving control volume, and deformable control volume, respectively. (1p.)

Problem 2 - Laminar Flow Between Parallel Plates (10 p.)

SAE 30W oil at a temperature of 20.0°C flows between two parallel plates separated by 40.0 mm. The plates have a length of 1.0 m in the flow direction and a width of 0.5 m. The flow between the plates is laminar. The lower plate is stationary, the upper plate moves at a constant velocity of 5.0 cm/s, and the pressure gradient in the flow direction is constant at -500.0 Pa/m.

(a) Derive an expression for the flow velocity distribution in the fluid between the parallel plates (5p.)

(b) Calculate the force needed to move the upper plate (3p.)

Theory questions related to the topic:

(c) Derive the momentum equation on differential form starting from the integral form (2p.)

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{cv} \mathbf{V}\rho d\mathcal{V}\right) + \sum (\dot{m}_i \mathbf{V}_i)_{out} - \sum (\dot{m}_i \mathbf{V}_i)_{in}$$

Problem 3 - Pipe Flow (10 p.)

Water at 10°C flows through an old rusty steel pipe (i.e. the pipe surface is not smooth). Measurements indicate that the friction factor remains constant at 0.0321 as long as the average velocity of the water flowing through the pipe exceeds 0.72 m/s.

(a) Based on the information given above, estimate the average height of the surface irregularities (the surface roughness) in the pipe (7p.)

Theory questions related to the topic:

(b) For fully developed laminar pipe flow, the velocity profile can be expressed as

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

Show that the average velocity in fully developed laminar pipe flow is half the maximum velocity. (2p.)

(c) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$? (1p.)

Problem 4 - Drag Force (10 p.)

The width, height, engine power, and drag coefficient of a typical truck and a typical car are given in the table below.

Vehicle	Width (m)	Height (m)	Engine power (kW)	C_D
Truck	2.59	4.12	410.13	0.60
Car	1.83	1.46	186.42	0.32

(a) When each vehicle is traveling at a speed of 105 km/h, what fraction of the engine power is used to overcome the aerodynamic drag? (4p.)

(b) Assuming that the aerodynamic drag force is the only important force on the vehicles, calculate the maximum possible velocity for the truck and the car respectively. (3p.)

(c) A 1:10 model of the truck is to be tested in a wind tunnel, will it be possible to establish a flow around the model-scale vehicle representative of the flow around the prototype scale truck traveling at a speed of 105 km/h? (justify your answer) (3p.)

Problem 5 - Flat-Plate Boundary Layer (10 p.)

Water at 20° flows past a smooth flat surface. The freestream velocity (the flow velocity away from the flat surface) is 50.0 mm/s. It can be assumed that transition from laminar to turbulent boundary layer occurs when the Reynolds number reaches 5.0×10^5.

(a) What is the flow velocity at a location 15.0 mm above the flat surface and 0.8 m downstream of the leading edge of the surface? (4p.)

(b) At what axial distance downstream of the leading edge will transition to turbulent boundary layer flow take place? (2p.)

(c) Calculate the thickness of the boundary layer at the transition location (1p.)

Theory questions related to the topic:

(d) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (1p.)

(e) Name two alternative ways to measure the boundary layer thickness than δ. How can these measures be interpreted physically? (2p.)

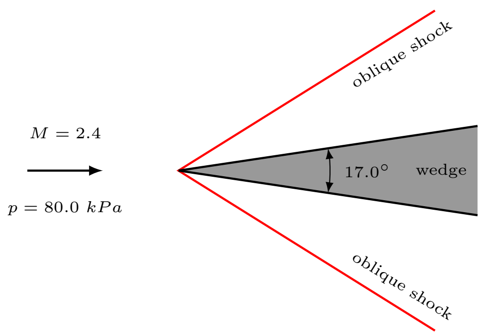
Problem 6 - Supersonic Flow Over a Wedge (10 p.)

Airflow at a Mach number of 2.4 and a pressure of 80.0 kPa impinges on a 17.0° wedge as shown in the figure below. Since the flow is supersonic, oblique shocks will form at the leading edge of the wedge in order to deflect the flow such that it follows the wedge surface. As you may recall from the Fluid Mechanics course, there are two possible solutions to this problem; one referred to as the *weak-shock* solution and the other referred to as the *strong-shock* solution. Of these two solutions, the *weak-shock* solution is the most common (i.e. most often seen in engineering applications) but the *strong-shock* solution is also valid and may occur under certain circumstances.

(a) Why are *weak-shock* solutions more common in engineering applications than *strong-shock* solutions? (1p.)

(b) For the conditions specified in the text above, calculate the shock angle for the *weak-shock* and the *strong-shock* solution, respectively. (6p.)

(c) Calculate the Mach number and pressure downstream of the oblique shock corresponding to the *weak-shock* and the *strong-shock* solution. (3p.)



P1) STEAM TURBINE

GIVEN:

TURBINE INLET :

$D_1 = 300.0$ mm
$V_{av,1} = 35.0$ m/s
$\rho_1 = 0.6$ kg/m^3
$h_1 = 5000.0$ kJ/kg

TURBINE OUTLET :

$D_2 = 500.0$ mm
$\rho_2 = 0.1$ kg/m^3
$h_2 = 3000.0$ kJ/kg

THE SYSTEM IS WELL INSULATED AND CAN BE ASSUMED TO BE ADIABATIC

EFFECTS OF GRAVITY CAN BE NEGLECTED.

a) CALCULATE THE AVERAGE FLOW VELOCITY IN THE OUTLET DUCT..

CONSERVATION OF MASS GIVES..

* FINITE NUMBER OF INLETS AND OUTLETS
* FIXED CONTROL VOLUME.

$$(3.27): \int_{CV} \frac{\partial \rho}{\partial t}\, dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

* STEADY STATE $\Rightarrow$ $\int_{CV} \frac{\partial \rho}{\partial t}\, dV = 0$

ONE INLET AND ONE OUTLET $\Rightarrow$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$\rho_1 V_1 \frac{\pi D_1^2}{4} = \rho_2 V_2 \frac{\pi D_2^2}{4}$$

$$V_2 = V_1 \left(\frac{D_1}{D_2}\right)^2 \left(\frac{\rho_1}{\rho_2}\right) = \underline{75.6 \text{ m/s}}$$

b) ESTIMATE THE POWER EXTRACTED BY THE TURBINE.

(LET'S ASSUME TURBULENT FLOW AND THUS KINETIC ENERGY CORRECTION FACTOR $\alpha = 1.0$)

$$(3.70) \quad h_1 + \frac{1}{2}V_1^2 + \cancel{g z_1} = h_2 + \frac{1}{2}V_2^2 + \cancel{g z_2} - \cancel{q} + w_s + \cancel{w_v}$$

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 + w_s$$

$$\Rightarrow w_s \approx 2000 \text{ kJ/kg}$$

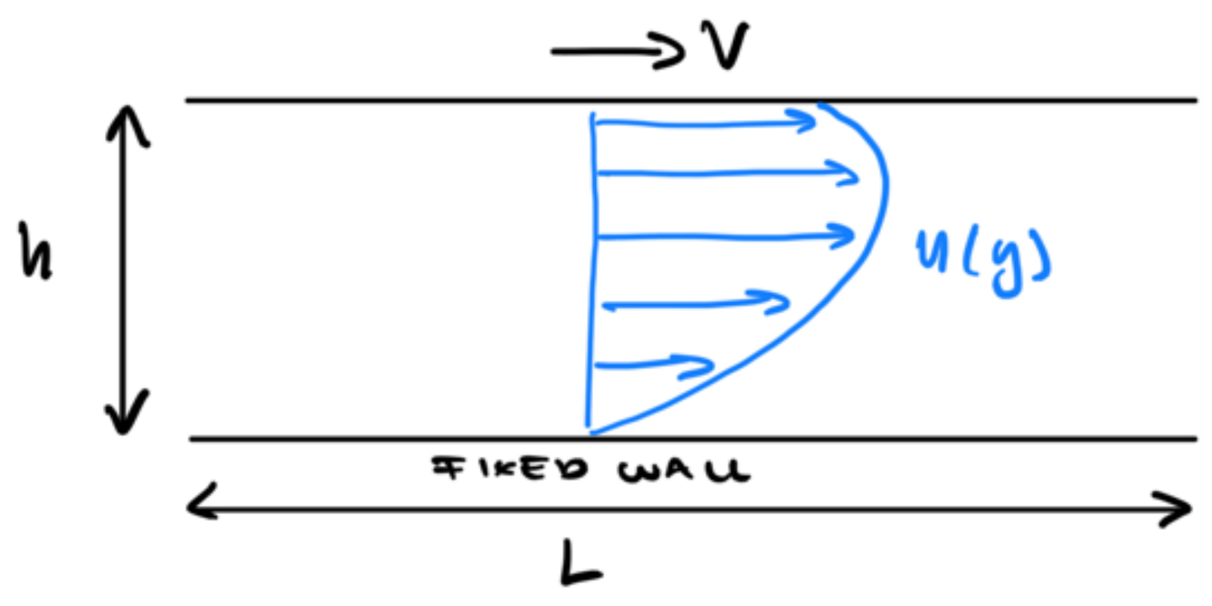
mass flow : $\dot{m} = \rho_1 V_1 A_1 = 1.48$ kg/s

$$\dot{W}_s = w_s \cdot \dot{m} \approx \underline{2.97 \text{ MW}}$$

P2 | LAMINAR FLOW BETWEEN PARALLEL PLATES.

GIVEN:

- SAE 30W OIL @ 20°C $\Rightarrow \mu = 0.29$ kg/m·s
- $h = 40.0$ mm
- $L = 1.0$ m
- $b = 0.5$ m
- LAMINAR FLOW
- $V = 5.0$ cm/s $= 0.05$ m/s
- $dp/dx = -500.0$ Pa/m



CONTINUITY :

(4.4) $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

ASSUME:

STEADY-STATE AND INCOMPRESSIBLE $\Rightarrow$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

ONLY FLOW IN X-DIRECTION $\Rightarrow$

$$\boxed{\frac{\partial u}{\partial x} = 0}$$

MOMENTUM (X-DIRECTION)

(4.38)

$$\rho\left(\underbrace{u\frac{\partial u}{\partial x}}_{=0} + \underbrace{v}_{=0}\frac{\partial u}{\partial y} + \underbrace{w}_{=0}\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \underbrace{\rho g_x}_{=0} + \mu\left(\underbrace{\frac{\partial^2 u}{\partial x^2}}_{=0} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\frac{\partial^2 u}{\partial z^2}}_{=0}\right)$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

INTEGRATE:

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1$$

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$$

APPLY BOUNDARY CONDITIONS:

\# $u(0) = 0$ (NO SLIP LOWER WALL)

$\Rightarrow C_2 = 0$

$u(h) = V$ (NO SLIP UPPER WALL)

$$\Rightarrow \frac{1}{2\mu}\frac{\partial p}{\partial x}h^2 + C_1 h = V$$

$$\Rightarrow C_1 = \frac{V}{h} - \frac{1}{2\mu}\frac{\partial p}{\partial x}h$$

a)

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}y(y-h) + \frac{Vy}{h}$$

$$\frac{\partial u}{\partial y} = \frac{1}{2\mu}\frac{\partial p}{\partial x}(2y-h) + \frac{V}{h}$$

b) CALCULATE THE FORCE NEEDED TO MOVE THE UPPER PLATE.

$$\tau_{top} = \mu\frac{\partial u}{\partial y}\bigg|_{y=h} = \frac{1}{2}\frac{\partial p}{\partial x}(2h-h) + \frac{V}{h}\mu$$

$$= \frac{h}{2}\frac{\partial p}{\partial x} + \frac{V}{h}\mu$$

$$F = \tau_{top}\cdot A = \tau_{top}\cdot L\cdot b = \underline{4{,}82\ N}$$

P3 | PIPE FLOW

GIVEN:

- WATER @ 10°C $\Rightarrow \mu = 1.307 \cdot 10^{-3}$ kg/m s
  $\rho = 1000.0$ kg/m^3
- OLD RUSTY PIPE (NOT SMOOTH)
- FRICTION FACTOR CONSTANT AT $f = 0.0321$ FOR $V > 0.72$ m/s

  $\Rightarrow$ FULLY ROUGH ..

a) ESTIMATE THE HEIGHT OF SURFACE IRREGULARITIES (ϵ)

DEFINITION OF FRICTION VELOCITY:

$$u^* = \sqrt{\frac{\tau_w}{\rho}} \qquad (1)$$

FRICTION FACTOR $\Rightarrow$

$$(6.11) \qquad f = \frac{8\tau_w}{\rho V^2} \Rightarrow \tau_w = \frac{\rho f V^2}{8} \qquad (2)$$

$$(2) \text{ in } (1) \Rightarrow \quad u^* = \sqrt{\frac{f}{8}}\, V$$

SURFACE ROUGHNESS:

$$\epsilon^+ = \frac{\epsilon u^*}{\nu} = \frac{\epsilon u^* \rho}{\mu}$$

FULLY ROUGH $\Rightarrow \epsilon^+ > 70$

LET'S SAY THAT $\epsilon^+ = 70$...

$$70 = \frac{\epsilon u^* \rho}{\mu} = \epsilon \sqrt{\frac{f}{8}} \frac{\rho V}{\mu}$$

$$= \epsilon \simeq \underline{2.0 \text{ mm}}$$

ALTERNATIVE SOLUTION (USING THE MOODY CHART)

$$f = 0.0321 \Rightarrow \epsilon/D \simeq 0.006$$

$$Re_D = \frac{\rho V D}{\mu}$$

$$\text{FULLY ROUGH} \Rightarrow Re_D \approx 2.5 \cdot 10^5$$

$$V = 0.72 \Rightarrow D = 0.45 \text{ m}$$

$$\epsilon / D = 0.006 \Rightarrow \underline{\epsilon = 2.7 \text{ mm}}$$

b) FULLY-DEVELOPED LAMINAR PIPE FLOW

$$u = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

SHOW THAT $U_{av} = \frac{U_{max}}{2}$

$$U_{av} = \frac{1}{A}\int_0^R u(r)\, 2\pi r\, dr =$$

$$= \frac{2\pi U_{max}}{\pi R^2}\int_0^R r - \frac{r^3}{R^2}\, dr =$$

$$= \frac{2 U_{max}}{R^2}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R =$$

$$= \frac{2 U_{max}}{R^2}\left[\frac{R^2}{2} - \frac{R^2}{4}\right] =$$

$$= \frac{2 U_{max}}{R^2}\,\frac{R^2}{4} = \frac{U_{max}}{2}$$

P4 | Drag Force

Given:

Vehicle	Width (m)	Height (m)	Power (kW)	C_D
Truck	2.59	4.12	410.13	0.60
Car	1.83	1.46	186.42	0.32

a) At 105 km/h what fraction of the total power is used to overcome drag for each of the vehicles?

$V = 105 \text{ km/h} \quad (29.17 \text{ m/s})$

Assume air @ 20°C $\Rightarrow \rho = 1.2 \text{ kg/m}^3$

$\mu = 1.8 \cdot 10^{-5} \text{ kg/ms}$

$$F_D = \frac{1}{2}\rho V^2 A_p C_D$$

$$P_D = F_D V = \frac{1}{2}\rho V^3 A_p C_D$$

$$A_p = W \cdot H \Rightarrow P_D = \frac{1}{2}\rho V^3 W H C_D$$

Truck

$$\frac{P_D}{P} = \left[\frac{1}{2}\rho V^3 W_{truck} H_{truck} C_{D_{truck}}\right] / P_{truck}$$

$$\approx 23.2\,\%$$

Car

$$\frac{P_D}{P} = \left[\frac{1}{2}\rho V^3 W_{car} H_{car} C_{D_{car}}\right] / P_{car}$$

$$\approx 6.8\,\%$$

b) Calculate the maximum velocity assuming that aerodynamic drag is the only important force on the vehicles.

$$P_D = P = \frac{1}{2}\rho V^3 W H C_D$$

$$\Rightarrow V = \sqrt[3]{\frac{2P}{\rho W H C_D}}$$

TRUCK:

$$V_{truck} = \sqrt[3]{\frac{2 P_{truck}}{\rho W_{truck} H_{truck} C_{D_{truck}}}}$$

$$\Rightarrow V_{truck} \approx 47.4 \text{ m/s}$$

$$(171 \text{ km/h})$$

CAR:

$$V_{car} = \sqrt[3]{\frac{2 P_{car}}{\rho W_{car} H_{car} C_{D_{car}}}}$$

$$\Rightarrow V_{car} \approx 71.4 \text{ m/s}$$

$$(257 \text{ km/h})$$

b) WILL IT BE POSSIBLE TO TEST A 1:10 MODEL OF THE TRUCK IN A WIND TUNNEL WITH A REPRESENTATIVE FLOW.

REPRESENTATIVE FLOW $\Rightarrow$ REYNOLDS SIMILARITY

$$Re = \frac{\rho V L}{\mu}$$

WITH ρ AND μ THE SAME FOR PROTOTYPE AND MODEL $\Rightarrow$

$$L_p = 10 L_m \Rightarrow U_m = 10 U_p$$

i.e. THE VELOCITY FOR THE MODEL SCALE TEST MUST BE 10 TIMES THE VELOCITY OF THE PROTOTYPE $\Rightarrow$ NOT POSSIBLE..

P5 | FLAT-PLATE BOUNDARY LAYER.

GIVEN:

- WATER @ 20°C $\Rightarrow \rho = 998\ kg/m^3$
  $\mu = 1.0 \cdot 10^{-3}\ kg/ms$
- $U_\infty = 50\ mm/s = 0{,}05\ m/s$
- $Re_{transition} = 5 \cdot 10^5$

a) FIND THE VELOCITY 15 mm ABOVE THE PLATE AT $x = 0{,}8\ m$

$$Re_x = \frac{U_\infty x \rho}{\mu} \approx 4.0 \cdot 10^4$$

$\Rightarrow$ LAMINAR FLOW.

$$(7.24): \quad \frac{\delta}{x} \approx \frac{5.0}{\sqrt{Re_x}}$$

$\Rightarrow \delta = 20\ mm \Rightarrow 15\ mm$

IS WITHIN THE BOUNDARY LAYER..

BLASIUS:

$$\eta = y\sqrt{\frac{U_\infty}{x\nu}} \approx 3.75$$

TABLE 7.1 $\Rightarrow u/U_\infty = 0{,}94$

$\Rightarrow u \approx 46.8\ mm/s$

b) AT WHAT DISTANCE FROM THE LEADING EDGE WILL TRANSITION TO TURBULENCE TAKE PLACE ?

$$Re_{transition} = 5.0 \cdot 10^5$$

$$Re_x = \frac{\rho U_\infty x}{\mu} = 5.0 \cdot 10^5 \Rightarrow$$

$\Rightarrow x = 10.02\ m$

c) CALCULATE THE BOUNDARY LAYER THICKNESS AT THE START OF TRANSITION..

ASSUME LAMINAR :

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} = \frac{5}{\sqrt{5 \cdot 10^5}}$$

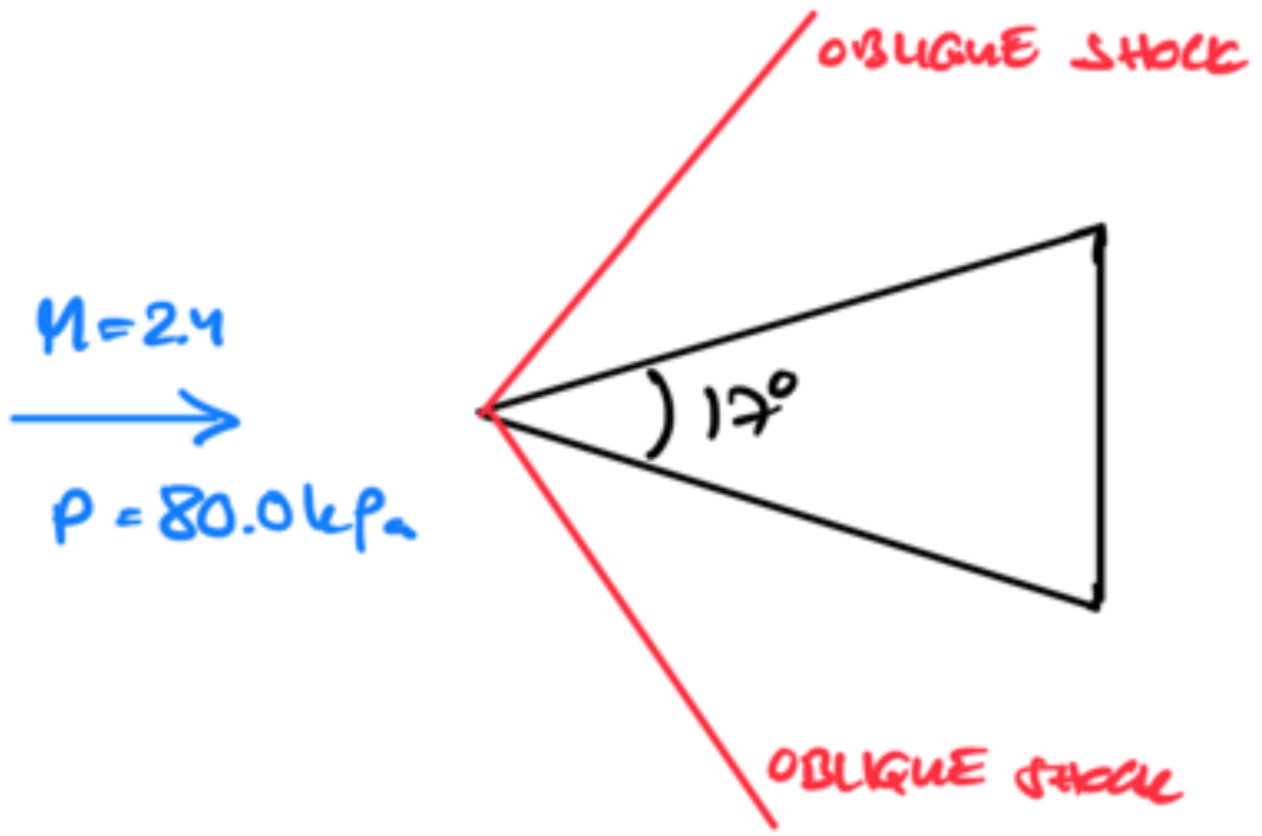
$$\Rightarrow \delta \approx 71 \text{ mm}$$

P6 | SUPERSONIC FLOW OVER A WEDGE.

a) WEAK SHOCKS INTRODUCE LESS LOSSES THAN STRONG SHOCKS AND THUS THE WEAK SHOCK SOLUTIONS ARE PREFERRED BY NATURE AND WILL ALWAYS BE THE SOLUTION IF POSSIBLE.

EXAMPLES WHERE THE STRONG-SHOCK SOLUTION WILL TAKE PLACE IS WHEN THE DEFLECTION ANGLE IS GREATER THAN THE MAXIMUM DEFLECTION POSSIBLE AT A GIVEN MACH NUMBER OR WHEN THE BACKPRESSURE IS TOO HIGH.

b) CALCULATE THE SHOCK ANGLE FOR THE WEAK AND STRONG SHOCK SOLUTION RESPECTIVELY.



$M_1 = 2.4$, $\theta = \frac{17^\circ}{2} = 8.5^\circ$

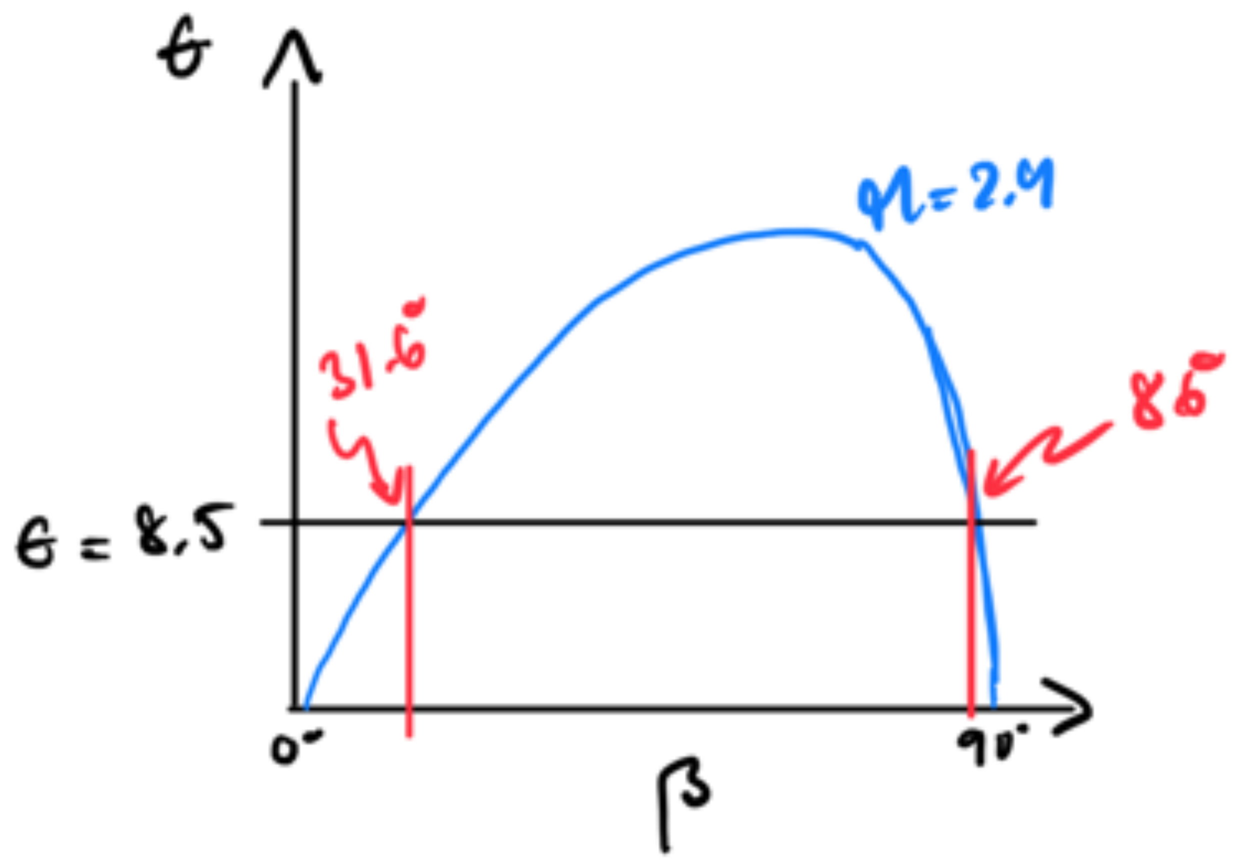
θ-β-M - RELATION (FIG 9.23) $\Rightarrow$

$\beta_{WEAK} \simeq 31.6^\circ$

$\beta_{STRONG} \simeq 86.1^\circ$

<u>ALSO:</u>

(eqn. 9.86) $\tan\theta = \dfrac{2\cot\beta\,(M_1^2\sin^2\beta - 1)}{M_1^2(\gamma + \cos(2\beta)) + 2}$



ESTIMATE FROM FIGURE AND VERIFY WITH EQN 9.86

ALT. SOLVE EQN 9.86 ITERATIVELY FOR

$M_1 = 2.4$ AND $\theta = 8.5°$

b) CALCULATE THE PRESSURE AND MACH NUMBER DOWNSTREAM OF THE WEAK AND STRONG SHOCK, RESPECTIVELY.

GENERAL SOLUTION: (ASSUME $\gamma = 1.4$)

(9.82) $$M_{n_1} = M_1 \sin(\beta) \quad (1)$$

$$M_{n_2} = M_2 \sin(\beta - \theta) \quad (2)$$

(9.57) $$M_{n_2} = \frac{(\gamma-1)M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma - 1)} \quad (3)$$

(1) $\Rightarrow M_{n_1}$

M_{n_1} in (3) $\Rightarrow M_{n_2}$

M_{n_2} in (2) $\Rightarrow M_2$

(9.55) $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1), \quad (4)$$

WITH M_{n_1} FROM (1) IN (4)

WE GET THE PRESSURE RATIO OVER THE SHOCK. p_1 IS GIVEN TO BE 80.0 kPa $\Rightarrow p_2$ ..

WEAK SHOCK SOLUTION: ($\beta = 31.6°$)

$M_2 = 2.06$ SUPERSONIC, OK!

$P_2 = 134.3$ kPa

STRONG SHOCK SOLUTION: ($\beta = 86.1°$)

$M_2 = 0.54$ SUBSONIC, OK!

$P_2 = 521.7$ kPa (WAY HIGHER THAN THE WEAK SHOCK SOLUTION)