MTF053 - Fluid Mechanics

2022-10-28 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

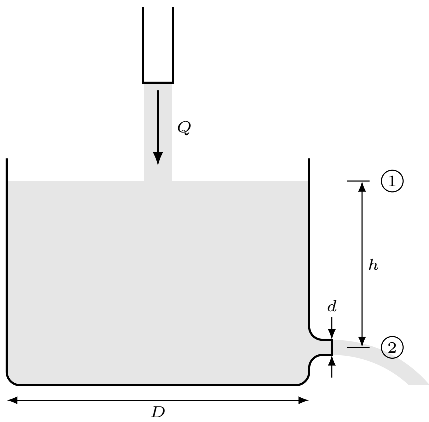
- In total 6 problems each worth 10p

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

PROBLEM 1 - TANK FLOW (10 P.)

The water tank in the figure below is evacuated through the pipe at the bottom and continuously filled from the top with a flow rate such that the level is constant at $h = 0.20m$. The pipe diameter is $d = 0.01m$. The water holds the temperature $T = 20°C$. The pressure at the pipe exit and at the water surface at the top of the tank is the atmospheric pressure.



(a) Tank problems are often solved assuming that the fluid velocity at the surface is zero. Obviously, the fluid velocity cannot be zero, but it is often a good assumption. The assumption gets better as the ratio d/D decreases. Calculate the ratio d/D such that the error is 0.01%. (6p.)

If Q is the flow rate obtained with $V_1 \neq 0$ (the velocity of a fluid particle at the surface) and Q_0 the flow rate that you get if you assume that $V_1 = 0$, the error is calculated as follows:

$$error = \left| \frac{Q - Q_0}{Q} \right|$$

(b) For the calculated ratio d/D, calculate the flow rate by which the tank is filled from above and the velocity at which water is leaving the tank at the bottom. (1p.)

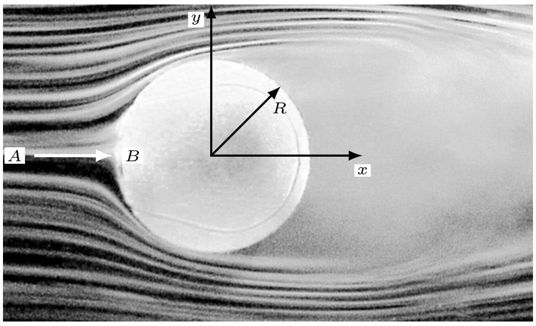
(c) Explain the difference between *streamline*, *pathline*, and *streakline*. Under what circumstances do these three line types coincide in a fluid flow? (1p.)

(d) When in use, the fluid in a fire extinguisher is flowing out from the tank through the hose. Describe the difference between

$$\left.\frac{dB}{dt}\right|_{system}, \text{ and } \frac{d}{dt}\left(\int_{cv} \beta\rho d\mathcal{V}\right)$$

if the extensive property B is fluid mass and if the control volume (CV) is fixed and aligned with the tank surface. (2p.)

Problem 2 - Tennis Ball (10 p.)



For the tennis ball in the figure, the velocity along the streamline $A \rightarrow B$ is given by

$$\mathbf{V} = u(x)\mathbf{e}_x = V_o\left(1 + \frac{R^3}{x^3}\right)\mathbf{e}_x$$

where R is the radius of the tennis ball and x is the coordinate from the center of the ball positive in the flow direction.

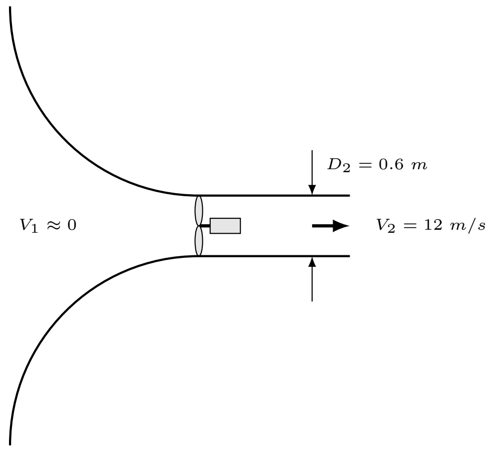
(a) Assuming the flow around the ball to be steady-state, derive an expression describing the acceleration experienced by a fluid particle along the streamline $A \rightarrow B$ (5p.)

(b) At what axial position will the magnitude of the acceleration be the biggest? (2p.)

(c) The diameter of a tennis ball is 2.7 inches (68.58 mm). When a professional tennis player serves, the velocity of the ball can reach velocities of up to 200.0 km/h. What is the maximum magnitude of acceleration that a fluid particle approaching the stagnation point at the front of the tennis ball will experience at this velocity? (2p.)
note: it is a ridiculously high value

(d) Finally a question related to another sport. Why do dimpled golf balls have lower pressure drag than golf balls with smooth surfaces? (1p)

PROBLEM 3 - NOT A BIG FAN (10 P.)



An axial-flow fan installed in an air ventilation system is driven by an electric motor marked 0.4 kW. The flow velocity in the air intake ahead of the fan can be assumed to be zero and after the fan, the fluid velocity is 12.0 m/s. The duct in which the fan is installed has the diameter $D = 0.6m$.

(a) Based on the data provided above and assuming that the flow is adiabatic and that the pressure is the atmospheric pressure both upstream and downstream of the fan, calculate the efficiency for the fan installation. (6p.)

hints:

- A fan can be represented as shaft work added to the flow in the same way as shaft work can represent a pump
- Efficiency (η) is defined as the energy that has been transferred to the flow by the fan divided by the energy consumed by the electric motor
- Head (dimension $[m]$) can be converted to flow power (dimension $[W]$) by multiplying with mass flow ($\dot{m}$ $[kg/s]$) times gravity constant (g $[m/s^2]$)

(b) What does Q and W in the energy equation represent? (1p.)

(c) What is the physical meaning of the terms in the energy equation on the form given below? (1p.)

$$\rho C_v \frac{dT}{dt} = k\nabla^2 T + \Phi$$

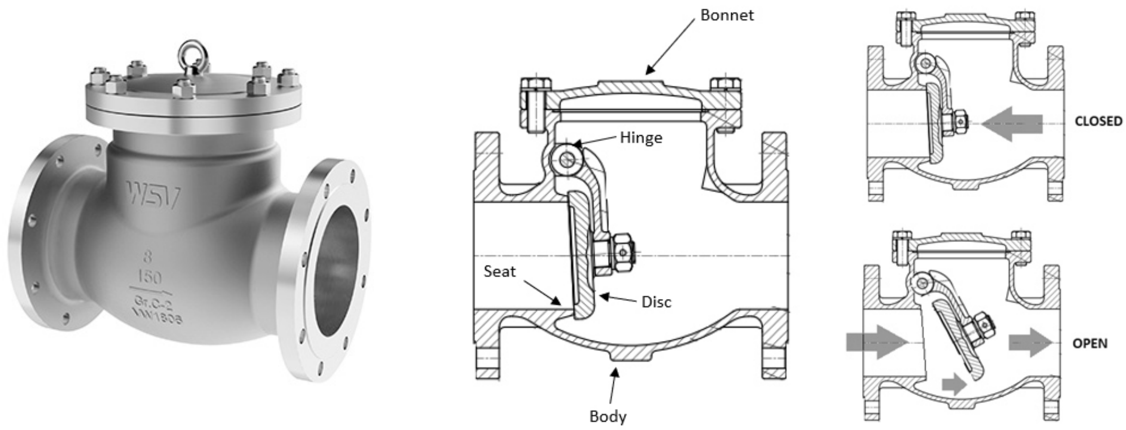
(d) The Bernoulli equation on the form given below is derived for steady-state, incompressible, frictionless flow along a streamline

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const$$

In what ways are the Bernoulli equation more limited than the energy equation on the form given below? (2p.)

$$\left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + \frac{\hat{u}_2 - \hat{u}_1 - q}{g}$$

Problem 4 - Valve Test (10 p.)



A large check valve designed to be installed in a water supply system is to be tested in model scale. The full-scale valve has an inlet diameter of $D_p = 0.6\ m$ and the water system is supposed to be able to handle water with the temperature $T = 20°C$ at a flow rate of $Q = 0.85\ m^3/s$. The reason for doing the test is to get estimates of forces on hinges and installation bolts. Geometric similarity and Reynolds number similarity is assumed to be sufficient to obtain dynamic similarity.

The following constraints are given by the test facility:

valve inlet diameter
$70\ mm \leq D_{inlet} \leq 140\ mm$
flow rate
$Q \leq 0.20\ m^3/s$
certified fluids
Water SAE 30W oil SAE 10W oil SAE 10W30 oil Ethylene glycol Ethanol

(a) Is it possible to design a model-scale valve for testing in the facility with the given constraints and obtain dynamic similarity? (6p)
(*your answer must be justified by calculations*)

(b) Explain the concepts *geometric similarity* and *dynamic similarity* (2p.)

(c) How does the fluid viscosity vary with temperature in liquids and gases, respectively? (2p.)

Problem 5 - Pipe Flow (10 p.)

(a) A sprinkler system used in case of fire is fed by a pipe made from galvanized iron that can be assumed to be in new condition. When the sprinkler system is activated the flow rate through the pipe should be 0.2 m^3/s. The total length of the pipe is 35 m and the head loss must not exceed 50 m. Calculate the smallest pipe diameter that fulfills the constraints. (6p.)

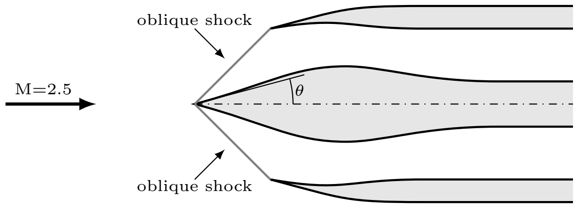
(b) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$? (1p.)

(c) Show that, for a laminar pipe flow, the friction factor f can be calculated as

$$f = \frac{64}{ReD}$$

(3p.)

Problem 6 - Engine Intake for Supersonic Flight (10 p.)



The center cone of the engine intake in the figure below has an angle θ that corresponds to 90% of maximum possible flow deflection at $M = 2.5$. This means that if the engine is operated at a freestream Mach number of 2.5, an oblique shock will be attached to the leading edge of the cone.

(a) Calculate the cone angle at the leading edge (θ) (2p)

(b) Calculate the Mach number downstream of the oblique shock attached to the leading edge if the freestream Mach number is 2.5 (2p.)

(c) Calculate the pressure and temperature downstream of the oblique shock if the upstream temperature and pressure is $T_1 = 20°C$ and $p_1 = 1.0\ bar$, respectively. (2p.)

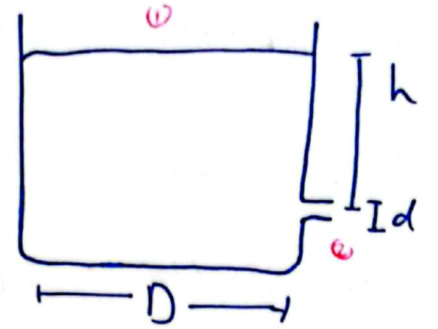
(d) Show schematically, what the flow in the in the vicinity of the engine intake would look like if the angle of the center cone was larger than the maximum deflection angle for the freestream Mach number. (2p.)

(e) What is required for a process to be isentropic? (1p.)

(f) Which of the properties h_o, T_o, a_o, p_o, and ρ_o are constants in a flow if the flow is adiabatic and isentropic, respectively? (1p.)

$h = 0.2$
$d = 0.01$
error $= 0.01\,\%$

Problem 1



Task ⓐ :-

Step ① : Q_0

$V_2 = \sqrt{2gh}$; 1

$V_2 = 1.981$ m/s ;

$Q_0 = V_2 A_2$

$Q_0 = 1.5558 * 10^{-4}$

Step ② , $V_1 \neq 0$ assumption

$V_1 A_1 = V_2 A_2$;

$V_1 = V_2 \left(\frac{d}{D}\right)^2$; Let $x = \frac{d}{D}$

$V_1 = V_2 \cdot x^2$;

$\Rightarrow \frac{\cancel{P_1}}{\cancel{\rho}} + \frac{1}{2} V_1^2 + g z_1 = \frac{\cancel{P_2}}{\cancel{\rho}} + \frac{1}{2} V_2^2 + g \cancel{z_2}$

$V_1^2 + 2gz = V_2^2$;

$V_2^2 x^4 + 3.924 = V_2^2$; 2

$3.924 = V_2^2 (1 - x^4)$;

$V_2 = \sqrt{\frac{3.924}{1 - x^4}}$

error $= \frac{Q - Q_0}{Q_0}$;

$0.0001\, Q = Q - 1.5558 * 10^{-4}$

$Q = \frac{1.5558 * 10^{-4}}{(1 - 0.0001)}$

$Q = V_2 A_2$

$\frac{1.5558 * 10^{-4}}{1 - 0.0001} = \sqrt{\frac{3.924}{1 - x^4}} * \frac{\pi}{4} * 0.01^2$

$3.92477 * (1 - x^4) = 3.924$

$x = 0.1185$ 2

$\frac{d}{D} = 0.1185$

(b) $Q_s = ??$ $\quad V_{out} = ??$

$Q_{in} = Q_{out}$

$$Q_{in} = V_2 A_2 = \sqrt{\frac{3.924}{1 - x^4}} * \frac{\pi}{4} * 0.01^2$$

$Q_{in} = 1.556 * 10^{-4} \; m^3/s$ ①

$$V_{out} = \sqrt{\frac{3.924}{1-x^4}} \Rightarrow V_{out} = 1.9811 \; m/s$$

(c) Streamline : tangent to the velocity vector everywhere at an instant of time.

Pathline : line traced by single fluid particle over time. ①

Streakline: line formed by multiple fluid particles that have all passed through a certain point.

(d) $\left.\frac{dB}{dt}\right|_{system}$ ① $\Rightarrow$ Lagrangian Approach; where B is an extensive property that describes the mass quantity into the extinguisher

Eularian Approach

$$\frac{d}{dt}\left(\int_{cv} \beta \rho \, dV\right)$$ ① $\Rightarrow$ β is an intensive property that describes the rate of change of mass over the arbitrary control volume.

Problem 2 . Problem 2

a) $$a = \frac{Dv}{Dt} = \cancel{\frac{\partial v}{\partial t}} + u \frac{\partial v}{\partial x}$$ 1 0

$$a = u \frac{\partial v}{\partial x} \; ;$$

$$\Rightarrow \quad u = V_0 \left(1 + \frac{R^3}{x^3}\right) \; ;$$ 1

$$\Rightarrow \frac{\partial v}{\partial x} = -3 \frac{V_0 R^3}{x^4} \; ;$$ 2

$$a = -3 V_0^2 R^3 \left(\frac{1}{x^4} + \frac{R^3}{x^7}\right)$$ 1

(b) Maximum acceleration $\frac{\partial a}{\partial x} = 0$

$$\frac{\partial a}{\partial x} = -\cancel{3 V_0} R^3 \left(\frac{-4}{x^5} + - \frac{7R^3}{x^8}\right) = 0$$ 1

$$-4 - \frac{7R^3}{x^3} = 0$$

$$-4x^3 = 7R^3$$

$$x^3 = \frac{-7}{4} R^3$$

$$\boxed{x = \sqrt[3]{\frac{-7}{4} R^3}}$$ 1

$$\hookrightarrow x = -0.04132 m$$

(c) $R = 0.03429$ m

$V_0 = 55.556$ m/s

$x = -0.04312$ m

$a = -5.48 * 10^4$ m/s^2 2

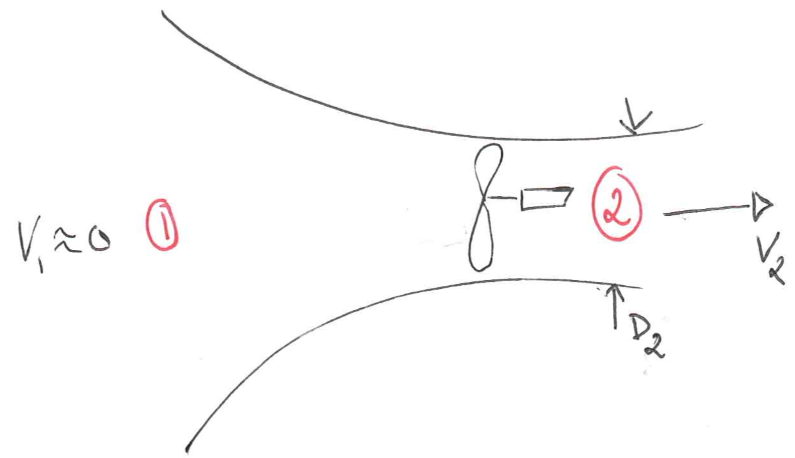
(d) turbulent boundary layer → delayed seperation

1

P3

Problem 3

Given: $P_{in} = 0.4\,kW$
$V_2 = 12\ m/s$
$V_1 \approx 0$
$D_2 = 0.6\ m$
Assume adiabatic
$P_1 = P_2 = P_{atm}$



a) Use Bernoulli with losses from 1 to 2

$$(3.70)\quad \cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_{turbine}} - \underbrace{h_{pump}}_{h_{fan}} + \cancel{h_{fric}}$$

$P_1 = P_2$ $\quad V_1 \approx 0$ $\quad z_1 = z_2$ $\quad$ no turbine $\quad$ assume frictionless due to short pipe

$$\Rightarrow 0 = \frac{V_2^2}{2g} - h_{fan} \quad \Rightarrow h_{fan} = \frac{V_2^2}{2g} = 7.34\,m$$

Given in hints: $h = \dfrac{P}{\dot m g}$

$P_{out} = h_{fan}\,\dot m\, g$

$$\dot m = \rho Q = \rho A V = \rho \frac{\pi D_2^2}{4} V_2 = \left[\begin{array}{l}\text{assume air @ } 20^\circ C \\ \rho = 1.2\ kg/m^3\ (A.2)\end{array}\right] = 1.2 \cdot 0.2826 \cdot 12$$

$\dot m = 4.07\ kg/s$

$P_{out} = 7.34 \cdot 4.07 \cdot 9.81 = 293\,W$

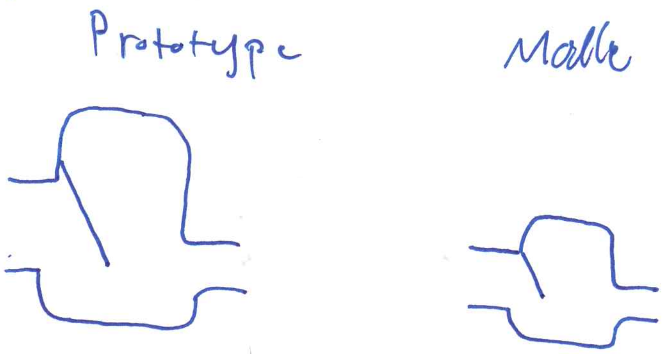
Given in hints: $\eta = \dfrac{P_{received}}{P_{used}} = \dfrac{P_{out}}{P_{in}}$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{293}{400} = 0.73$$

The efficiency of the fan installation is 73%

P4)

Problem 4



Known:

$D_p = 0.6 m$

$Q_p = 0.85 m^3/s$

Water @ 20°C $\begin{cases} \rho = 998 kg/m^3 \\ \mu = 1.0 \cdot 10^{-3} \frac{kg}{m \cdot s} \end{cases}$

a)

Task: Is it possible to design model scale with geometric and Re-similarity? → dynamic similarity

For Re Similarity: $\boxed{Re_p = Re_M}$

(1.24) $Re = \frac{\rho U L}{\mu}$, For cylindric cross section use $L = D$, and $U = \frac{Q}{A} = \frac{4Q}{\pi D^2}$ $\Big\} \Rightarrow$

$$Re = \frac{\rho}{\mu} \cdot \left(\frac{4Q}{\pi D^2}\right) \cdot D = \frac{4 \rho Q}{\mu \pi D} \qquad [Re_p = 1.8 \cdot 10^6]$$

$$Re_p = Re_M \Rightarrow \frac{4 \rho_p Q_p}{\mu_p \pi D_p} = \frac{4 \rho_M Q_M}{\mu_M \pi D_M} = \left\{ \begin{array}{l} \text{Assume Water in model scale} \\ \rho_p = \rho_M, \mu_p = \mu_M \end{array} \right.$$

$$\Rightarrow \frac{Q_p}{D_p} = \frac{Q_M}{D_M} \Rightarrow Q_M = \frac{Q_p}{D_p} D_M \Rightarrow [0.07 \le D_M \le 0.14] \Rightarrow$$

$Q_M = [0.1 ; 0.19] m^3/s$, ok for the lab

b) Geometric similarity → Scale geometry. $L_p = \alpha L_M$

Dynamic Similarity → Scale Force vectors, req geom sim and kin sim

c) Liquids: Decreasing with temperature

Gases: Increasing with teperature

Increasing temp

Problem 5 - Pipe Flow (10 P.)

(a) $Q = 0.2\ m^3/s$, $L = 35m$

loss head $\leq 50m$, d ?

↳ Solution

— From table 6.1 for galvanized iron new

$\varepsilon = 0.15\ mm$

Assume at the limit that $h = 50m$

0.5p

— Write the diameter in term of friction factor:

$$(6.10)\quad h_f = f\,\frac{L}{d}\,\frac{V^2}{2g} \Rightarrow \left(V = \frac{4Q}{\pi d^2}\right) \Rightarrow$$

$$h_f = f\,\frac{L}{d}\,\frac{16Q^2}{2g\pi^2 d^4} \Rightarrow f = \frac{\pi^2 g\, h_f\, d^5}{8LQ^2}$$

or

$$d^5 = f\,\frac{L}{h_f}\,\frac{8Q^2}{\pi^2 g} \quad (1)$$

[1 P]

- Write Reynolds number & roughness ratio in terms of "d":

$$Re_d = \frac{\rho V d}{\mu} = \frac{4\rho Q d}{\mu \pi d^2} = \frac{4\rho Q}{\mu \pi d} \quad (2)$$

$$\frac{\varepsilon}{d} = ? \quad (3)$$

[0.5p]

- Explain process of calculation [2p]

1) Guess f
2) Compute d from (1)
3) Compute Re_d from (2)
4) Compute better f from Moody chart or Eq. 6.48

* Do one iteration and get reasonable number [1.5p]

** Do two iterations and get better number [2p]

- Initial guess of $f = 0{,}03$ → middle of graph for turbulent

Equation (1) → $d = 0{,}147$ m
↓
Equation (2) → $Re = 575.000$
↓
Moodychart (Eq 6.48) → $f_{new} \simeq 0{,}023$

1st iteration

. . . . . .

Equation (1) → $d = 0{,}139$ m
↓
Equation (2) → $Re = 606\,384$
↓
Moodychart (Eq. 6.48) → $f_{new} = 0{,}02$

2nd iteration

Eq. (1) → $d = 0{,}135$m → Eq. (2) → $Re = 623573$
↓
Moody (Eq. 6.48)
↓
$f_{new} = 0{,}021$

Converged

$$\boxed{d = 0{,}137 \text{ m}}$$

(b) Due to not having reliable friction factors [0.5p] in this range, which corresponds to transition from laminar to turbulent flow [0.5p]

(c) Start with equation 6.11

[0.5p] $f = \frac{8 \tau_w}{\rho V^2}$ (1)

[1p] $\tau_{w_{lam}} = \left| \mu \frac{\partial u}{\partial r} \right|_{r=R}$

[0.5p] $u = u_{max} \left(1 - \frac{r^2}{R^2}\right)$ $\Rightarrow \tau_{w_{lam}} = \frac{4 \mu V}{R} \overset{(1)}{\Rightarrow}$

[0.5p] $V = \frac{u_{max}}{2}$

$$\Rightarrow f_{lam} = \frac{8 \cdot 4 \mu V}{\rho V^2 R} \overset{R = \frac{D}{2}}{=} \frac{64 \mu}{\rho V d} \Rightarrow$$

$$\Rightarrow f_{lam} = \frac{64}{Re_d}$$ [0.5p]

Problem 6 – Engine Intake for Supersonic Flight (10 p)

(a) - From figure 9.1 max deflection for

$M = 2.5 \rightarrow \theta_{max} = 30°$ [1 p]

- To get 90 % $\rightarrow$ $\theta = 0.9\,\theta_{max} = 27°$ [1 p]

(b) - From figure 9.1 $\rightarrow M = 2.5$ & $\theta = 27°$

we get $\beta \simeq 55°$ [0.5 p]

- Use 9.82 to get M_{n_1} :

$M_{n_1} = M_1 \sin\beta = 2{,}047$ [0.5 p]

- Use 9.57 to get M_{n_2} :

$$M_{n_2}^2 = \frac{(\gamma-1)M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma-1)} \overset{\gamma = 1.4,\, air}{\Longrightarrow} M_{n_2} = 0{,}57$$

[0.5 p]

- Use 9.82 to get M_2 :

$M_2 = M_{n_2} / \sin(\beta - \theta) \Rightarrow M_2 = 1{,}21$ [0.5 p]

(c) Use equation 9.55 (or table B.2):

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \Rightarrow$$

$$\Rightarrow P_2 = P_1\left(1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)\right) \Rightarrow$$

$$\Rightarrow P_2 = 4{,}72 \text{ bar} \quad [1p]$$

Use equation 9.58 (or table B.2):

$$\frac{T_2}{T_1} = \left(2 + (\gamma-1) M_{n_1}^2\right) \frac{2\gamma M_{n_1}^2 - (\gamma-1)}{(\gamma+1) M_{n_1}^2} \Rightarrow$$

* equation from formula sheet

$$\Rightarrow \frac{T_2}{T_1} = 4.13 \Rightarrow T_2 = 1210\ °K \quad [1p]$$

$$(937\ °C)$$

if using table $\frac{T_2}{T_1} \simeq 1.72 \rightarrow T_2 = 503.9\ °K$

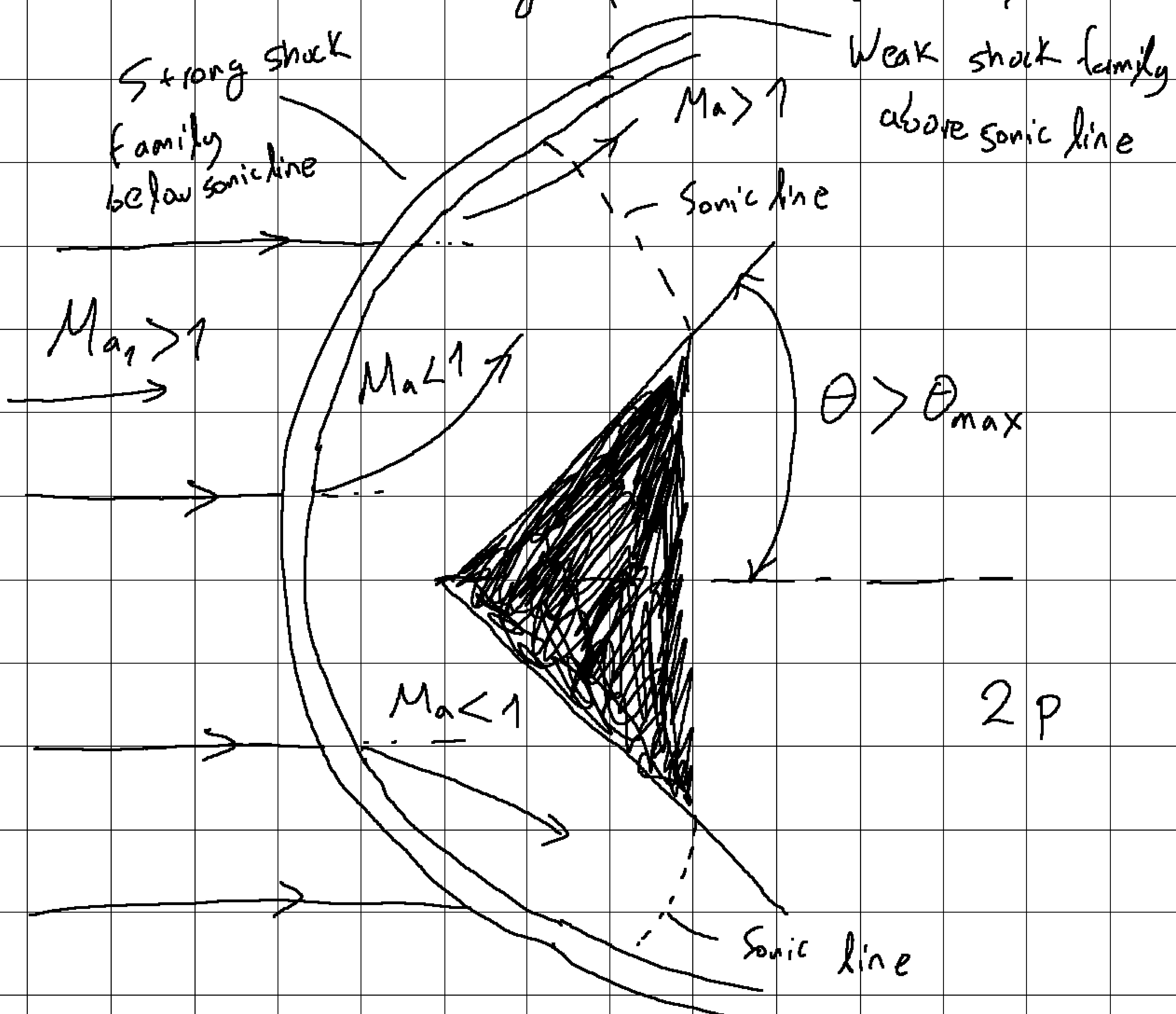
$(230\ °C)$

⚠ Both will be taken as correct!

(d) The answer is found in Fig 9.22b

Main points for case $\theta > \theta_{max}$:

- attached oblique shock is impossible
- broad curved detached shock forms
- discontinuous deflection of flow with $\theta < \theta_{max}$
- flow curves → expands and deflects subsonically around wedge (sonic → supersonic, via corner)



2 p

(e) It has to be adiabatic (not Q) [0.5p]

and reversible [0.5p]

(f) All stagnation properties are constants in an isentropic flow [0.5p]

h_0, T_0 & a_0 are constants in an adiabatic but not necessarily P_0 & ρ_0 [0.5p]