MTF053 - Fluid Mechanics

2022-01-05 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

PROBLEM 1 - FLOW IN DUCTS (10 P.)

(a) Gasoline 20°C is pumped through a 180.0 mm diameter pipe at a flow rate of 0.2 m^3/s. The pipe is made of cast iron and is 16.0 km long. Estimate the power delivered to the pump if the pump efficiency is $\eta = 0.75$ (note: the pump power is $\rho g Q$ times the pump head). (8p.)

(b) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re_D < 4000$? (0.5p.)

(c) How is the hydraulic diameter defined and how can it be used for calculation of the friction factor f for laminar and turbulent flow in non-circular ducts? (0.5p)

(d) Explain the closure problem related to the Reynolds-averaged flow equations. (1p.)

PROBLEM 2 - SKIN FRICTION DRAG (10 P.)

(a) The figure below shows a schematic representation of a sailboat. Assume that the boat moves at 3 knots (1.54 m/s) in sea water with a temperature of 4 degrees Celsius, what is the skin friction drag from the keel?

In your calculations, you can assume that the keel can be approximated to be a flat plate with dimensions as indicated in the figure and that transition takes place at $Re_x = 10^6$.

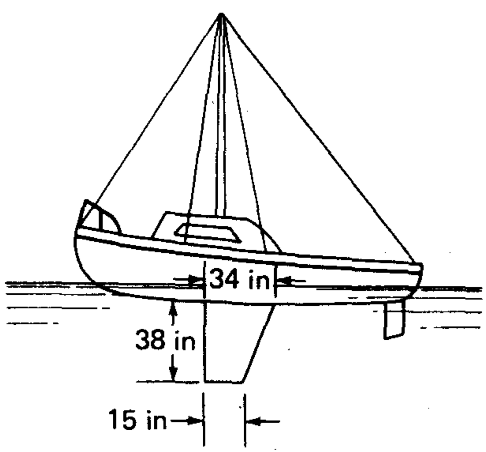
Please note that the keel dimensions are given in inches in the figure – 1.0*in is* 25.4*mm.* (7p.)

(b) Make a schematic sketch of the flow over a cylinder at $Re_D = 10^5$. Indicate the stagnation point, separation points and the wake region. (1p.)

(c) Name two alternative to δ as measures the boundary layer thickness. How can these measures be interpreted physically? (1p.)

(d) In what way is the transition location effected by (assume other properties to be constant) (1p.)

  (a) increased freestream velocity U for a given $Re_{x,tr}$
  (b) surface roughness ε
  (c) freestream turbulence
  (d) positive pressure gradient



PROBLEM 3 - FLOW RATE AND MASSFLOW (10 P.)

(a) A flow nozzle is a device inserted into a pipe as shown in the illustration below. As shown in the figure, a mercury manometer (manometer fluid density: ρ_{Hg}) is used to measure the pressure drop over the flow nozzle. A If A_1 and A_2 are the inlet and exit areas of the flow nozzle and the density of the fluid flowing through the flow nozzle is ρ, show that for incompressible flow, the flow rate, Q, can be obtained as

$$Q = C_d \left[\frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2\rho_{Hg} g \Delta h}{\rho}} \right]$$

where C_d is a *discharge coefficient* that accounts for viscous losses or losses related to secondary flow and is defined as

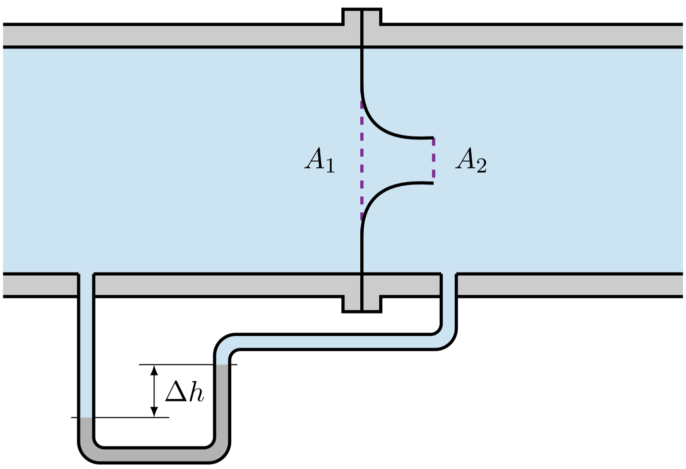
$$C_d = \frac{Q_{real}}{Q_{ideal}}$$

The value of the discharge coefficient is usually obtained experimentally

(8p.)

(b) Show how the volume flow Q and massflow $\dot{m}$ over a control volume surface can be calculated in a general way. (1p.)

(c) Give examples of when it is appropriate to use fixed control volume, moving control volume, and deformable control volume, respectively. (1p.)



PROBLEM 4 - THRUST REVERSER (10 P.)

(a) A so-called thrust reverser is used for reducing the forward speed of an aircraft at landing. In the specific case illustrated below, the aft-going flow is turned by the thrust reverser, when deployed, such that the flow leaves the engine at an angle of 20° from the vertical direction (both upwards and downwards), i.e. a weakly forward oriented flow. The massflow through the engine is 70 $[kg/s]$. Air enters the engine at 100 $[m/s]$ and leaves the engine with a velocity of 450$[m/s]$. It can be assumed that the flow velocity at the exit is unchanged when the thrust reverser is deployed. Calculate the engine mount force under normal operating conditions (no thrust reverser) and when the thrust reverser is deployed.
(7p.)

(b) How can we simplify the continuity equation on integral form under the following circumstances (assuming that the control volume is fixed)? (1p.)

$$\int_{cv} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{cs} \rho(\mathbf{V} \cdot \mathbf{n}) dA = 0$$

  (a) inlets and outlets can be assumed to be one-dimensional
  (b) steady-state flow
  (c) incompressible flow

(c) Explain the physical meaning of each of the terms I, II, and III in the momentum equation on integral form. (1p.)

$$\underbrace{\sum \mathbf{F}}_{I} = \underbrace{\frac{d}{dt}\left(\int_{cv} \mathbf{V}\rho d\mathcal{V}\right)}_{II} + \underbrace{\int_{cs} \mathbf{V}\rho(\mathbf{V}_r \cdot \mathbf{n}) dA}_{III}$$

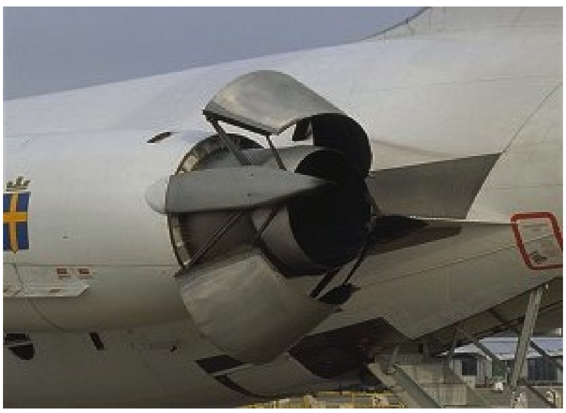
(d) The Bernoulli equation is a simplified form of the energy equation.

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const$$

In what ways are the Bernoulli equation above more limited than the energy equation on the form given below?

$$\left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + \frac{\hat{u}_2 - \hat{u}_1 - q}{g}$$

(1p.)



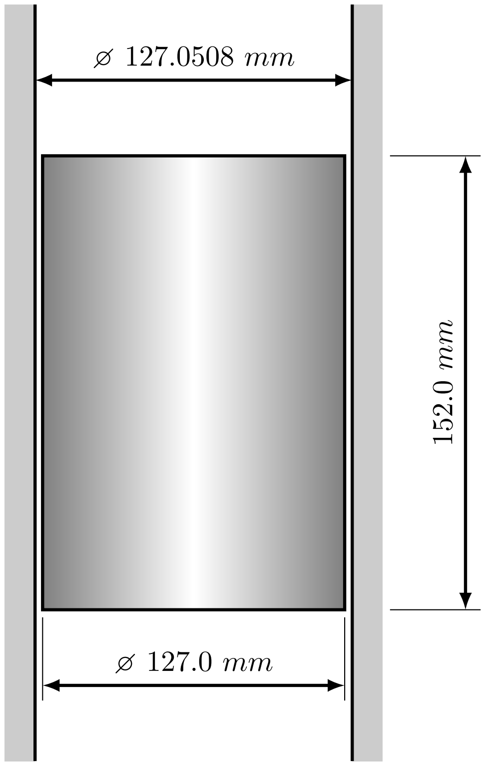
PROBLEM 5 - VISCOSITY (10 P.)

(a) A piston of weight 9.5 kg slides in a lubricated vertical pipe (see figure below). The clearance between the piston and the pipe is 0.0254 mm. If the piston decelerates at 0.64 m/s^2 when the speed is 6.4 m/s, what is the viscosity of the fluid used for lubrication? (7p.)

(b) What does it mean that a fluid is Newtonian? (1p.)

(c) How does the fluid viscosity vary with temperature in liquids and gases, respectively. (1p.)

(d) How does the turbulence viscosity μ_t compare to the fluid viscosity μ in the viscous sublayer and in the fully turbulent region, respectively? (1p.)



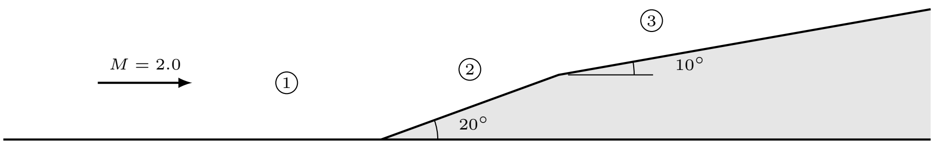
Problem 6 - Wedge Flow (10 p.)

A 20° wedge with a 10° shoulder (depicted in the figure below) is situated in a flow with a free stream Mach number of $M = 2.0$.

(a) draw a schematic sketch of the important flow features in the flow over the wedge. (2p.)

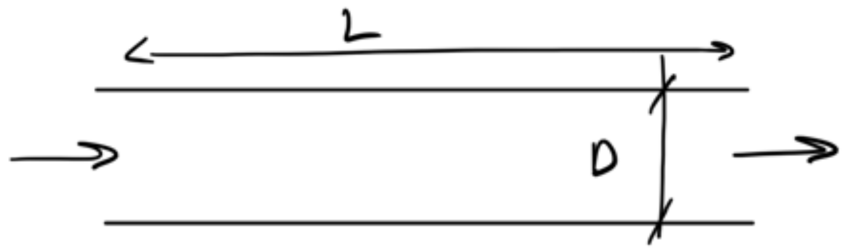
(b) calculate the Mach numbers in regions 2 and 3 (7p.)

(c) assume that the flow would pass a simple 10° wedge (without the shoulder), the resulting flow direction would be the same. Would the total pressure in the directed flow be greater or lower than in the case with the shoulder? (justify your answer) (1p.)



PROBLEM 1 - FLOW IN DUCTS

a)



GIVEN: GASOLINE @ 20°C
$D = 180.0$ mm
$Q = 0{,}2\ m^3/s$
$L = 16.0$ km
Pump efficiency : $\eta = 0{,}75$
Pipe material : cast iron

Calculate power delivered to the pump.

Equ. (3.73)

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_2 + h_t - h_p + h_f$$

$$p_1 = p_2,\ V_1 = V_2,\ z_1 = z_2,\ h_t = 0 \Rightarrow$$

$$\Rightarrow h_p = h_f$$

Equ. (6.10)

$$h_f = f \frac{L}{d} \frac{V^2}{2g}$$

Need to estimate the friction factor (f)

Cast iron $\Rightarrow$ $\varepsilon = 0.26 \Rightarrow \varepsilon/D = 0.0014$

$$Re_D = \frac{\rho V D}{\mu} = \left\{ \begin{array}{l} \rho = 680 \text{ kg/m}^3 \text{ (tab A3)} \\ \mu = 2.92 \cdot 10^{-4} \text{ kg/ms (tab. A3)} \\ V = Q/A = \frac{4Q}{\pi D^2} = 7.86 \text{ m/s} \end{array} \right\}$$

$$= 3.3 \cdot 10^6 \text{ (turbulent flow)}$$

$$\left.\begin{array}{l} Re_D = 3.3 \cdot 10^6 \\ \varepsilon/D = 0.0014 \end{array}\right\} \Rightarrow \text{(Moody Chart)} \Rightarrow f = 0.021$$

$$h_p = h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$$P_p = (h_p \rho g Q)/\eta = 10.8 \text{ MW}$$

b)

FOR THE SPECIFIED RANGE OF REYNOLDS NUMBERS ($2000 < Re_D < 4000$)

THERE WILL BE A TRANSITION FROM LAMINAR FLOW TO TURBULENT FLOW.

THE TRANSITION PROCESS DEPENDS ON EXTERNAL CONDITIONS.

THERE IS NO RELIABLE THEORY GOVERNING THE TRANSITION AND THUS THE VALUES GIVEN IN THE MOODY CHART FOR THESE REYNOLDS NUMBERS ARE NOT RELIABLE AND THEREFORE THIS RANGE OF REYNOLDS NUMBERS SHOULD BE AVOIDED.

c) HYDRAULIC DIAMETER:

$$D_h = \frac{4A}{P}$$

WHERE A IS THE CROSS-SECTION AREA AND P IS THE WETTED PERIMETER.

REYNOLDS NUMBER : $Re_{D_h} = \frac{V D_h}{\nu}$

FRICTION FACTOR : $f = \frac{C}{Re_{D_h}}$

WHERE C IS OBTAINED FROM TABLE.

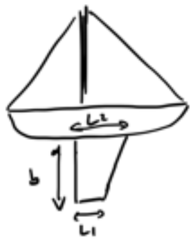
d) WHEN APPLYING REYNOLDS-AVERAGING TO THE GOVERNING EQUATIONS, NEW UNKNOWNS ARE ADDED TO THE EQUATIONS (THE REYNOLDS STRESSES)

WITH THE SAME NUMBER OF EQUATIONS AND MORE UNKNOWNS, IT IS NOT POSSIBLE TO SOLVE THE EQUATIONS

THIS IS KNOWN AS THE CLOSURE PROBLEM

PROBLEM 2 - SKIN FRICTION DRAG

a)



$L_1 = 15$ inch
$L_2 = 34$ inch
$b = 38$ inch
(1 inch = 25,4 mm)

$\Rightarrow L_1 = 0,381$ m
$L_2 = 0,864$ m
$b = 0,9652$ m

Given: $V = 3$ knots (1,54 m/s)
SEA WATER @ 4°C
$Re_{tr} = 10^6$

ESTIMATE THE KEEL FRICTION DRAG..

TABLE A3 :

$\Rightarrow$ SEAWATER @ 20°C

$\rho = 1025$ kg/m³
$\mu = 1.07 \cdot 10^{-3}$ kg/ms
$C = 7,28$

$$\frac{\mu_{4°C}}{\mu_{20°C}} = e^{\left[C\left(\frac{293}{277} - 1\right)\right]}$$

$$\Rightarrow \mu_{4°C} = 1.68 \cdot 10^{-3} \text{ kg/m}^3$$

$$Re_{L,max} = \frac{\rho V L_2}{\mu} = 8,37 \cdot 10^5 < 10^6$$

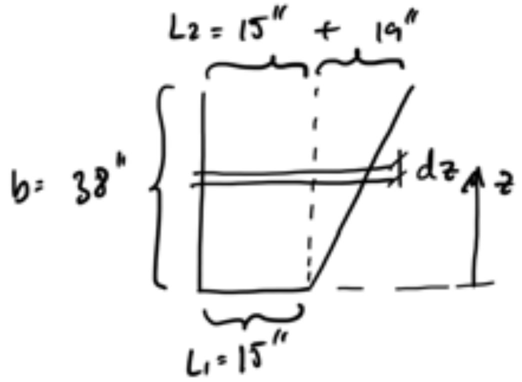
$\Rightarrow$ LAINAR FLOW OVER THE KEEL..

Eqn (7.27) :

$$D(L) \approx \frac{1,328}{\sqrt{Re_L}} \frac{1}{2} \rho V^2 b L$$

L IS NOT CONSTANT $\Rightarrow$

APPROACH I – INTEGRAL



$$D(z) = \frac{1{,}328}{\sqrt{\frac{V L(z)}{\upsilon}}} \frac{1}{2} \rho V^2 dz\, L(z) =$$

$$= \frac{1{,}328 \rho V^2}{2\sqrt{V/\upsilon}} \sqrt{L(z)}\, dz$$

$$D = \frac{1{,}328 \rho V^2}{2\sqrt{V/\upsilon}} \int_0^b (0{,}381 + 0{,}5 z)^{1/2} dz$$

VARIABEL-SUBSTITUTION:

$$0{,}381 + 0{,}5 z = y \Rightarrow dy = 0{,}5 dz$$

or $dz = 2 dy$

$$D = \underbrace{\frac{1{,}328 \rho V^2}{2\sqrt{V/\upsilon}}}_{A} 2 \int_{0{,}381}^{0{,}8636} y^{1/2} dy = A \left[\frac{2}{3} y^{3/2}\right]_{0{,}381}^{0{,}8636}$$

$$= \underline{\underline{2{,}48\ N}}$$

APPROACH II – AVERAGE LENGTH:

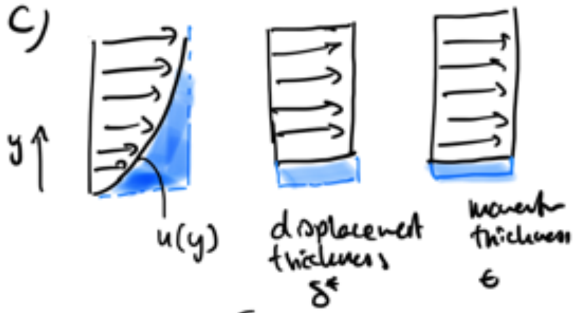
$$L_{AV} = \frac{1}{2}(L_1 + L_2)$$

$$Re_{L_{AV}} = \frac{\rho V L_{AV}}{\mu} = 6.02 \cdot 10^5$$

$$D = \frac{1.328}{\sqrt{Re_{L_{AV}}}} \frac{1}{2} \rho V^2 b L_{AV} = \underline{\underline{2.50\,N}}$$

b) FLOW OVER CYLINDER @ $Re_D = 10^5$
(LAMINAR SEPARATION.)

c)



$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$u(y=\delta) = U$$

(δ^*) THE DISPLACEMENT THICKNESS IS THE DISTANCE THAT GIVES THE MASS FLOW THAT CORRESPONDS TO THE DEFICITE OF MASS FLOW DUE TO THE PRESENCE OF THE BOUNDARY LAYER..

(θ) THE MOMENTUM THICKNESS IS THE DISTANCE THAT GIVES THE MOMENTUM THAT CORRESPONDS TO THE DEFICITE OF MOMENTUM DUE TO THE PRESENCE OF THE BOUNDARY LAYER..

b) TRANSITION LOCATION CHANGES:

a) EARLIER

b) EARLIER

c) EARLIER

d) LATER.

FOR THE SPECIFIED FLOW NOZZLE, SHOW THAT

$$Q = C_D \left[\frac{A_2}{\sqrt{1-(A_2/A_1)^2}} \sqrt{\frac{2\rho_{Hg}\, g\,\Delta h}{\rho}} \right]$$

WHERE $C_D = \dfrac{Q_{REAL}}{Q_{IDEAL}}$

SET UP THE BERNOULLI EQN (3.54) OVER THE FLOW NOZZLE.

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + \cancel{z_1} = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + \cancel{z_2}$$

WHERE: $u_1 = \dfrac{Q}{A_1}$ AND $u_2 = \dfrac{Q}{A_2}$

$$\Rightarrow \frac{p_1}{\rho g} + \frac{Q^2}{2gA_1^2} = \frac{p_2}{\rho g} + \frac{Q^2}{2gA_2^2}$$

$$\frac{Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) = \frac{1}{\rho g}(p_2 - p_1)$$

$$Q^2\left(\frac{A_2^2 - A_1^2}{A_1^2 A_2^2}\right) = \frac{2}{\rho}(p_2 - p_1)$$

$$Q^2\left(\frac{(A_2/A_1)^2 - 1}{A_2^2}\right) = \frac{2}{\rho}(p_2 - p_1)$$

$$Q^2 = \frac{A_2^2}{(A_2/A_1)^2 - 1}\,\frac{2}{\rho}(p_2 - p_1)$$

$$Q^2 = \frac{A_2^2}{(A_2/A_1)^2 - 1} \frac{2}{\rho} (p_2 - p_1)$$

$$Q^2 = \frac{A_2^2}{1 - (A_1/A_2)^2} \frac{2}{\rho} (p_1 - p_2)$$

$$Q = \frac{A_2}{\sqrt{1 - (A_1/A_2)^2}} \sqrt{\frac{2}{\rho} (p_1 - p_2)}$$

THE MANOMETER READING GIVES:

$$p_1 - \rho_{Hg}\, g\, \Delta h = p_2$$

$$\Rightarrow p_1 - p_2 = \rho_{Hg}\, g\, \Delta h$$

(eqn. (2.14))

$\Rightarrow$

$$Q = \frac{A_2}{\sqrt{1 - (A_1/A_2)^2}} \sqrt{\frac{2 \rho_{Hg}\, g\, \Delta h}{\rho}}$$

THIS IS THE IDEAL VOLUME FLOW AND THUS..

$$Q = C_D \left[\frac{A_2}{\sqrt{1 - (A_1/A_2)^2}} \sqrt{\frac{2 \rho_{Hg}\, g\, \Delta h}{\rho}} \right]$$

V.S.V.

b)

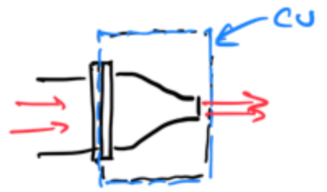
VOLUME FLOW : $Q = \int_{CS} (\mathbf{V} \cdot \mathbf{n})\, dA$

MASS FLOW : $\dot{m} = \int_{CS} \rho (\mathbf{V} \cdot \mathbf{n})\, dA$

c)

FIXED CONTROL VOLUME.

ANALYZING THE FLOW THROUGH A STATIONARY VOLUME (EXAMPLE: NOZZLE FLOW ANALYSIS)



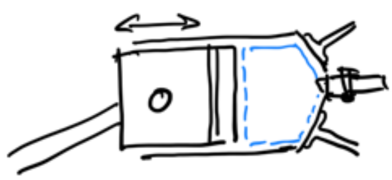
MOVING CONTROL VOLUME.

ANALYZING THE FLOW AROUND A MOVING OBJECT (EXAMPLE: BOAT MOVING AT CONSTANT SPEED)

DEFORMABLE CONTROL VOLUME:

ANALYZING THE FLOW IN A VOLUME THAT CHANGES OVER TIME

EXAMPLE: FLOW IN THE COMBUSTION CHAMBER OF AN INTERNAL COMBUSTION ENGINE



PROBLEM 4 – THRUST REVERSER

EQN. (3.40)

$$\sum \mathbb{F} = \frac{d}{dt}\left(\int_{cv} \mathbf{V}\, \rho\, dU\right) + \sum_i (\dot{m}_i \mathbf{V}_i)_{out} + $$

$$- \sum_i (\dot{m}_i \mathbf{V}_i)_{in}$$

STEADY-STATE FLOW $\Rightarrow$

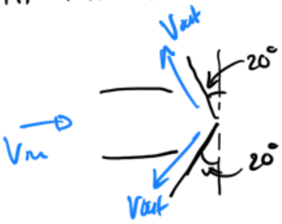
$$\sum \mathbb{F} = \sum_i (\dot{m}_i \mathbf{V}_i)_{out} - \sum_i (\dot{m}_i \mathbf{V}_i)_{in}$$

NO THRUST REVERSER $\Rightarrow$

$$F_x = \dot{m}\left(V_{out} - V_{in}\right) = 24.5\ kN$$

$$F_y = mg \quad (\text{not known})$$

WITH THRUST REVERSER DEPLOYED:



AT THE OUTLET THE MASSFLOW IS ASSUMED TO BE DIVIDED IN TWO EQUAL STREAMS (ON UP AND ONE DOWN)

$$F_x = \dot{m}\left(-\frac{V_{out}}{2}\sin 20^\circ - \frac{V_{out}}{2}\sin 20^\circ + \right.$$

$$\left. - V_{in}\right)$$

$$\Rightarrow F_x = \dot{m}\left(-V_{out}\sin 20^\circ - V_{in}\right)$$

$$= -17.8\ kN$$

$$F_y = mg + \dot{m}\left(\frac{V_{out}}{2}\cos 20^\circ - \frac{V_{out}}{2}\cos 20^\circ\right)$$

$$= mg \quad (\text{not known})$$

NOTE! THE OUTLET VELOCITY IS ACTUALLY HIGH ENOUGH TO RESULT IN SUPERSONIC EFFECTS WHICH IS A RESULT OF A TYPO FROM MY SIDE. IF YOU REALIZED THAT AND STRUGGLED TO FIND A SOLUTION TO THE PROBLEM, YOU GET 7p ON THIS PROBLEM... SORRY FOR THE CONFUSION..

b) $$\int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} \rho (\mathbb{V} \cdot \mathbb{n}) dA = 0$$

(FIXED CONTROL VOLUME)

a) INLETS AND OUTLETS CAN BE ASSUMED TO BE ONE DIMENSIONAL.

$$\boxed{\int_{CV} \frac{\partial \rho}{\partial t} dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in}}$$

b) STEADY-STATE FLOW:

$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow$$

$$\boxed{\int_{CS} \rho (\mathbb{V} \cdot \mathbb{n}) dA = 0}$$

c) INCOMPRESSIBLE FLOW:

$$\rho = \text{CONST} = \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow \int_{CS} \rho (\mathbb{V} \cdot \mathbb{n}) dA = 0$$

$$= \{\rho = \text{const}\} = \rho \int_{CS} (\mathbb{V} \cdot \mathbb{n}) dA = 0$$

$$\Rightarrow \boxed{\int_{CS} (\mathbb{V} \cdot \mathbb{n}) dA = 0}$$

c) $$\underbrace{\sum \mathbb{F}}_{\text{I}} = \underbrace{\frac{d}{dt}\left(\int_{cv} \mathbb{V} \rho \, dV\right)}_{\text{II}} + \underbrace{\int_{cs} \mathbb{V} \rho (\mathbb{V}_r \cdot \mathbb{n}) dA}_{\text{III}}$$

I: SUM OF ALL FORCES

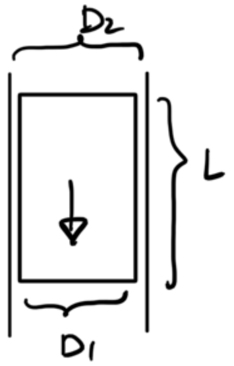
II: RATE OF CHANGE OF MOMENTUM WITHIN THE CONTROL VOLUME (CV)

III: THE NET FLUX OF MOMENTUM OVER THE CONTROL VOLUME SURFACE (CS)

d) THE BERNOULLI EQUATION IS DERIVED WITH THE ASSUMPTION OF FRICTIONLESS FLOW ALONG A STREAMLINE.

IT RESEMBLES THE ENERGY EQUATION BUT IT DOES NOT INCLUDE VISCOUS WORK AND CHANGES IN INTERNAL ENERGY DUE TO HEAT ADDITION.

PROBLEM 5 – VISCOSITY.

a) 

PISTON WEIGHT : 9.5kg
CLEARANCE : 0,0254 mm
DECELERATION : 0,64 m/s²
SPEED : 6,4 m/s

ESTIMATE THE VISCOSITY OF THE FLUID USED FOR LUBRICATION:

THE CLEARANCE IS MUCH SMALLER THAN THE RADIUS OF THE PISTON

$$\left(\frac{D_1}{2}\right) << \left(\frac{D_2 - D_1}{2}\right)$$

$\Rightarrow$ IT IS POSSIBLE TO USE CARTESIAN COORDINATES LOCALLY.

THE FORCE ON THE PISTON SHOULD BE BALANCED BY THE FRICTION FROM THE LUBRICATION FLUID.

$$m(g+a) = \tau_w A \qquad (1)$$

WHERE: $m = 9.5$ kg
$g = 9.81$ m/s²
$a = -0.64$ m/s²
$A = \pi D L = 6.06 \cdot 10^{-2}$ m²

$$\tau_w = \mu \frac{\partial u}{\partial y} = \mu \frac{u}{\Delta h} \quad (2)$$

WHERE: $u = 6.4\ m/s$

$\Delta h = 0.0254\ mm$

(1) AND (2) $\Rightarrow$

$$m(g-a) = \mu \frac{u}{\Delta h} \pi D L$$

$$\Rightarrow \mu = \underline{6.5 \cdot 10^{-3}\ Ns/m^2}$$

b) IN A NEWTONIAN FLUID, THE SHEAR STRESS IS PROPORTIONAL TO THE VELOCITY GRADIENT.

c)

LIQUIDS:

THE VISCOSITY <u>DECREASES</u> WITH INCRESED TEMPERATURE

GASES:

THE VISCOSITY <u>INCREASES</u> WITH INCREASED TEMPERATURE

d) IN THE VISCOUS SUBLAYER THE FLOW IS DOMINATED BY THE MOLECULAR VISCOSITY

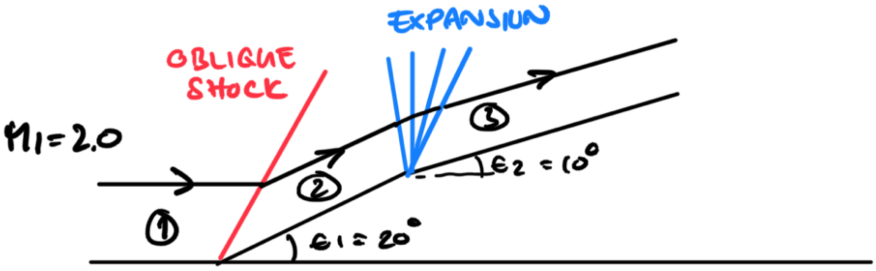
$$\mu \gg \mu_t$$

IN THE FULLY TURBULENT REGION THE FLOW IS DOMINATED BY THE TURBULENT VISCOSITY.

$$\mu_t \gg \mu$$

PROBLEM 6 - WEDGE FLOW

a) SCHEMATIC SKETCH



b) CALCULATE THE MACH NUMBER IN REGIONS 2 AND 3

$1 \rightarrow 2$: OBLIQUE SHOCK $(M_1 = 2.0,\ \theta = 20^\circ)$

THE OBLIQUE SHOCK SHOULD DEFLECT THE $M = 2.0$ FLOW 20°.

θ-β-M-RELATION (Fig 7.1) GIVES

$\beta = 58^\circ$

$$\left.\begin{aligned} M_{n_1} &= M_1 \sin(\beta) \\ M_{n_2} &= M_2 \sin(\beta - \theta) \end{aligned}\right\} (9.82)$$

$\Rightarrow M_2 = 1.21$ //

$2 \rightarrow 3$: PRANDTL-MEYER EXPANSION

$(9,99) \Rightarrow \omega_1 = \omega(M_2) = 3,81^\circ$

$\omega_2 = \omega_1 + \Delta\theta = 3,81 + 10^\circ = 13,81^\circ$

(9,99) (OR INTERPOLATION IN TABLE B5)

WITH $\omega_2(M_3) = 13,81^\circ$

$\Rightarrow \underline{\underline{M_3 = 1,56}}$

c)

A 20-DEGREE FLOW DEFLECTION RESULTS IN A STRONGER SHOCK THAN A 10-DEGREE FLOW DEFLECTION AND THUS MORE LOSSES AND CONSEQUENTLY LOWER TOTAL PRESSURE IN THE DIRECTED FLOW.

$1 \rightarrow 2$ WITH $M_1 = 2.0$ AND $\theta = 10^\circ$

$\Rightarrow \beta = 39.3$

$M_2 = 1,6$

(A SIGNIFICANTLY WEAKER SHOCK)