MTF053 - Fluid Mechanics 2021-10-29 08.30 - 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- Beta Mathematics Handbook for Science and Engineering
- Physics Handbook : for Science and Engineering
- Graph drawing calculator with cleared memory

Exam Outline:

 $-\,$ In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24 - 35	36 - 47	48-60
grade	3	4	5

PROBLEM 1 - COUETTE-POISEUILLE (10 P.)

A Couette flow is established in the fluid between two parallel plates if the plates move with different velocity as in the left figure below where the lower plate is fixed and the upper plate moves to the right with the constant velocity V. The right figure shows a Poiseuille flow generated as a fluid between two fixed parallel plates is exposed to a constant pressure gradient.



Now, let's combine the two elementary flows above, i.e. flow between two parallel plates of which the upper is moving with a constant velocity V and the fluid is exposed to a constant pressure gradient dp/dx.

- (a) Derive an expression for the velocity profile u(y) starting from the momentum equations on partial differential form (Eqn. 4.38). The vertical distance between the plates is 2h as in the figures above (4p)
- (b) Find the velocity V as a function of the height (h), the fluid viscosity (μ) , and the pressure gradient (dp/dx) such that the wall-shear stress at the upper wall is zero (2p)
- (c) Find the vorticity at the center of the channel when the wall-shear stress at the upper wall is zero (2p)
- (d) What is the physical interpretation of fluid viscosity? (1p)
- (e) What does it mean that a fluid is Newtonian? (1p)

PROBLEM 2 - FLOW DEFLECTION (10 P.)

A jet strikes an inclined fixed plate and the jet flow is divided into two jets (as indicated in the picture below). The jet flow velocity is unchanged and the volume flow Q is separated such that the volume flow of the jet going upwards is αQ , $\alpha \in [0, 1]$ and consequently the volume flow of the jet going in the opposite direction is $(1 - \alpha)Q$.

- (a) Find α as a function of the deflection angle θ such that the tangential force F_t is zero (7p)
- (b) Explain the physical meaning of each of the terms in Reynolds transport theorem (2p)

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt} \left(\int_{cv} \beta \rho d\mathcal{V} \right) + \int_{cs} \beta \rho(\mathbf{V}_r \cdot \mathbf{n}) dA$$

(c) Explain the physical meaning of the local acceleration term and the convective acceleration term (1p)



PROBLEM 3 - SURFACE ROUGHNESS (10 P.)

In order to estimate the surface roughness of a badly corroded pipe, pressure is measured at two positions in the pipe as water with a temperature of $20^{\circ}C$ flows through the pipe at a flow rate $Q = 20 \ m^3/h$. The inner diameter of the pipe is 5.0 cm and the pipe slopes downward at an angle of 8°.

station	pressure [kPa]	z-coordinate [m]
1	420	12
2	250	3

- (a) Estimate the average surface roughness ε (6p)
- (b) Estimate the percent change in head loss if the pipe were smooth (same flow rate) (2p)
- (c) What does critical Reynolds number mean for a pipe flow? (1p)
- (d) Why does the Moody chart not give reliable values in the Reynolds number range 2000 < Re < 4000? (1p)

PROBLEM 4 - BOUNDARY LAYER FLOW (10 P.)

A stagnation tube is mounted on a flat plate as shown in the figure below. The entrance of the tube is located at the axial distance L = 0.5 m from the leading edge of the plate. The vertical distance from the flat plate surface to the center of the orifice of the stagnation tube is h = 2.0 mm. The freestream velocity is $U_{\infty} = 15 m/s$. The fluid is air at 20°C and atmospheric pressure. The stagnation tube is attached to water-manometer.

- (a) Calculate Δh if the boundary layer is laminar (8p)
- (b) Explain the closure problem related to the Reynolds-averaged flow equations (1p)
- (c) How does the turbulence viscosity ν_t compare to the kinematic viscosity ν in the viscous sublayer and in the fully turbulent region, respectively? (1p)



PROBLEM 5 - PUMP (10 P.)

The construction schematically represented in the figure below is used to pump up water from a reservoir. The pipe between the pump and the reservoir has the diameter $D_1 = 30.0 \ cm$. At the height $h = 10.0 \ m$ above the water level in the reservoir, a nozzle with the exit diameter $D_2 = 15.0 \ cm$ is attached to the pipe. The water leaves the nozzle with an average velocity of $V_{exit} = 5.0 \ m/s$. The friction losses in the pipe system can be approximated as V_{exit}^2/g and the flow can be assumed to be turbulent in all pipes.

- (a) Calculate the efficiency of the pump $(\eta = power_{out}/power_{in})$ if 20 kW delivered to the pump (8p)
- (b) Why is the kinetic energy correction factor larger for laminar flows than for turbulent flows ($\alpha_{lam} = 2.0, \alpha_{turb} \approx 1.0$)? (1p)
- (c) Give three examples of sources of local losses in a pipe system (1p)



PROBLEM 6 - ENGINE INLET (10 P.)

Engine inlets designed for supersonic operation often feature inlet cones for gradual deceleration of the flow by setting up a system of oblique shocks. In the schematic figure below, two engine inlets are compared. The engine inlet to the left has an inlet cone were the flow angle is changed in two discrete steps, which will produce two oblique shocks. In each of the two steps, the flow is bent 8 degrees. After passing the two oblique shocks the flow passes a normal shock when reaching the engine nacelle. In the example to the right, the flow is decelerated by a single normal shock at the engine inlet face.

In reality, the engine inlets are circular but for simplicity let's assume that it is possible to analyse the flow in two dimensions.



- (a) Considering that the oblique shock formed at the tip of the cone needs to deflect the flow an angle of 8 degrees, make an estimate of the lowest Mach number for which the engine inlet will function as intended (2p)
- (b) Calculate the Mach number of the flow entering the engine in the two cases if the freestream Mach number is 3.0 (6p)
- (c) Explain why the engine inlet design with the oblique shock system (left figure) would be more efficient than the an engine inlet design with a single normal shock at the inlet plane (right figure) (2p)

Problem 1. Couette-Poiseuille (10p)

Suggested answer (based on "Fluid Mechanics", Frank M. White, 8th edition):

- a) [4/10]
 - Implement continuity equation to show that u=u(y) [1p]:
 - i. State assumptions for simplifying continuity equation [0.5p]:

2D incompressible flow $\rightarrow \frac{\partial}{\partial z} = 0$

Plates very long and very wide $\rightarrow v = 0, w = 0$

Since the flow is incompressible, continuity equation (4.4) simplified as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

ii. Implement the rest of the assumptions listed above and derive u=u(y) [0.5p]:

$$\frac{\partial u}{\partial x} = 0 \to u = u(y)$$

- Implement linear momentum equation (4.38) and calculate the velocity profile [3p]:
 - i. State assumptions to simplify equation [0.5p]:

2D incompressible flow $\rightarrow \frac{\partial}{\partial z} = 0$

Plates very long and very wide $\rightarrow v = 0, w = 0$

Neglect gravity effects, steady pressure gradient, constant viscosity (μ):

$$\frac{\partial p}{\partial y} = 0, \frac{\partial p}{\partial z} = 0 \rightarrow \frac{\partial p}{\partial x} = \frac{dp}{dx} = const < 0$$

$$\rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x} + \frac{\partial^2 u}{\partial y} + \frac{\partial^2 u}{\partial z} \right)$$
$$= \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

ii. Use the listed assumptions to simplify the above equation and calculate the velocity profile with constants [1.5p]:

$$0 - \frac{dp}{dx} + \mu \left(0 + \frac{\partial^2 u}{\partial y} + 0 \right) = \rho \left(0 + u \cdot 0 + 0 \cdot \frac{\partial u}{\partial y} + 0 \right) \Rightarrow$$

$$\Rightarrow \left(\frac{\partial^2 u}{\partial x}\right) = \frac{1}{\mu} \frac{dp}{dx} \xrightarrow{\text{integrate twice}}$$
$$\Rightarrow u(y) = \frac{1}{2\mu} \frac{dp}{dx} y^2 + c_1 y + c_2, \text{for} - h \le y \le h - [eq. 1]$$

iii. Employ boundary conditions to get the correct velocity profile [1p]:

No-slip conditions at upper and lower plates:

$$u(-h) = 0$$
 and $u(+h) = V$

Insert boundary conditions in eq.1:

$$c_2 = -\frac{1}{2\mu} \frac{dp}{dx} h^2 + \frac{V}{2}$$
$$c_1 = \frac{V}{2h}$$

The final expression for the velocity profile is:

$$u(y) = \frac{1}{2\mu} \frac{dp}{dx} y^2 + \frac{V}{2h} y + -\frac{1}{2\mu} \frac{dp}{dx} h^2 + \frac{V}{2} , -h \le y \le h - [eq. 2]$$

- b) [2/10]
 - Implement the correct equations for calculating the shear stress [1p]:

The assumption of Newtonian fluid (viscous stresses proportional to the element strain rates and viscosity coefficient) along with incompressible flow, enable usage of (4.37):

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x}, \tau_{yy} = 2\mu \frac{\partial v}{\partial y}, \tau_{zz} = 2\mu \frac{\partial w}{\partial z}$$
$$\tau_{xy} = \tau_{yx} = \left(\mu \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tau_{xz} = \tau_{zx} = \left(\mu \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$
$$\tau_{yz} = \tau_{zy} = \left(\mu \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

• Arrive at the correct result [1p]:

Based on earlier assumptions, the only stress term that is non-zero is τ_{xy} :

$$\tau_{xy} = \left(\mu \frac{\partial u}{\partial y}\right) \stackrel{eq.2}{\Longrightarrow}$$
$$\tau_{xy} = \frac{dp}{dx}y + \frac{\mu V}{2h} \stackrel{y=h,\tau_{xy}=0}{\Longrightarrow}$$
$$0 = \frac{\partial p}{\partial x}h + \frac{\mu V}{2h} \Rightarrow V = -\frac{2h^2}{\mu} \frac{dp}{dx}, [eq.3]$$

c) [2/10]

• Implement the correct equations (4.109-4.111) for calculating the vorticity [1p]:

$$\zeta = \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix}$$

* *i*, *j* and *k* are unit vectors along *x*, *y* and *z* axes

• Arrive at the correct value [1p].

Calculating the vorticity vector, considering assumptions made, gives:

$$\begin{aligned} \zeta &= i \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) - j \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + k \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \Rightarrow \\ \zeta &= i (0 - 0) - j (0 - 0) + k \left(0 - \frac{\partial u}{\partial y} \right) \stackrel{eq.2}{\Longrightarrow} \\ \zeta &= -k \left(\frac{1}{\mu} \frac{dp}{dx} y + \frac{V}{2h} \right) \stackrel{y=0}{\Longrightarrow} \\ \zeta &= -k \left(\frac{V}{2h} \right) \stackrel{eq.3}{\Longrightarrow} \end{aligned}$$

[0.5p up to this point if question (b) is not used]

$$\zeta = -k \left(\frac{1}{2h} \left(-\frac{2h^2}{\mu} \frac{dp}{dx} \right) \right) \Rightarrow$$
$$\zeta = k \left(\frac{h}{\mu} \frac{dp}{dx} \right)$$

- d) [1/10]
 - Viscosity is a quantitative measure of a fluid's resistance to flow. It determines fluid's strain rate generated by a given applied shear stress.
- e) [1/10]
 - The applied shear stress is proportional to the velocity gradient for a Newtonian fluid. The constant of proportionality is the viscosity coefficient. This is expressed by equation (1.23).

2. Flow Deflection

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(a) Find α as a function of the deflection angle $\theta {\rm such}$ that the tangential force F_t is zero



Conservation of linear momentum with finite 1D inlets and outlets (3.40)

$$\sum \mathbf{F} = \underbrace{\frac{d}{dt} \int_{cv} \mathbf{V} \rho dV}_{\text{Steady state}} + \sum_{i} (\dot{m}_i \mathbf{V}_i)_{out} - \sum_{i} (\dot{m}_i \mathbf{V}_i)_{im}$$

In tangential direction

$$F_t = \dot{m}_2 \mathbf{V}_2 + \dot{m}_3 \mathbf{V}_3 - \dot{m}_1 \mathbf{V}_1$$

$$F_t = 0, \ \dot{m} = \rho Q$$

$$0 = \rho Q_2 \mathbf{V}_2 + \rho Q_3 \mathbf{V}_3 - \rho Q_1 \mathbf{V}_1$$

$$Q_1 = Q, \ Q_2 = \alpha Q, \ Q_3 = (1 - \alpha)Q \text{ and } \mathbf{V}_1 = \cos \theta V, \ \mathbf{V}_2 = V, \ \mathbf{V}_3 = -V$$

$$0 = \alpha Q \mathcal{V} - (1 - \alpha) Q \mathcal{V} - \cos \theta Q \mathcal{V}$$

$$0 = 2\alpha - 1 - \cos \theta$$
$$\alpha = \frac{1 + \cos \theta}{2}$$

(b) Explain the physical meaning of each of the terms in the Reynolds transport theorem

$$\underbrace{\frac{d}{dt}(B_{syst})}_{I} = \underbrace{\frac{d}{dt}(\int_{cv}\beta\rho dV)}_{II} + \underbrace{\int_{cs}\beta\rho(\mathbf{V}_{r}\cdot\mathbf{n})dA}_{III}$$

- I, Change of B in system over time
- II, Change of B in control volume over time
- III, Change of B over control surface, i.e in- and outflow of B into control volume

(c) Explain the physical meaning of the local acceleration and the convective acceleration term

$$\frac{Du}{Dt} = \underbrace{\frac{\partial u}{\partial t}}_{\text{Local}} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Convective}}$$

The local term is the change of velocity with respect to time at a given point in the flow field. The convective term is the rate of change of velocity due to change of position of fluid particle in fluid flow field, for example due to geometry changes.



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- $\begin{array}{l} & \omega \ \text{ATEV2} \ 20^{\circ} \text{C} \ , \ \text{g} = 978 \ \text{kg/m}^{\text{s}} \\ & \mu = 0.001 \ \text{kg/m}_{\text{s}} \\ & \mathcal{Q} = 20.0 \ \text{m}^{\text{s}}/\text{h} \\ & d = 0.05 \ \text{m} \\ & \mathcal{X} = 8^{\circ} \\ & P_1 = 920 \ \text{LPa} \\ & \mathcal{Z}_1 = 12 \ \text{m} \\ & P_2 = 250 \ \text{Le} \ \text{Pa} \\ & \mathcal{Z}_2 = 3 \ \text{ma} \end{array}$
- a) Estimate THE SURFACE REMAINED OF THE PIPE (2)
- D THE CHANGE (IN PERCENT) IN HEAD LESS IF THE PIPE WAS STREETH AND THE FROM BATE WAS KEPT THE SAME.

9)
$$Q = 20 n^3/h = \frac{20}{3600} = 0.00556 m^3/s$$

THE AVENINGE VELOCITY

(ADD MYMING INCOMPRESSIBLE STEADY-STATE FLOW)

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = 2.83 \text{ m/s}$$

PIPE LENGTH (DENOTED L IN THE FIGURE)

$$L = \frac{\Delta 7}{8146} = 67,7 m$$

$$\left(\frac{P}{89} + \frac{V^2}{29} + \frac{P}{2}\right)_{1} = \left(\frac{P}{89} + \frac{V^2}{23} + \frac{P}{2}\right)_{2} + hf$$

NOUMPLESSIBLE, STEADY-STATE =>

$$v_1 = v_2 = v_1$$

$$= \sum \left(\frac{P}{s_3} + \frac{1}{2}\right)_1 = \left(\frac{P}{s_5} + \frac{1}{2}\right)_2 + hf$$

WE CAN NON CALLWATE UP TO ZE. hf = 26.36 mTHE FRICTION FACTOR of CAN RE CALCULATED $hf = l \frac{L}{d} \frac{V^2}{2g} = 2 f = 0.05$ THE REYNCLOS NUMBER 13: $R_{e_p} = \frac{gVd}{g_1} \approx 1.4.5$

FROM THE TROODY CHART, WE CAN NON ESTIMATE THE NON - DIFTENSIONAL SURFACE ROUGHNED TO BE

$$\frac{z}{d} \approx 0.021$$

WHICH YLEANS THAT

2 = 1.05 ~~

WITH VOLUME FLOW KEPT, THE HEAD LOLS IS A LINEAR FUNCTION OF THE FRICTION FACTUR

ЬЈ

$$hf = \int \frac{L}{d} \frac{V^2}{2g} = \int const$$

FUR THE BOUGH PIPE, WE HAVE FUND THE FRICTION FACTOR TO BE

f= 0,05

FOR A SPACOTH PIPE AND THE SAME BEGINGLOS NUMBER, THE MODY CHART GIVES :

THE CHAMLE IN HEAD Less:



GIVEN:

L = 0, 5 m h = 0,002 m $U_{00} = |5 \text{ m}/s$ Find: air @ 20°C => $\beta = 1.2 \text{ kg/m}^{2}$ $qL = 1.8 \cdot 10^{-5} \text{ kg/ms}$ MANUMETER FLUD WATER@ 20°C => $S_{m} = 998 \text{ kg/ms}^{2} / t = 10^{-3} \text{ kg/ms}$ LANINAL ROUNDARY USER. ESTIMATE THE MANUMETER READING: Δh STARTIMA POINT: THE BERNOW' ROUATON (3.55) $P_{1} + 3 \frac{V_{1}^{2}}{2} + 33^{2} = P_{2} + 3 \frac{V_{2}^{2}}{2} + 33^{2} =$ $=> 8 \frac{V_{1}^{2}}{2} = P_{2} - P_{1}$ $=> V_{1}^{2} = 2 \frac{P_{2} - P_{1}}{P} = 2 \frac{\Delta P}{8}$ (1) THE MANUMETER READING WILL BE PREACETIONAL TO DP.

$$\Delta p = g_{w}g \Delta h$$
 (2)

TO BE ADLE TO OBTAIN DN, WE NEED THE VELOCITY VI

SINCE IT IS A LAMINANL BOUMDARY VAYER. WE CAN USE THE PUSSIS PROFILE.

$$2 = 3\sqrt{\frac{40}{2}} = h\sqrt{\frac{40}{2}} = 2.83$$
$$= 2.83$$
$$= \frac{1}{2}\sqrt{\frac{1}{2}} = 0.8126 \quad (4)$$
$$(4) \quad (3) = 26h = 9.1m.$$

5. Pump

(a) Calculate the efficiency of the pump $(\eta = Power_{out}/Power_{in})$ if 20kW is delivered to the pump

Known:

 $\begin{array}{ll} D_1 = 0.3 \ [\mathrm{m}] & h_f = \frac{V_{exit}^2}{g} \ [\mathrm{m}] \\ D_2 = 0.15 \ [\mathrm{m}] & P_i n = 20 \ [\mathrm{kW}] \\ V_{exit} = 5 \ [\mathrm{m/s}] & h = 10 \ [\mathrm{m}] \\ \mathrm{Assume \ the \ water \ is \ } 20^\circ, \ \mathrm{this \ gives} \ \rho = 998 \ [\mathrm{kg/m^3}] \end{array}$

The energy equation for steady state, incompressible flow (3.73)

$$\begin{split} \left(\underbrace{\frac{P}{\not g}}_{\approx P_{atm}} + \underbrace{\frac{V^2}{2g}}_{\text{large tank}} + z^{*0}\right)_1 &= \left(\underbrace{\frac{P}{\not g}}_{=P_{atm}} + \frac{V^2}{2g} + \underbrace{z}_h\right)_2 + \underbrace{h_{tarbine}}_{h_{tarbine}} - h_{pump} + h_f \\ h_{pump} &= \underbrace{h_f}_{V_{exit}^2/g} + \frac{V_{exit}^2}{2g} + h \\ h_{pump} &= \frac{V_{exit}^2}{g} + \frac{V_{exit}^2}{2g} + h = \frac{3V_{exit}^2}{2g} + h = 13.8[m] \\ P_{out} &= \underbrace{F}_{P\cdot A} \cdot V = \underbrace{\rho g h_{pump}}_{P} \cdot \underbrace{Q}_{A\cdot V} = \rho g h_{pump} \cdot \frac{\pi D_2^2}{4} \cdot V_{exit} = 11957W \\ \eta &= \frac{P_{out}}{P_{in}} = \frac{11957}{20000} = 0.5979 \end{split}$$

The pump has an efficiency of approximately 60%

(b) Why is the kinetic energy correction factor larger for laminar flows than for turbulent flows ($\alpha_{lam} = 2.0, \ \alpha_{turb} \approx 1.0$)?

A laminar flow has a parabolic velocity distribution where $V_{ave} = 1/2V_{max}$, but for turbulent flow the maximum velocity is much closer to the average due to the irregular flow.

(c) Give three examples of sources of local losses in a pipe system

For example:

- Valves
- Bends
- Junctions
- Expansions and contractions
- Entrances

Problem 6. Engine Inlet (10p)

Suggested answer (based on "Fluid Mechanics", Frank M. White, 8th edition):

a) [2/10]

An estimate of the lowest Mach number for which the engine inlet can deflect the flow 8 degrees cab be made via Figure 9.1.
Following the dash-dotted line which indicates the maximum deflection angles θ_{max} for each Mach number we get:

$$Ma_{min} \approx 1.4$$

- b) [6/10]
 - Solve the oblique shocks case [4p]:

To make use of the equations listed in the equation sheet, one must state that the flow is assumed isentropic between shock waves and adiabatic through shocks. Moreover, assume that the medium (air) is perfect gas with k=1.4.

Calculate flow after the first oblique shock [1.5p]:

- Deflection angle is θ=8 degrees, M₁=3. To get the shock angle use Figure 9.1 by assuming the weak shock solution. Considering the above take β=25 degrees.
- Implement equation (9.82):

$$M_{n1} = 1.267$$

Implement equation (9.57) for the normal velocity components:

$$M_{n2} = 0.8$$

Implement equation (9.82):

$$M_2 = 2.746$$

Calculate flow after the second oblique shock [1.5p]:

 \circ Deflection angle is θ=8 degrees, M₂=2.746. To get the shock angle use Figure 9.1 by assuming the weak shock solution.

Considering the above take β =28 degrees.

• Implement equation (9.82):

$$M_{n2} = 1.28$$

Implement equation (9.57) for the normal velocity components:

$$M_{n3} = 0.791$$

• Implement equation (9.82): $M_3 = 2.3$

Calculate flow after the normal shock [1p]:

• Implement equation (9.57) for the normal velocity components:

$$M_4 = M_{entry} = 0.53$$

• Solve the normal shock case [1p]:

• Implement equation (9.57)

$$M_{entry} = 0.4753$$

c) [2/10]

• From the calculations computed in the previous question it was found that the oblique shock system needs to decelerate the flow at the last step (normal shock) a significant lesser amount compared to the single normal shock case.

Thereby the system losses accompanying this normal shock are expected to be less for the oblique shock system, while bringing the inlet flow at approximately the same magnitude.

A system of weaker oblique shocks always results in less losses than a strong single normal shock. That's why engine intakes for supersonic operation are usually designed to generate a system of oblique shocks rather than a normal shock.