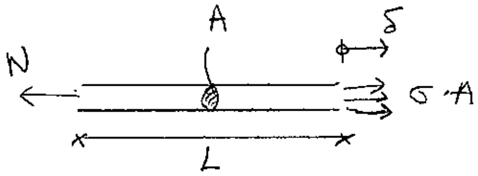
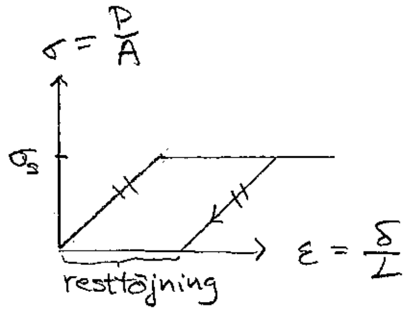


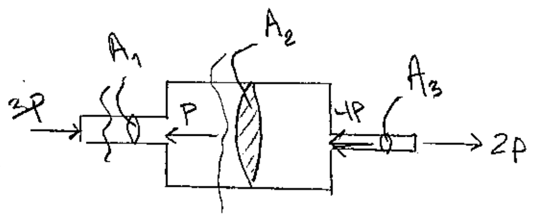
2011-03-22
Tisdag Lv1

$$\left.\begin{aligned} \sigma &= \frac{N}{A} \\ \varepsilon &= \frac{\delta}{L} \\ \sigma &= E\varepsilon \end{aligned}\right\} \quad N = \sigma A = EA\varepsilon = \underline{\frac{EA}{L}\cdot\delta} = N \qquad (2\text{-}15)$$

$$\delta = \frac{NL}{EA} \qquad (2\text{-}14)$$

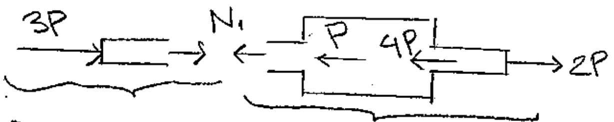


1.7



$A_1 = 75\ mm^2$
$A_2 = 100\ mm^2$
$A_3 = 50\ mm^2$
$P = 2kN$

Snitt genom del 1



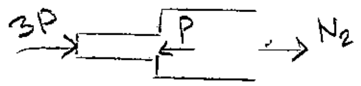
$\rightarrow: \; 3P + N_1 = 0$
$N_1 = -3P$

$\leftarrow: \; N_1 + P + 4P - 2P = 0$
$N_1 = -3P$

$$\sigma_1 = \frac{N_1}{A_1} = \frac{-3P}{A_1} = -80\cdot 10^6\, Pa = \underline{-80\ MPa}$$

(tryck)

Snitt genom del 2:

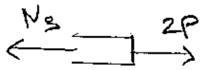


$\longrightarrow$: $N_2 + 3P - P = 0$

$N_2 = -2P$

$\sigma_2 = \dfrac{-2P}{A_2} = \underline{-40\,\text{MPa}}$

Snitt genom del 3:

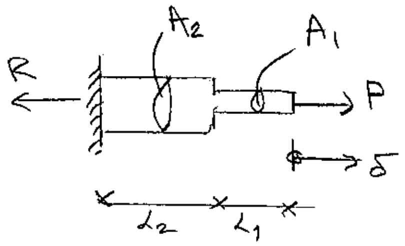


$\longrightarrow$: $N_3 = 2P$

$\sigma_3 = \dfrac{2P}{A_3} = \underline{80\,\text{MPa}}$

(drag)

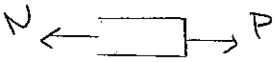
1.3



$\longrightarrow$: $R = P$

Bestäm spänningarna i delarna samt δ

Normalkraften blir samma i båda delarna.
Snitta någonstans:



$N = P$ (båda delarna)

Del 1: $\underline{\sigma} = \dfrac{N}{A_1} = \underline{\dfrac{P}{A_1}}$

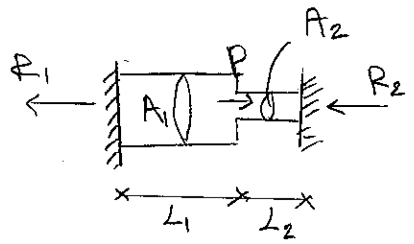
Del 2: $\underline{\sigma = \dfrac{P}{A_2}}$

Förlängning av del 1: $\delta_1 = \dfrac{NL_1}{EA_1}$

——— " ——— del 2: $\delta_2 = \dfrac{NL_2}{EA_2}$

$$\delta = \delta_1 + \delta_2 = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right) \qquad (P = N)$$

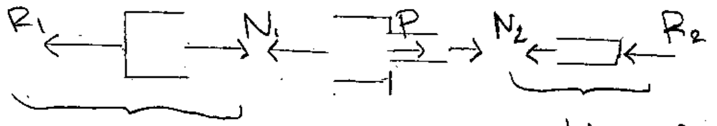
2.2



Bestäm normalkrafterna.

$\rightarrow$: $P - R_1 - R_2 = 0$ (1)

Snitta genom delarna:



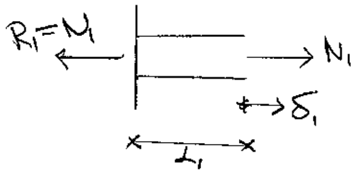
$\rightarrow$: $N_1 = R_1$ (2) $\qquad N_2 = -R_2$ (3)

$\rightarrow$: $P + N_2 - N_1 = 0$ (4)

Ej lineärt oberoende: (2) & (3) insatt i (4) ger (1)

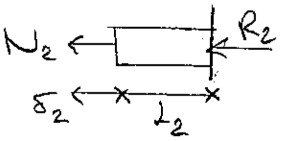
Kompatibilitet: $\delta = 0$ (stångens förlängning)

Del 1:



$$\delta_1 = \frac{N_1 L_1}{EA_1} = \frac{R_1 L_1}{EA_1}$$

Del 2:



$$\delta_2 = \frac{N_2 L_2}{EA_2} = \frac{-R_2 L_2}{EA_2} \overset{(5)}{=} \frac{(R_1 - P)L_2}{EA_2}$$

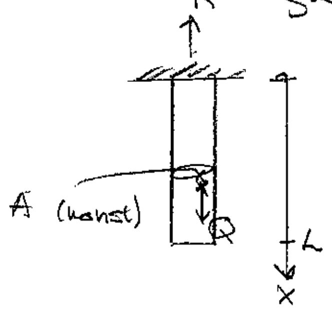
$$\delta = \delta_1 + \delta_2 = 0 \Rightarrow \frac{R_1 L_1}{EA_1} + \frac{R_1 L_2}{EA_2} - \frac{P L_2}{EA_2} = 0$$

$$\underline{R_1 = \frac{P L_2 A_1}{L_2 A_1 + L_1 A_2} = N_1}$$

(5) $$-R_2 = -P + R_1 = \underline{\frac{-P L_1 A_2}{L_2 A_1 + L_1 A_2} = N_2}$$

1.9

$R = \rho L A g$



$\rho = 7800 \text{ kg/m}^3$ $\quad E = 210 \text{ GPa}$

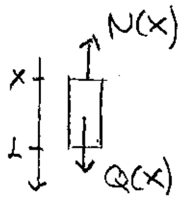
$\sigma_B = 370 \text{ MPa}$

$g = 9{,}82 \text{ m/s}^2$

$Q = \rho L A g$

a) Bestäm L_{max} s.a. $\sigma < \sigma_B$

Snitta vid godtyckligt x:



$Q(x) = \rho (L-x) A g$

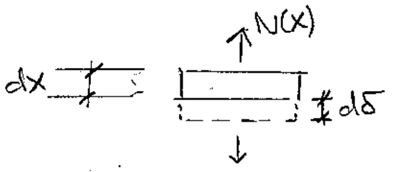
$\uparrow: \; N(x) = Q(x) = \rho g A (L-x)$

$$\sigma_{max} = \left(\frac{N}{A}\right)_{max} = \frac{N_{max}}{A} = \frac{N(0)}{A} = \rho g L < \sigma_B$$

$$\Rightarrow \; L < \frac{\sigma_B}{\rho g} = \underline{\underline{4{,}83 \text{ km}}}$$

b) Bestäm δ om $L = \dfrac{\sigma_B}{2\rho g}$

Betrakta en lamell med tjocklek dx:



$\varepsilon = \dfrac{d\delta}{dx}$

$\sigma(x) = \dfrac{N(x)}{A} = \rho g (L-x)$

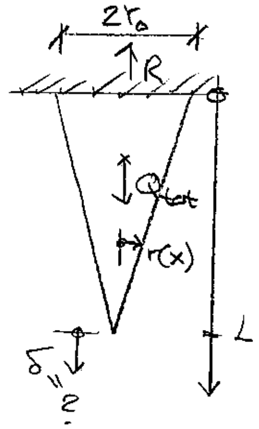
$\varepsilon = \dfrac{\sigma}{E} = \dfrac{\rho g}{E}(L-x)$

$$d\delta = \varepsilon dx = \frac{\rho g}{E}(L-x)dx$$

$$\delta = \int_0^L d\delta = \frac{\rho g}{E}\int_0^L (L-x)dx = \frac{-\rho g}{2E}\left[(L-x)^2\right]_0^L = \frac{\rho g L^2}{2E}$$

$$= \left\{ L = \frac{\sigma_B}{2\rho g} \right\} = \frac{\sigma_B^2}{8E\rho g} = \underline{\underline{1{,}06 \text{ m}}}$$

1.19



E, ς givna

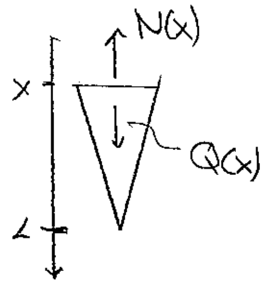
$r(x) = r_0 \left(1 - \frac{x}{L}\right)$

$A(x) = \pi\, r(x)^2$

Snitta vid godtyckligt x:

$Q(x) = A(x)(L-x)\,\frac{1}{3}\varsigma g$

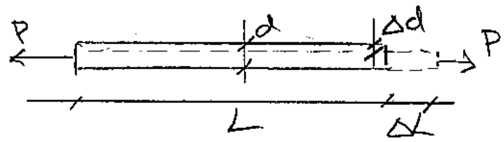
$\uparrow: N(x) = \frac{\varsigma g A(x)(L-x)}{3}$



......

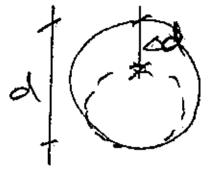
(som förut) $\quad \delta = \int_0^L \varepsilon\, dx = \underline{\underline{\frac{\varsigma g L^2}{6E}}}$

1.15



$L = 200$ mm

$\Delta L = 0{,}3$ mm



ν givet, bestäm $\frac{\Delta V}{V}$

$\varepsilon = \frac{\Delta L}{L} \qquad \varepsilon_{tvärs} = \frac{-\Delta d}{d} = -\nu\varepsilon$

$V = L\frac{\pi d^2}{4}$

$$V+\Delta V = (L+\Delta L)\frac{\pi}{4}(d-\Delta d)^2 = \frac{\pi}{4}\big(\overbrace{Ld^2}^{V}+L(\Delta d)^2 - 2L d\cdot\Delta d + \Delta L\cdot d^2 + \Delta L\cdot(\Delta d)^2 - 2d\Delta L\cdot\Delta d\big)$$

$$= \left\{\Delta d,\ \Delta L \ll L, d\right\} = V + \underbrace{\frac{\pi}{4}d\,(d\cdot\Delta L - 2L\cdot\Delta d)}_{\Delta V}$$

$$\Delta V = \frac{\pi}{4}\cdot d\Big(d\cdot\varepsilon\cdot L + 2L\underbrace{\varepsilon_{tvärs}}_{-\nu\varepsilon}d\Big) =$$

$$= \underbrace{\frac{\pi}{4}d^2L}_{V}(1-2\nu)\varepsilon$$

$$\Rightarrow \quad \frac{\Delta V}{V} = (1-2\nu)\varepsilon$$

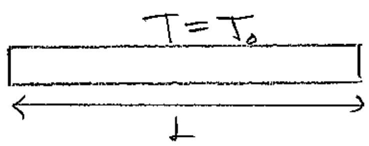
a) $\nu = 0{,}3 \Rightarrow \frac{\Delta V}{V} = (1-2\cdot 0{,}3)\underbrace{\frac{0{,}3\,\text{mm}}{200\,\text{mm}}}_{\varepsilon=\frac{\Delta L}{L}} = \underline{\underline{0{,}6\cdot 10^{-3}}}$

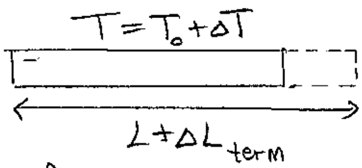
b) $\nu = 0{,}5 \Rightarrow \frac{\Delta V}{V} = 0$ (inkomp.)

/and tis

Termoelasticitet (5.3)

2011-03-25
Fredag Lv1





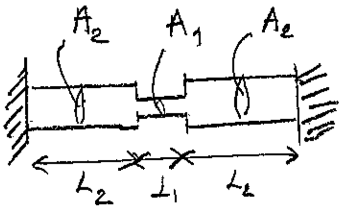
$\Delta L = \alpha\Delta T L$, α = längdutvidgningskoeff.

$$\varepsilon_{term} = \frac{\Delta L_{term}}{L} = \alpha\Delta T$$

$$\sigma = \frac{N}{A} = E\varepsilon_{max}$$

$$\varepsilon_{max} = \frac{N}{EA} \qquad \varepsilon = \varepsilon_{mek} + \varepsilon_{term}$$

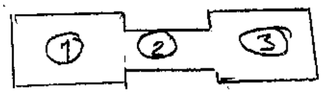
2.6)



$E = 210$ GPa
$\alpha = 12{,}5 \cdot 10^{-6}/°C$
$\Delta T = 30°C$

$L_1 = 10$ cm, $L_2 = 20$ cm, $A_1 = 1$ cm^2, $A_2 = 2$ cm^2
$\sigma = 0$ vid $T = T_0$. Bestäm σ vid $T = T_0 + \Delta T$

Lösn



$\varepsilon_{term} = \alpha \Delta T$
$\Delta L_{term} = \varepsilon_{term}(L_1 + 2L_2)$

Del 1 och 3: $\sigma_1 = \sigma_3 = \dfrac{N}{A_2}$

$$\varepsilon_1 = \varepsilon_3 = \frac{\sigma_1}{E} = \frac{N}{EA_2}$$

Del 2: $\sigma_2 = \dfrac{N}{A_1}$ $\quad \varepsilon_2 = \dfrac{\sigma_2}{E} = \dfrac{N}{EA_1}$

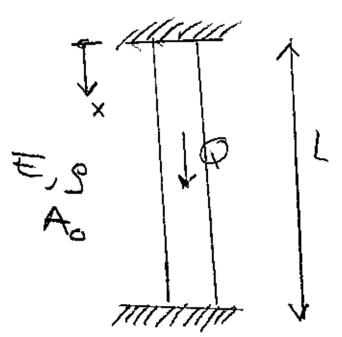
$$\Delta L_{mek} = \sum_i \varepsilon_i L_i = \frac{2NL_2}{EA_2} + \frac{NL_1}{EA_1}$$

$$\Delta L = \Delta L_{mek} + \Delta L_{term} =$$
$$= N\left(\frac{L_1}{EA_1} + \frac{2L_2}{EA_2}\right) + \alpha \Delta T (L_1 + 2L_2) = 0$$

kompatibilitet

$$\Rightarrow N \Rightarrow \sigma_{max} = \frac{N}{A_1}$$

2.8



Beräkna normalspänningen $\sigma(x)$ av egentyngden.

Lösn: $Q = \rho V g \qquad K_x = \frac{Q}{V} = \rho g$

$$-\frac{d}{dx}\Big[EA \underbrace{\frac{du}{dx}}_{\varepsilon}\Big] = K_x A$$

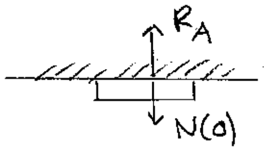
$$\begin{cases} -\dfrac{d^2u}{dx^2} = \dfrac{\rho g}{E} & 0 < x < L \\ u(0) = 0 \\ u(L) = 0 \end{cases} \Longrightarrow u = \frac{-\rho g x^2}{2E} + C_1 x + C_2$$

$$u(0) = 0 \Rightarrow C_2 = 0$$

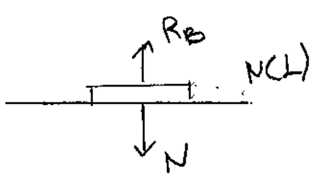
$$u(L) = 0 \Rightarrow -\frac{\rho g L^2}{2E} + C_1 L = 0$$

$$\Rightarrow u(x) = \frac{\rho g L}{2E} x - \frac{\rho g}{2E} x^2$$

$$\sigma(x) = E\varepsilon = E\frac{du}{dx} = \frac{\rho g L}{2} - \rho g x = \frac{\rho g L}{2}\left(1 - 2\frac{x}{L}\right)$$

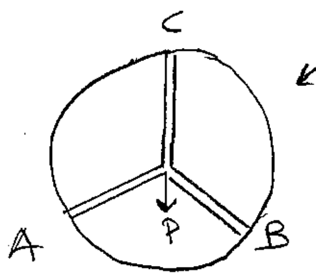


$$R_A = N(0) = \frac{\rho g L A}{2}$$



$$R_B = \frac{\rho g L A}{2}$$

2.9



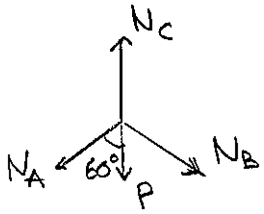
Ekrarna har axialstyvheten

$k = \frac{EA}{L}$

Beräkna navets vertikala förskjutning Δ.

Lösn:

Frilägg:



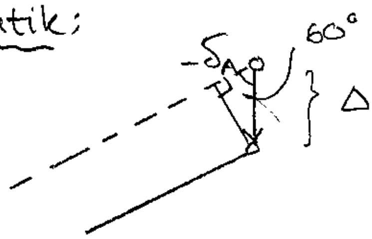
$\leftarrow$: $N_A \sin 60° - N_B \sin 60° = 0$, $N_A = N_B$ (1)

$\downarrow$: $P - N_C + N_A \cos 60° + N_B \cos 60° = 0$

$N_C - (N_A + N_B)\frac{1}{2} = P$ (2)

$N_C - N_A = P$ (3)

Kinematik:



$\Delta = \delta_C \ll L$

$-\delta_A = \Delta \cos 60°$, $\delta_A = -\frac{\Delta}{2}$

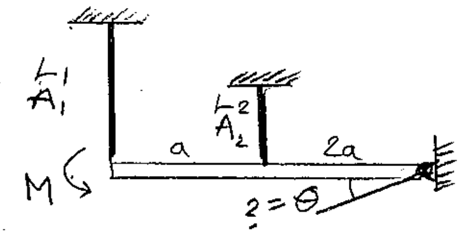
$N_A = \frac{EA}{L}\delta_A = \frac{-EA}{2L}\cdot\Delta$ (4)

$N_C = \frac{EA}{L}\delta_C = \frac{EA}{L}\cdot\Delta$ (5)

(4) och (5) in i (3) $\Longrightarrow$

$\Longrightarrow \left(\frac{EA}{L} + \frac{EA}{2L}\right)\Delta = P$, $\Delta = \frac{2PL}{3EA}$

2.20

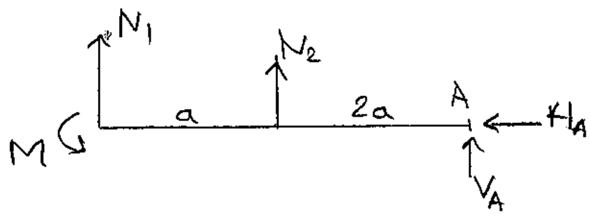


E given.

Hur mycket vrider sig bommen då den belastas med momentet M?

Lösn.

Frilägg:

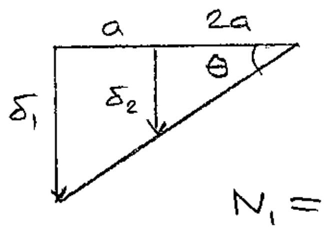


$$\overset{\curvearrowright}{A}: \quad N_1 \cdot 3a + N_2 \cdot 2a - M = 0$$

$$3N_1 + 2N_2 = \frac{M}{a} \qquad (1)$$

$$\left(\leftarrow: \quad H_A = 0 \quad , \quad \uparrow: \quad V_A + N_1 + N_2 = 0 \right)$$

Deformationssamband:



$$\tan\theta = \frac{\delta_1}{3a} = \frac{\delta_2}{2a}$$

$$\delta_1 = 3\theta a \quad , \quad \delta_2 = 2\theta a$$

$$N_1 = \frac{EA_1}{L_1}\delta_1 = \frac{3EA_1}{L_1}\theta a \qquad (2)$$

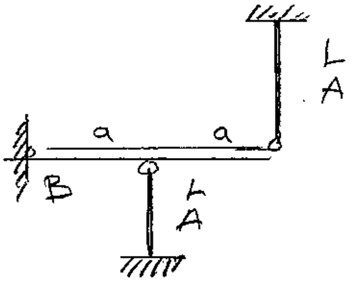
$$N_2 = \frac{EA_2}{L_2}\delta_2 = \frac{2EA_2}{L_2}\theta a \qquad (3)$$

(2) och (3) in i (1) ger:

$$\left(\frac{9EA_1}{L_1} + \frac{4EA_1}{L_2} \right)\theta a = \frac{M}{a}$$

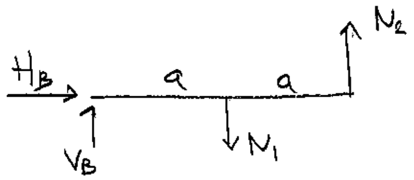
$$\theta = \frac{ML_1L_2}{Ea^2(9A_1L_2 + 4A_2L_1)} \qquad \text{(dimension O.K.)}$$

2.21



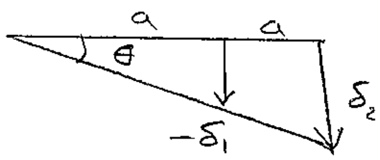
Hur mycket förskjuts balkens högra ände om båda stängerna upphettas T?
Längdutvidgn.koeff. α given
Elasticitetsmodul E given

Lösn: Frilägg i



$$B\curvearrowright : N_1 a - N_2 \cdot 2a = 0 \Rightarrow N_1 - 2N_2 = 0 \quad (1)$$

Deformationssamband:



$\theta << 1$

$$\tan\theta \approx \theta = \frac{\delta_2}{2a} = \frac{-\delta_1}{a}$$

$$\delta_1 = \frac{-\delta_2}{2}$$

$$\delta_2 = \frac{N_2 L}{EA} + \alpha T L$$

$$N_2 = \frac{EA}{L}\delta_2 - \alpha TEA \quad (2)$$

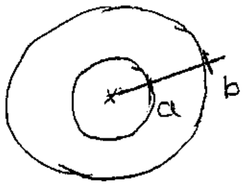
$$\delta_1 = \frac{N_1 L}{EA} + \alpha T L = \frac{-\delta_2}{2}$$

$$N_1 = -\frac{EA}{2L}\delta_2 - \alpha TEA \quad (3)$$

(2) och (3) i (1) ger: $\left(-\frac{EA}{2L} - \frac{2EA}{L}\right)\delta_2 - \alpha TEA + 2\alpha TEA = 0$

$$\Rightarrow \delta_2 = \alpha TEA \cdot \frac{2}{5} \cdot \frac{L}{EA} = \underline{\underline{\frac{2\alpha TL}{5}}}$$

/end fre Lv 1



$$\varphi = \frac{M_v L}{GK} \qquad (6-11)$$

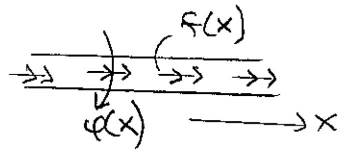
$$\tau = \frac{M_v r}{K} \qquad (6-13)$$

2011-03-29
Tisdag Lv 2

$$K = \frac{\pi}{2}(b^4 - a^4) \qquad (6-12)$$

Balkens diff.ekv.

$$-\frac{d}{dx}\Big[\underbrace{GK\frac{d\varphi}{dx}}_{M_v(x)}\Big] = f(x)$$

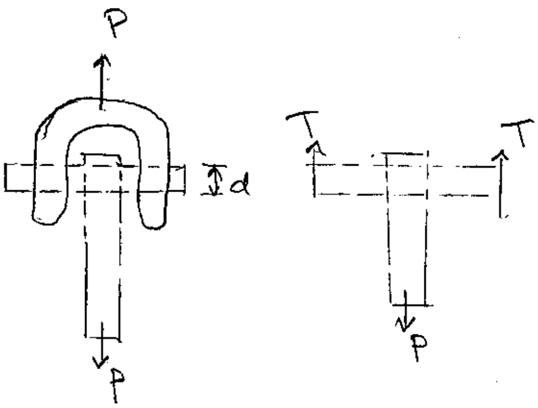


$$K = 2\pi a^3 h \qquad (6-7)$$

$$\varphi = \frac{M_v L}{2\pi G a^3 h} \qquad (6-6)$$

$$\tau = \frac{M_v}{2\pi a^2 h} \qquad (6-4)$$

4.1.

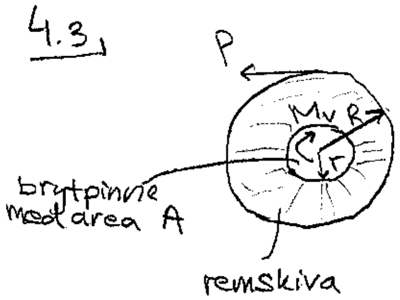


$$\uparrow: T = \frac{P}{2} \qquad (2T - P = 0)$$

$$\bar{\tau} = \frac{T}{A} = \frac{P/2}{\pi \frac{d^2}{4}} = \underline{6.4 \text{ MPa}}$$

$$\left(T = \int_A \tau \, dA = \bar{\tau} A\right)$$

4.3



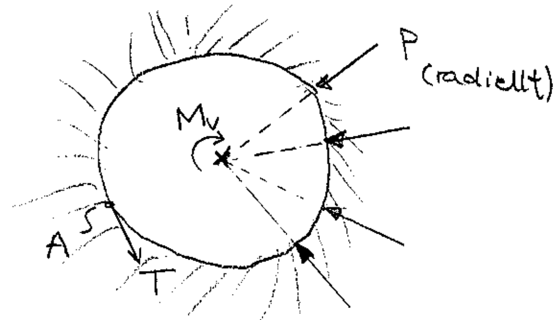
$R = 200$ mm, $r = 50$ mm

Ingen friktion mellan axel och skiva.

Bestäm A på brytpinnen så att den går av då

$M_v = 200$ Nm, $\tau_s = 200$ MPa.

Lösn: Frilägg axeln:

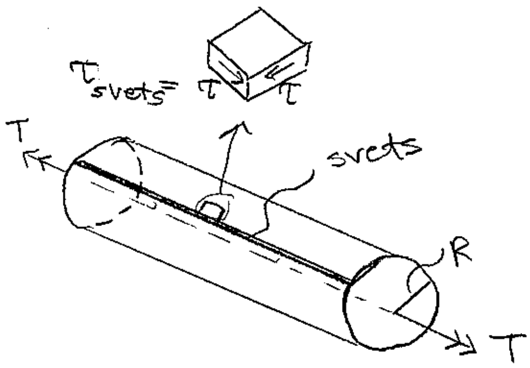


$\curvearrowright$: $M_v - T \cdot r = 0$

$$\tau = \frac{T}{A} = \tau_s$$

$$A = \frac{M_v}{r\tau_s} = 20 \cdot 10^{-6}\,\text{m}^2 = 20\,\text{mm}^2$$

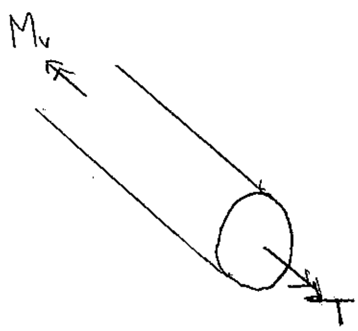
4.5



$h = 5$ mm

$R = 100$ mm

Bestäm T_{max} om $\tau_{svets}^{(max)} = 200$ MPa.



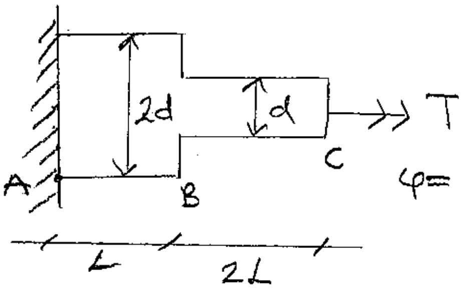
$M_v = T$ i hela röret

(6-4): $\tau = \dfrac{M_v}{2\pi R^2 h}$

$$T = 2\pi R^2 h\tau = \{\tau = \tau_{svets}\} =$$

$$= 63\ \text{kNm}$$

4.7



$\varphi = ?$

$L = 200$ mm
$d = 80$ mm
T så att
$\tau_{max} = 50$ MPa
$G = 85$ GPa

Bestäm den fria ändens rotation φ.

(6-13): $$\tau_{max} = \frac{M_v b}{\frac{\pi}{2}(b^4 - \underbrace{a^4}_{=0})} = \{a=0\} = \frac{2M_v}{\pi b^3} =$$

$$= \{ b = \tfrac{d}{2} \text{ ger max} \} = \frac{16 M_v}{\pi d^3}$$

$$M_v = T = \frac{\pi d^3}{16} \cdot \tau_{max} \quad (\approx 5 \text{ kNm})$$

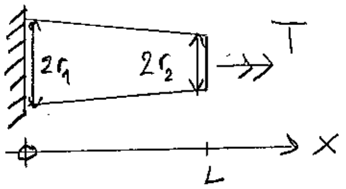
Konstant i hela axeltappen.

(6-11). $$\varphi_{AB} = \frac{M_v \cdot L}{G \frac{\pi}{2} d^4} = \frac{\tau_{max} L}{8 G d}$$

$$\varphi_{BC} = \frac{M_v \cdot 2L}{G \frac{\pi}{2} (\frac{d}{2})^4} = \frac{4 \tau_{max} L}{G d}$$

$$\varphi = \varphi_{AB} + \varphi_{BC} = \frac{33}{8} \frac{\tau_{max} L}{G d} \approx 6{,}066 \cdot 10^{-3} \text{ rad} \approx 0{,}35^\circ$$

4,13



$r(x) = r_1 - \frac{x}{L}(r_1 - r_2)$

G given. Bestäm $\varphi(x)$

$b = r(x)$, $a = 0$ $\quad$ (6-12): $K = \frac{\pi}{2}\left(r_1 - \frac{x}{L}(r_1 - r_2)\right)^4$

$$\begin{cases} -\frac{d}{dx}\left[GK\frac{d\varphi}{dx}\right] = 0 & 0 < x < L \\ \varphi(0) = 0 & (1) \\ \left.\frac{d\varphi}{dx}\right|_{x=L} = \frac{T}{GK(L)} & (2) \end{cases}$$

$\overset{M_v(L)}{\twoheadleftarrow}\!\!]\ \overset{T}{\twoheadrightarrow}$ $\qquad \twoheadrightarrow$: $M_v(L) = T$, $\qquad M_v(x) = GK\frac{d\varphi}{dx}$

Integrera d.e.: $GK\frac{d\varphi}{dx} = C_1$, $\frac{d\varphi}{dx} = \frac{C_1}{GK(x)}$

(2): $\frac{C_1}{GK(L)} = \frac{T}{GK(L)}$ $\qquad C_1 = T \Rightarrow \frac{d\varphi}{dx} = \frac{2T}{G\pi\left(r_1 - \frac{x}{L}(r_1 - r_2)\right)^4}$

Integrera igen:

$$\varphi(x) = \frac{2TL}{3G\pi\left(r_1 - \frac{x}{L}(r_1 - r_2)\right)^3 (r_1 - r_2)} + C_2$$

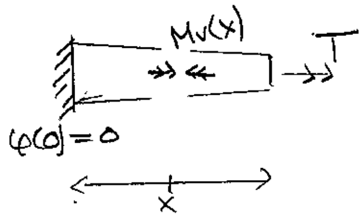
(1): $\frac{2TL}{3G\pi r_1^3 (r_1 - r_2)} + C_2 = 0$

$$\varphi(x) = \frac{2TL}{3G\pi(r_1 - r_2)}\left(\frac{1}{\left(r_1 - \frac{x}{L}(r_1 - r_2)\right)^3} - \frac{1}{r_1^3}\right)$$

Speciellt: $\varphi(L) = \dfrac{2TL}{3\pi G(r_1 - r_2)}\left(\dfrac{1}{r_2^3} - \dfrac{1}{r_1^3}\right) =$

$$= \frac{2TL(r_1^3 - r_2^3)}{3\pi G(r_1 - r_2) r_1^3 r_2^3} = \frac{2TL(r_1^2 + r_1 r_2 + r_2^2)}{3\pi G\, r_1^3\, r_2^3}$$

<u>Alt</u>:

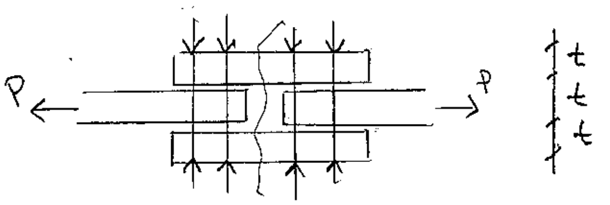


$M_v(x) = T$ konstant

$GK\dfrac{d\varphi}{dx} = T$

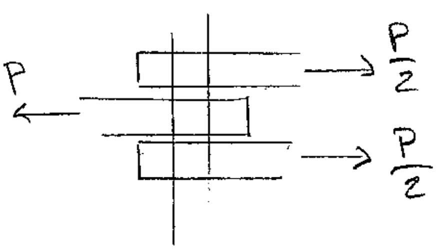
$\dfrac{d\varphi}{dx} = \dfrac{T}{GK}$

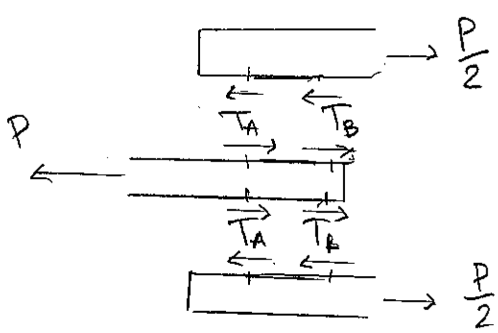
<u>4.6</u>



Bredd B

Bestäm $\dfrac{A_A}{A_B}$ s.a. skjuvsp. blir samma i båda skruvar.

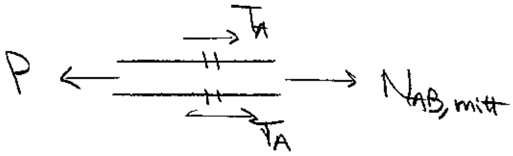




$\leftarrow$: $P - 2T_A - 2T_B = 0$

$$\Rightarrow T_B = \frac{P}{2} - T_A \quad (1)$$

Mittplåten mellan A o B:

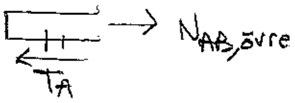


$\rightarrow$: $N_{AB,mitt} = P - 2T_A$

$$\delta_{AB,mitt} = \frac{(P-2T_A)L}{E\,Bt} \qquad (2)$$

(2-14)

Övre plåten mellan A o B:



$\rightarrow$: $N_{AB,övre} = T_A$

$$\delta_{AB,övre} = \frac{T_A L}{E \cdot Bt} \qquad (3)$$

$$T_A = \frac{P}{2} - T_B$$

(2) = (3) om skruvarnas deformation kan försummas.

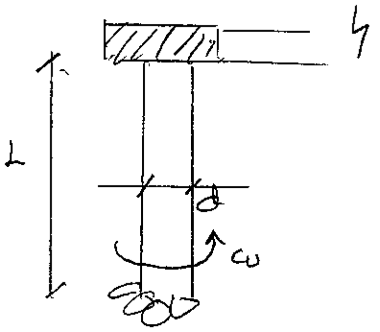
$\Longrightarrow$ $P - 2T_A = T_A$, $T_A = \frac{P}{3}$

Insatt i (1): $T_B = \frac{P}{2} - \frac{P}{3} = \frac{P}{6}$

$$\bar{\tau}_A = \frac{T_A}{A_A}, \qquad \bar{\tau}_B = \frac{T_B}{A_B}$$

$$\bar{\tau}_A = \bar{\tau}_B \Rightarrow \frac{T_A}{A_A} = \frac{T_B}{A_B} \Rightarrow \frac{A_A}{A_B} = \frac{T_A}{T_B} = \underline{\underline{2}}$$

Exempel



$P = 1\ \text{MW}$

$n = 350$ varv/min

E, ν givet

$\sigma_s = 410$ MPa

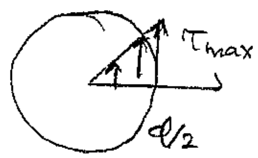
Bestäm d s.a. säkerheten mot plasticering är $s = 5$.

$$\tau_{max} = \frac{\tau_s}{5} \approx \frac{\sigma_s}{10} \qquad \omega = n \cdot \frac{2\pi}{60}$$

$$\text{Effekt} = \frac{\text{Arbete}}{\text{Tid}} = \frac{\text{moment} \cdot \text{vinkel}}{\text{tid}} = M_v \omega = P$$

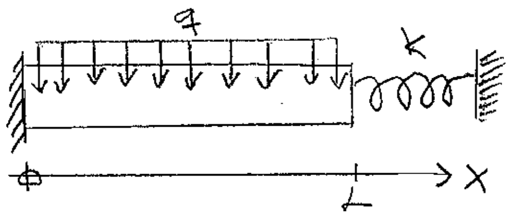
$$(6\text{-}13): \tau_{max} = \frac{M_v \cdot \frac{d}{2}}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{16}{\pi d^3} \cdot \frac{30 P}{n\pi} = \frac{\sigma_s}{10}$$

$$d = \left(\frac{30 \cdot 16 \cdot 10 \cdot P}{n \cdot \pi^2 \sigma_s}\right)^{1/3} \approx 150 \text{ mm}$$



/end tis Lv 2

4.14,



(6-7): $K = 2\pi r^3 t$

$$\begin{cases} -\dfrac{d}{dx}\left[GK\dfrac{d\varphi}{dx}\right] = qr & 0<x<L \\ \varphi(0) = 0 & (1) \\ \left.\dfrac{d\varphi}{dx}\right|_{x=L} + \dfrac{k}{GK}\varphi(L) = 0 & (2) \end{cases}$$

$\underline{x = L:}$ $M_v(L)$, $\varphi(L)$, $k\cdot\varphi(L)$

$\twoheadleftarrow$: $M_v(L) + k\varphi(L) = 0$

$M_v = GK\dfrac{d\varphi}{dx}$

$$\frac{d\varphi}{dx} = \frac{-qrx}{GK} + C_1, \quad \varphi(x) = \frac{-qrx^2}{2GK} + C_1 x + C_2$$

(1): $C_2 = 0$

(2): $\dfrac{-qrL}{GK} + C_1 + \dfrac{k}{GK}\left(C_1 L - \dfrac{qrL^2}{2GK}\right)$

$$\Rightarrow C_1 = \frac{qrL}{2GK} \cdot \frac{2 + \frac{kL}{GK}}{1 + \frac{kL}{GK}}$$

$$M_v(x) = GK\frac{d\varphi}{dx} = C_1 GK - qrx = \frac{qrL}{2} \cdot \frac{2 + \frac{kL}{GK}}{1 + \frac{kL}{GK}} - qrx \quad (3)$$

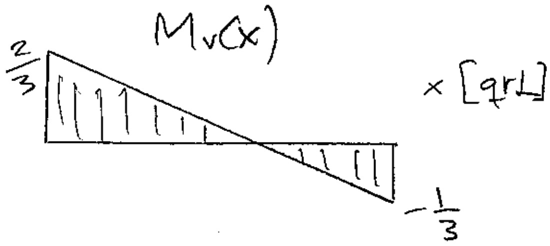
a) Bestäm $M_v(x)$ om $k = 4G\pi r^3 t / L$

$$\frac{kL}{GK} = \frac{4G\pi r^3 t}{G \cdot 2\pi r^3 t} = 2.$$

(3) ger då:

$$M_v(x) = \frac{2qrL}{3} - qrx = \underline{qrL\left(\frac{2}{3} - \frac{x}{L}\right)}$$

$M_v(0) = \frac{2qrL}{3}$, $M_v(L) = \frac{-qrL}{3}$



b) Bestäm k så att $|\tau|_{max}$ minimeras.

$$\tau = \frac{M_v}{2\pi r^2 t} \qquad (6-4)$$

τ blir störst där $|M_v|$ är störst. $M_v(x)$ varierar lineärt $\Rightarrow$ $|M_v|_{max} = M_v(0)$ eller $M_v(L)$

Vi har också att $M_v(0) - M_v(L) = qrL$

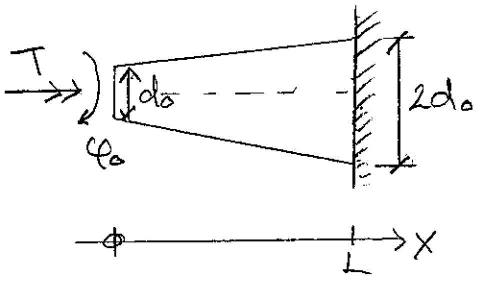
Minst $|M_v|_{max}$ fås då $M_v(0) = -M_v(L)$.

$$(3): \underbrace{\frac{qrL}{2} \cdot \frac{2+\frac{kL}{GK}}{1+\frac{kL}{GK}}}_{M_v(0)} = qrL - \frac{qrL}{2} \cdot \frac{2+\frac{kL}{GK}}{1+\frac{kL}{GK}}$$

$$\Rightarrow \underbrace{\frac{2+\frac{kL}{GK}}{1+\frac{kL}{GK}}}_{=1}$$

$$\therefore \frac{\frac{2}{kL} + \frac{1}{GK}}{\frac{1}{kL} + \frac{1}{GK}} = 1 \Rightarrow \underline{kL \rightarrow \infty}$$

Exempel (gammal tentauppgift)



$d(x) = d_0\left(1+\frac{x}{L}\right)$, Massivt tvärsnitt.

$$K(x) = \frac{\pi}{2}\left(b(x)^4 - a^4\right) = \underbrace{\frac{\pi d_0^4}{32}}_{K_0}\left(1+\frac{x}{L}\right)^4 = K_0\left(1+\frac{x}{L}\right)^4$$

φ_0 given. Bestäm T.

$$\frac{d\varphi}{dx} = \frac{C_1}{GK} = \frac{C_1}{GK_0(1+\frac{x}{L})^4} \qquad (1)$$

$$\varphi(x) = \frac{-C_1}{3GK_0(1+\frac{x}{L})^3} + C_2 = \left\{ C_3 = \frac{-C_1 L}{3GK_0} \right\} = \frac{C_3}{(1+\frac{x}{L})^3} + C_2$$

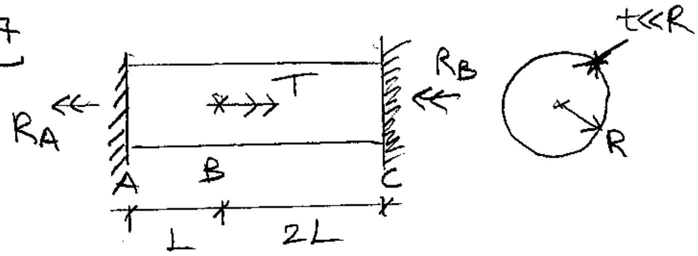
$$\left.\begin{aligned} \varphi(0) = \varphi_0 &\Rightarrow C_3 + C_2 = \varphi_0 \\ \varphi(L) = 0 &\Rightarrow \frac{C_3}{8} + C_2 = 0 \end{aligned}\right\} \Rightarrow C_2 = \frac{-\varphi_0}{7}, \quad C_3 = \frac{8\varphi_0}{7}$$

$$\Rightarrow C_1 = \frac{-3GK_0C_3}{L} = \frac{-24}{7}\frac{GK_0}{L}\varphi_0$$

$$M_v(x) = GK\frac{d\varphi}{dx} = C_1 = \frac{-24}{7}\frac{GK_0}{L}\cdot\varphi_0$$

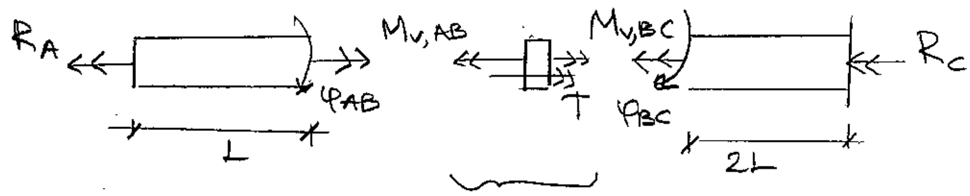
$\underline{\underline{x=0}}$ $\quad T \twoheadrightarrow [\twoheadrightarrow M_v(0)$, $T = -M_v = \frac{24}{7}\frac{GK_0}{L}\cdot\varphi_0 = \underline{\underline{\frac{3\pi}{28}\frac{Gd_0^4}{L}\varphi_0}}$

4.17



$t = 5\,mm$
$R = 100\,mm$
$L = 1\,m$
$G = 77\,GPa$, $\tau_s = 100\,MPa$

a) Bestäm $T = T_s$ som ger begynnande plasticering, samt vridningsvinkeln φ.



$$\rightarrow: \; T + M_{v,BC} - M_{v,BA} = 0$$

$$M_{v,BA} = T + M_{v,BC} \quad (1)$$

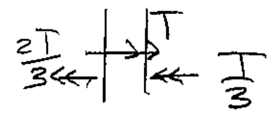
(6-6) $\varphi_{AB} = \dfrac{M_{v,AB} \cdot L}{G \cdot 2\pi R^3 t}$ (2), $\varphi_{BC} = \dfrac{-M_{v,BC} \cdot 2L}{G \cdot 2\pi R^3 t}$ (3)

Kompatibilitet: $\varphi_{AB} = \varphi_{BC} \Rightarrow M_{v,AB} + 2M_{v,BC} = 0$ (4)

(1) in i (4) ger

$$T + 3M_{v,BC} = 0, \quad M_{v,BC} = \frac{-T}{3}$$ insatt i (1) ger $M_{v,AB} = \dfrac{2T}{3}$

$M_{v,AB} > |M_{v,BC}| \Rightarrow$ delen AB plasticerar först



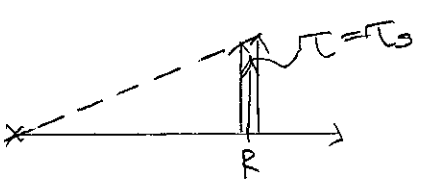
(6-4): $\tau = \dfrac{M_v}{2\pi R^2 t} \Rightarrow \tau_s = \dfrac{\frac{2T_s}{3}}{2\pi R^2 t}$

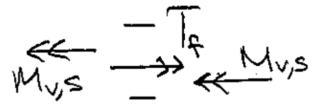
$$T_s = 3\pi R^2 t \tau_s = \underline{47\ kNm}$$

(2) ger då $\varphi = \varphi_{AB} = \dfrac{\frac{2T_s}{3} \cdot L}{G \cdot 2\pi R^3 t} = \{ T_s = 3\pi R^2 t \cdot \tau_s \}$

$$= \frac{\tau_s L}{GR} \approx 12{,}99 \cdot 10^{-3}\ \text{rad} \approx \underline{0{,}74^\circ}$$

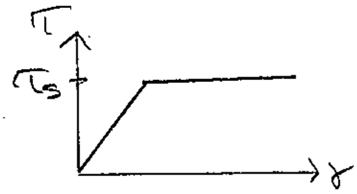
b) Bestäm $T = T_f$ som ger kollaps och motsvarande φ.





$\twoheadrightarrow$: $T_f = 2M_{v,s} = 4\pi R^2 t \tau_s$
$\approx \underline{\underline{68\ \text{kNm}}}$

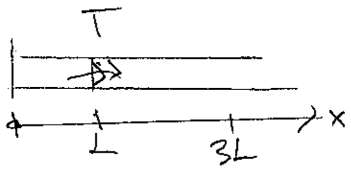
(6-4) : $\tau_s = \dfrac{M_{v,s}}{2\pi R^2 t}$, $M_{v,s} = 2\pi R^2 t \tau_s$



(3) ger då $\varphi = \dfrac{M_{v,s} \cdot 2L}{G \cdot 2\pi R^3 t} = \dfrac{2\tau_s L}{GR} \approx 25{,}97 \cdot 10^{-3}\ \text{rad}$
$\approx \underline{\underline{1{,}49^\circ}}$

"om en konstruktion är statiskt obestämd kollapsar den inte om en liten del går sönder"

c) $\beta = \dfrac{T_f - T_s}{T_s} = \dfrac{4\pi R^2 t \tau_s}{3\pi R^2 t \tau_s} - 1 = \dfrac{1}{3} \approx \underline{\underline{33\%}}$



/end fre Lv 2

$$\begin{cases} -\dfrac{d}{dx}\Big[\underbrace{GK\dfrac{d\varphi}{dx}}_{M_v}\Big] = T\delta(x-L) \\ \varphi(0) = 0 \\ \varphi(3L) = 0 \end{cases}$$

$$\left.\begin{aligned}
\varepsilon_x &= \frac{1}{E}\left(\sigma_x - \nu(\sigma_y+\sigma_z)\right)\\
\varepsilon_y &= \frac{1}{E}\left(\sigma_y - \nu(\sigma_x+\sigma_z)\right)\\
\varepsilon_z &= \frac{1}{E}\left(\sigma_z - \nu(\sigma_x+\sigma_y)\right)\\
\gamma_{ij} &= \frac{\tau_{ij}}{G}
\end{aligned}\right\}$$

L.B. ek 10-4-9
F.s. sid 14

$\det(\underset{\sim}{S} - \sigma \underset{\sim}{I}) = 0$ ger huvudspänningarna σ_1, σ_2 & σ_3
om en huvudspänning känd $(\sigma_z = \sigma_3)$

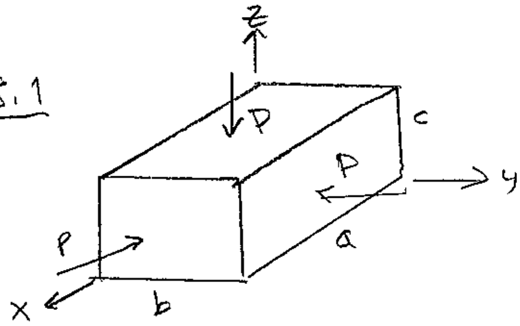
$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (9-49)$$

$$\sigma_e^T = \max\left(|\sigma_1-\sigma_2|, |\sigma_2-\sigma_3|, |\sigma_1-\sigma_3|\right)$$

$$\sigma_e^M = \sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2-\sigma_x\sigma_y-\sigma_x\sigma_z-\sigma_y\sigma_z+3(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2)}$$

$$= \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$

8.1



Bestäm $\frac{\Delta V}{V_0}$, där $V_0 = abc$.
E, ν givna

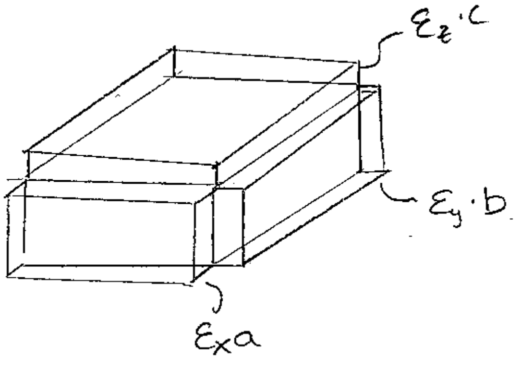
$$V = (a + a\varepsilon_x)(b + b\varepsilon_y)(c + c\varepsilon_z) = abc(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) =$$
$$= abc(1 + \varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_x\varepsilon_y + \varepsilon_x\varepsilon_z + \varepsilon_y\varepsilon_z + \varepsilon_x\varepsilon_y\varepsilon_z) =$$
$$= \{\text{antag } \varepsilon_i \ll 1\} \approx \underbrace{abc}_{V_0}(1 + \varepsilon_x + \varepsilon_y + \varepsilon_z)$$

$$\frac{\Delta V}{V_0} = \frac{V - V_0}{V_0} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_v \quad \text{(volymetrisk töjning)}$$

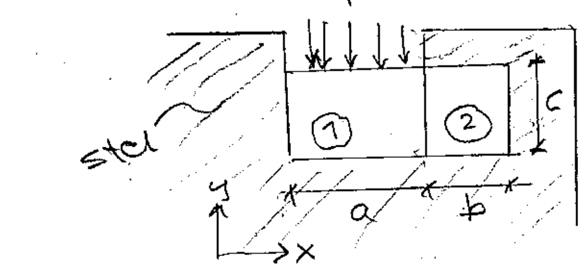
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \{\sigma_x = \sigma_y = \sigma_z = -p\} =$$
$$= \frac{-p}{E}(1 - 2\nu) \qquad \varepsilon_x = \varepsilon_y = \varepsilon_z$$

$$\frac{\Delta V}{V_0} = \frac{-3p}{E}(1 - 2\nu)$$

$\nu = \frac{1}{2} \Rightarrow$ inkompressibelt



8.4



① : $E_1 = 10$ GPa, $\nu_1 = 0{,}4$

② : $E_2 = 30$ GPa, $\nu_2 = 0{,}4$

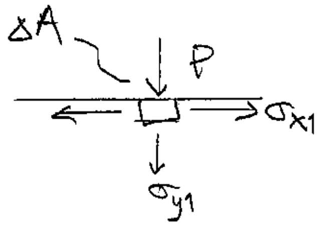
$(\nu = 0{,}4)$

$a = 20$ mm, $b = 10$ mm, $c = 20$ mm, $p = 100$ MPa

Ingen friktion $\Rightarrow \tau_{ij} = 0$.

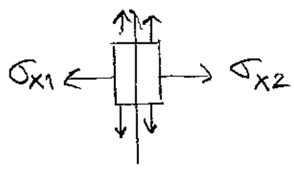
Fri utvidgn. i z-led $\Rightarrow \sigma_z = 0$

a) $\sigma_{x1}, \sigma_{y1}, \sigma_{x2}, \sigma_{y2}$?



$\downarrow: p \cdot \Delta A + \sigma_{y1} \cdot \Delta A = 0$

$\sigma_{y1} = -p = \underline{-100 \text{ MPa}}$



$\rightarrow: \sigma_{x1} = \sigma_{x2} = \sigma_x$

$$\varepsilon_{y2} = \frac{1}{E_2}\Big(\sigma_{y2} - \nu(\underbrace{\sigma_{x2}}_{=\sigma_x} + \underbrace{\sigma_{z2}}_{=0})\Big), \quad \varepsilon_{y2} \cdot c = 0 \Rightarrow \sigma_{y2} = \nu\sigma_x$$

$$\varepsilon_{x1} = \frac{1}{E_1}\Big(\underset{\sigma_x}{\sigma_{x1}} - \nu(\underset{=-p}{\sigma_{y1}} + \underset{=0}{\sigma_{z1}})\Big) = \frac{1}{E_1}(\sigma_x + \nu p)$$

$$\varepsilon_{x2} = \frac{1}{E_2}\Big(\underset{\sigma_x}{\sigma_{x2}} - \nu(\underset{\nu\sigma_x}{\sigma_{y2}} + \underset{=0}{\sigma_{z2}})\Big) = \frac{\sigma_x}{E_2}(1 - \nu^2)$$

$$a\varepsilon_{x1} + b\varepsilon_{x2} = 0 \Rightarrow \sigma_x = \frac{-\nu p a}{b(1-\nu^2) + \frac{E_2}{E_1}a} \cdot \frac{E_2}{E_1} =$$

$$= \frac{-20}{57}p \approx \underline{-35\ \text{MPa}} = \sigma_{x1} = \sigma_{x2}$$

$$\sigma_{yz} = \nu\sigma_x = \frac{-8}{57}p \approx \underline{-14\ \text{MPa}}$$

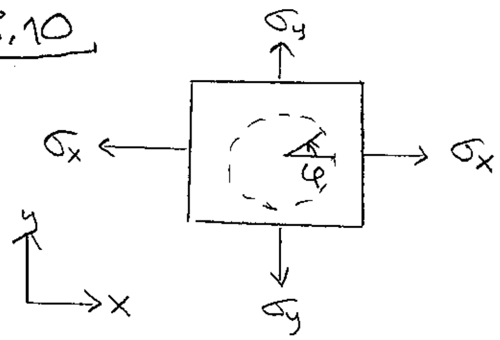
b) $$\delta_y = c\varepsilon_y = \frac{c}{E_1}\left(\sigma_x - \nu(\sigma_{y1} + \cancel{\sigma_{z1}})\right) = \frac{cP}{E_1}\left(-\frac{20}{57} + \nu\right) =$$

$$= \frac{-49pc}{57E_1} \approx \underline{-0{,}17\ \text{mm}}$$

$$\delta_x = a\varepsilon_{x1}\ (= -b\varepsilon_{x2}) = \frac{a}{E_1}\left(\sigma_x - \nu(\sigma_{y1} + \sigma_{z1})\right) =$$

$$= \frac{14pa}{285E_1} \approx \underline{0{,}01\ \text{mm}}$$

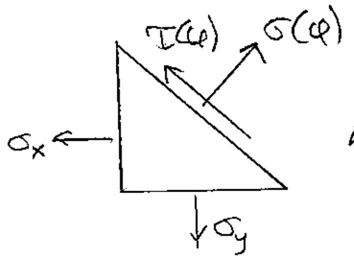
<u>8.10</u>

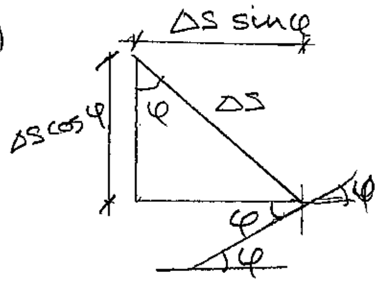


$\sigma_x = \sigma_y = \sigma$

Övriga spänningar noll

Bestäm spänningarna längs en cirkulär snittyta.





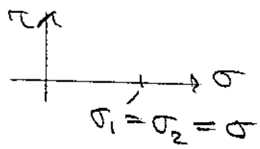
t = tjocklek i z-led

$\nearrow$: $\sigma(\varphi) \cdot t \cdot \Delta s - \sigma_x t \Delta s \cos\varphi \cos\varphi - \sigma_y t \Delta s \sin\varphi \sin\varphi = 0$

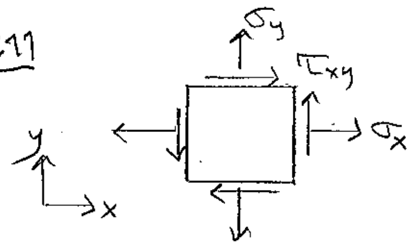
$\sigma(\varphi) = \sigma_x \cos^2\varphi + \sigma_y \sin^2\varphi = \sigma(\cos^2\varphi + \sin^2\varphi) = \underline{\underline{\sigma}}$

$\nwarrow$: $\tau(\varphi) \cdot t \cdot \Delta s + \sigma_x t \Delta s \cos\varphi \sin\varphi - \sigma_y t \Delta s \sin\varphi \cos\varphi = 0$

$\tau(\varphi) = (\sigma_y - \sigma_x) \cos\varphi \sin\varphi = \underline{\underline{0}}$



<u>8.11</u>



$\sigma_x = 120$ MPa, $\sigma_y = 70$ MPa
$\tau_{xy} = 60$ MPa

Plant spänningstillstånd: $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

$\tau_{xz} = \tau_{yz} = 0 \Rightarrow \sigma_z = 0$ är en huvudspänning

(9-4a): $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} =$

$= (95 \pm 65)\text{ MPa} = \begin{cases} 160 \text{ MPa} = \sigma_1 \\ 30 \text{ MPa} = \sigma_2 \end{cases}$

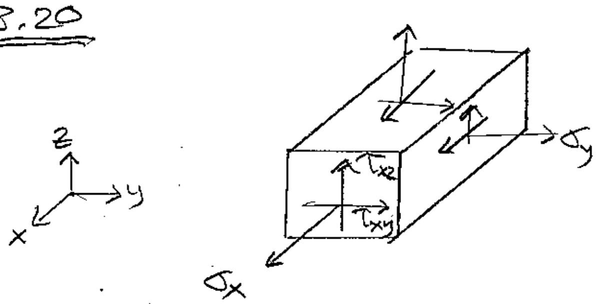
$\sigma_3 = 0$

$$\sigma_e^T = \max\left(|\sigma_1-\sigma_3|, |\sigma_2-\sigma_3|, |\sigma_1-\sigma_2|\right) = \underline{160 \text{ MPa}}$$

$$\sigma_e^M = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \approx \underline{147 \text{ MPa}}$$

$(\sigma_e^T \geq \sigma_e^M$ alltid$)$

8.20



$\sigma_x = -90$ MPa

$\sigma_y = 150$ MPa

$\sigma_z = 120$ MPa

$\tau_{xy} = 90$ MPa, $\tau_{xz} = \tau_{yz} = 0$

$\tau_{xy} = 90$ MPa, $\tau_{xz} = \tau_{yz} = 0$

$$\det(\underset{\sim}{S} - \sigma \underset{\sim}{I}) = 0 \Rightarrow \begin{vmatrix} \sigma_x - \sigma & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y - \sigma & 0 \\ 0 & 0 & \sigma_z - \sigma \end{vmatrix} =$$

$$= (\sigma_z - \sigma)\underbrace{\left[(\sigma_x-\sigma)(\sigma_y-\sigma) - \tau_{xy}^2\right]}_{=0} = 0$$

$\sigma = \sigma_z = 120$ MPa är en huvudsp.

$$\sigma^2 - (\sigma_x + \sigma_y)\sigma + \sigma_x\sigma_y - \tau_{xy}^2 = 0$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 - \sigma_x\sigma_y + \tau_{xy}^2} =$$

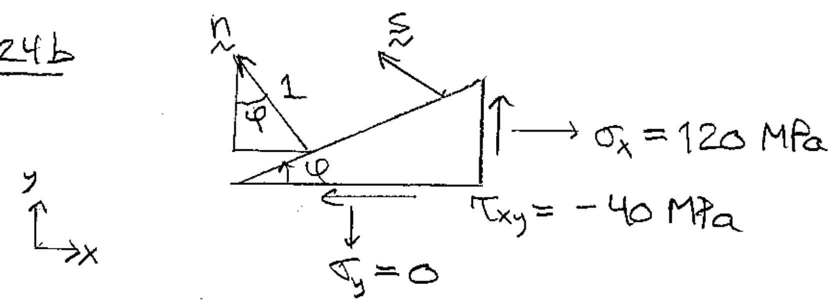
$$= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = (30 \pm 150)\ \mathrm{MPa}$$

$\sigma_1 = 180$ MPa, $\sigma_2 = 120$ MPa, $\sigma_3 = -120$ MPa

$$\sigma_e^T = \max_{ij}(|\sigma_i - \sigma_j|) = \underline{300\ \mathrm{MPa}}$$

$$\sigma_e^M = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2} = \underline{275\ \mathrm{MPa}}$$

<u>8.24b</u>



$\sigma_z = -100$ MPa, $\tau_{yz} = \tau_{xz} = 0$, $\varphi = 30°$

$$\underset{\sim}{n} = \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix}$$

$$\underset{\sim}{s} = \underset{\sim}{S}\,\underset{\sim}{n} = \begin{bmatrix} n_x\sigma_x + n_y\tau_{xy} + n_z\tau_{xz} \\ n_x\tau_{xy} + n_y\sigma_y + n_z\tau_{yz} \\ n_x\tau_{xz} + n_y\tau_{yz} + n_z\sigma_z \end{bmatrix} =$$

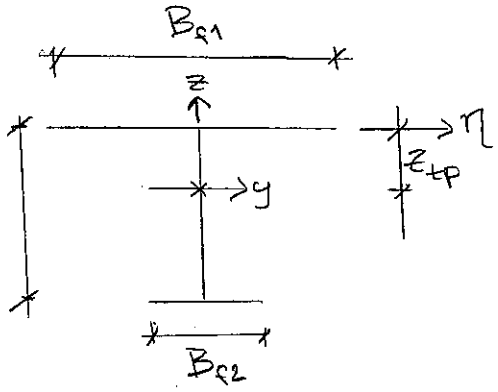
$$= \begin{bmatrix} -\sigma_x \sin\varphi + \tau_{xy}\cos\varphi \\ -\tau_{xy}\sin\varphi \\ 0 \end{bmatrix} = \begin{bmatrix} -94{,}64 \\ 20 \\ 0 \end{bmatrix} \text{ MPa}$$

$$\sigma(\varphi) = \underset{\sim}{n}^T \underset{\sim}{s} = \sigma_x \sin^2\varphi - \tau_{xy}\cos\varphi\sin\varphi - \tau_{xy}\sin\varphi\cos\varphi =$$
$$= \underline{\underline{64{,}6 \text{ MPa}}}$$

$$\tau(\varphi) = \sqrt{|\underset{\sim}{s}|^2 - \sigma(\varphi)^2} = \sqrt{(-94{,}64)^2 + 20^2 - 64{,}6^2} \text{ MPa} \approx$$
$$\approx \underline{\underline{72{,}0 \text{ MPa}}}$$

lörd tis Lv 3

5.4



2011-04-08
Fredag Lv3

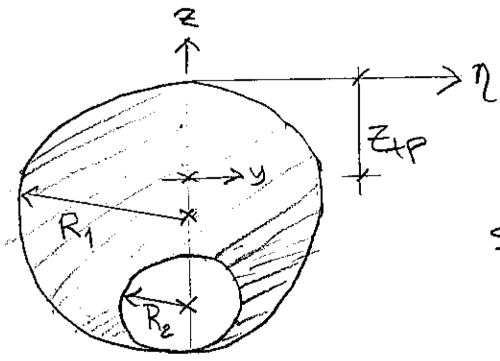
$t \ll H, B_{f1}, B_{f2}$
(godstjocklek)

$$S_\eta = z_{tp} \cdot A = \sum_i A \cdot z_{tpi}$$

$$z_{tp} = \frac{t \cdot B_{f2} H + tH\frac{H}{2} + t \cdot B_{f1} \cdot 0}{t(H + B_{f1} + B_{f2})} = \underline{\underline{\frac{H(H+2B_{f2})}{2(H+B_{f1}+B_{f2})}}}$$

$$B_{f1} = B_{f2} = B \Rightarrow z_{tp} = \frac{H(H+2B)}{2(H+2B)} = \underline{\frac{H}{2}}$$

5.6



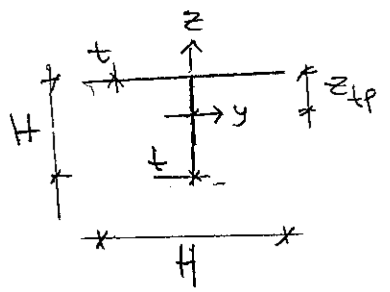
$$A = \pi\left(R_1^2 - R_2^2\right)$$

$$S_\eta = z_{tp} A =$$

$$= \pi R_1^2 \cdot R_1 - \pi R_2^2 \left(2R_1 - R_2\right)$$

$$z_{tp} = \frac{R_1^3 + R_2^3 - 2R_1R_2^2}{R_1^2 - R_2^2} = \underline{\frac{\cancel{(R_1 - R_2)}\left(R_1^2 - R_2^2 + R_1R_2\right)}{\cancel{(R_1-R_2)}\left(R_1 + R_2\right)}}$$

5.9



$t \ll H$.
Bestäm I_y o I_z.

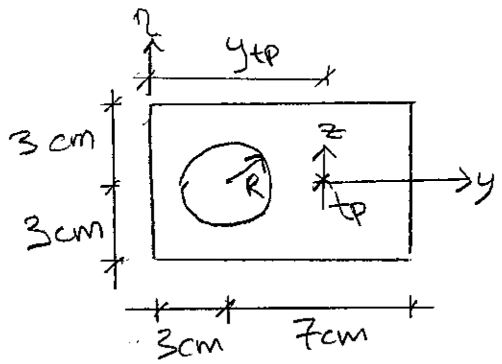
$$z_{tp} A = H \cdot t \frac{H}{2} + H \cdot t \cdot 0, \quad A = 2Ht, \quad \underline{z_{tp} = \frac{H}{4}}$$

$$I_y = \int_A z^2 dA = \underbrace{\frac{Ht^3}{12} + Ht \cdot z_{tp}^2}_{\text{flänsen}} + \underbrace{\frac{tH^3}{12} + Ht \cdot \left(\frac{H}{2} - z_{tp}\right)^2}_{\text{livet}} =$$

$$= \underbrace{\frac{t^3H}{12}}_{\approx 0} + \frac{tH^3}{16} + \frac{tH^3}{12} + \frac{tH^3}{16} = \underline{\underline{\frac{5tH^3}{24}}}$$

$$I_z = \int_A y^2 dA = \underline{\underline{\frac{tH^3}{12}}} + \underbrace{\cancel{\frac{Ht^3}{12}}}_{\approx 0}$$

5.17



R = 2cm.

Bestäm I_y o I_z

$$A = 10\,cm \cdot 6\,cm - \pi R^2 \approx 47{,}434\ cm^2$$

$$S_\eta = A \cdot y_{tp} = 6 \cdot 10\,cm^2 \cdot 5\,cm - \pi R^2 \cdot 3\,cm = 262{,}30\ cm^3$$

$$y_{tp} \approx 5{,}53\ cm$$

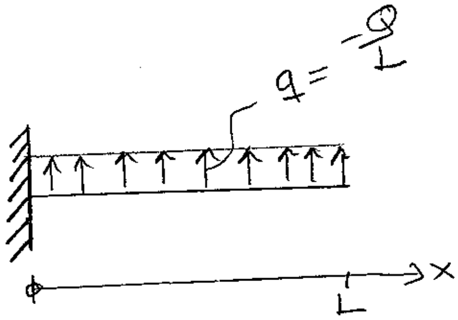
$$I_y = \frac{10\,cm \cdot (6\,cm)^3}{12} - \frac{\pi (2R)^4}{64} \approx \underline{\underline{167{,}43\ cm^4}}$$

F.s sid 6

$$I_z = \frac{6\,cm \cdot (10\,cm)^3}{12} + 6 \cdot 10\,cm^2 \left(y_{tp} - 5\,cm\right)^2 -$$

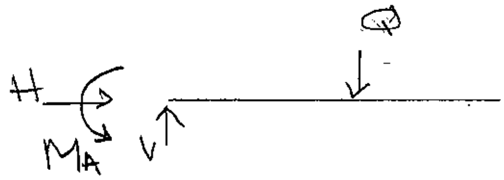
$$- \left(\frac{\pi (2R)^4}{64} + \pi R^2 (y_{tp} - 3\,cm)^2 \right) \approx \underline{\underline{423{,}85\ cm^4}}$$

6.3



Q = kraftresultanten

ab) Frilägg :



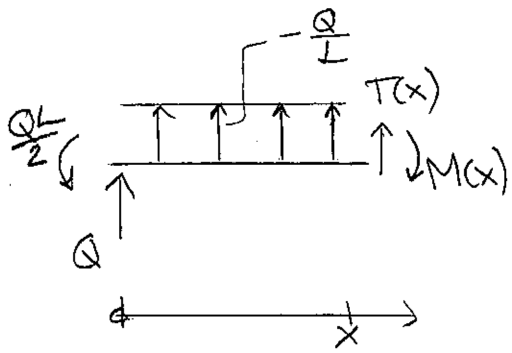
$\rightarrow$: $\underline{H = 0}$

$\uparrow$: $\underline{V = Q}$

$\curvearrowleft A$: $M_A - Q \cdot \frac{L}{2} = 0$

$\underline{M_A = Q\frac{L}{2}}$

Snitta vid x :



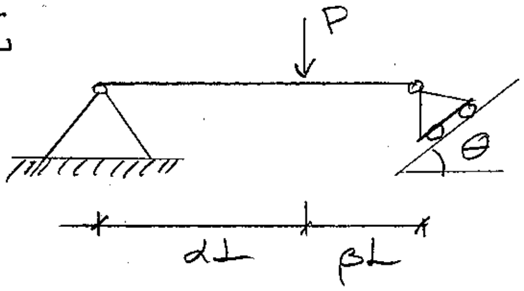
$$\uparrow : T(x) + Q - \frac{Q}{L} \cdot x = 0, \quad T(x) = \underline{\underline{Q\left(\frac{x}{L} - 1\right)}}$$

$$\curvearrowright x : M(x) - \frac{QL}{2} + Qx - \frac{Q}{L} \cdot x \cdot \frac{x}{2} = 0$$

$$\underline{\underline{M(x) = \frac{QL}{2}\left(1 - 2\frac{x}{L} + \left(\frac{x}{L}\right)^2\right)}}$$

c) $T(x) = \frac{dM}{dx} = \frac{QL}{2}\left(0 - \frac{2}{L} + \frac{2x}{L^2}\right) = \underline{\underline{Q\left(\frac{x}{L} - 1\right)}}$

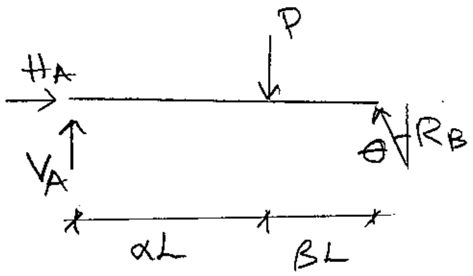
6.5



$\alpha + \beta = 1$

$M(x)$ e $T(x)$ söks

Frilägg:



$\curvearrowleft A$: $R_B \cos\theta \cdot L - P \cdot \alpha L = 0$

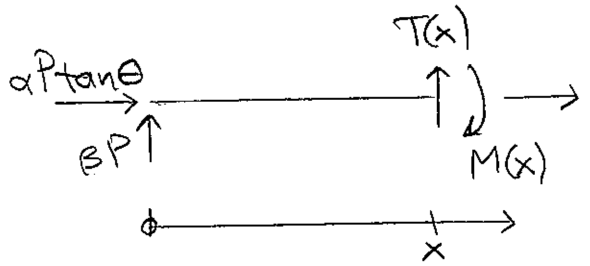
$$R_B = \frac{\alpha P}{\cos\theta}$$

$\rightarrow$: $H_A - R_B \sin\theta = 0$

$$H_A = \alpha P \tan\theta$$

$\curvearrowright B$: $V_A L - P \cdot \beta L = 0, \quad V_A = \beta P$

$0 < x < \alpha L$:



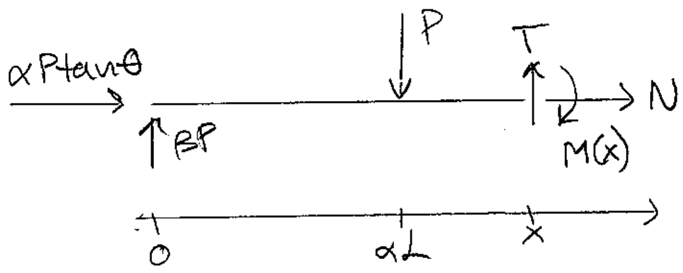
$\rightarrow N = -H_A$

$\uparrow$: $T + \beta P = 0, \quad T = -\beta P$

$\curvearrowright x$: $M(x) + \beta P \cdot x = 0$

$$M(x) = -\beta P x$$

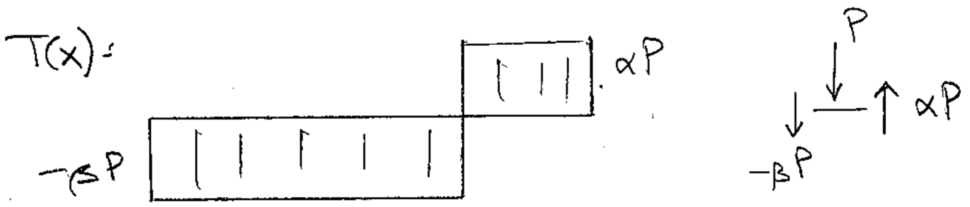
$\alpha L < x < L$

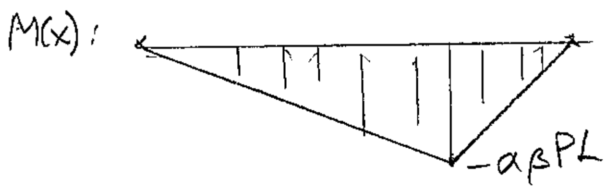


$\uparrow$: $T - P + \beta P = 0, \quad T = P(1-\beta) = \underline{\alpha P}$

$\curvearrowright x$: $M(x) + \beta P x - P(x - \alpha L) = 0$

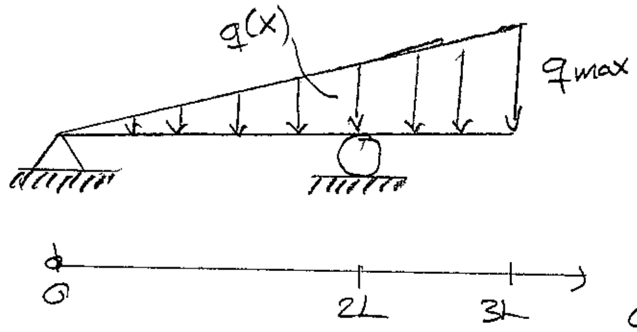
$M(x) = Px(1-\beta) - \alpha P L = \underline{\alpha P (x - L)}$





$M(\alpha L) = \alpha P \underbrace{(\alpha - 1)}_{-\beta} L$

6.8

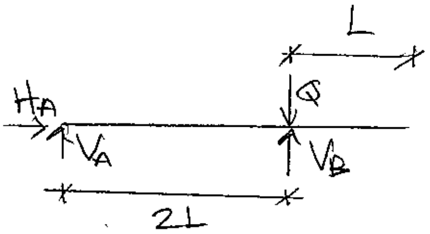


$$q(x) = q_{max}\frac{x}{3L} = \frac{2Qx}{9L^2}$$

$$Q = \int_0^{3L} q\,dx = \frac{q_{max}\cdot 3L}{2}$$

$$q_{max} = \frac{2Q}{3L}$$

Frilägg:



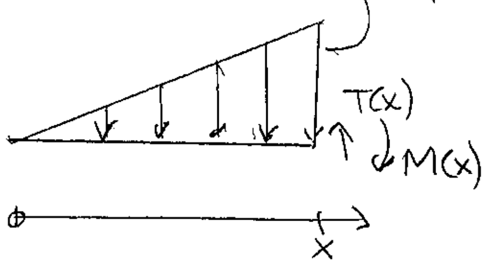
$\rightarrow$: $H_A = 0$

$\curvearrowleft A$: $V_B \cdot 2L - Q(3L - L) = 0$

$\underline{V_B = Q}$

$\uparrow$: $V_A + V_B - Q = 0$, $\underline{V_A = 0}$

$0 < x < 2L$



$\uparrow$: $T(x) = \frac{2Qx}{9L^2}\cdot x\cdot\frac{1}{2} = 0$, $T(x) = \frac{Qx^2}{9L^2}$ (behövs inte)

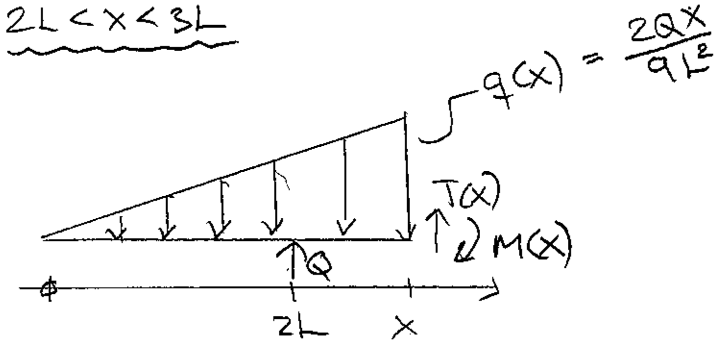
$\curvearrowright x$: $M(x) - \frac{2Qx}{9L^2}\cdot x\cdot\frac{1}{2}\cdot\frac{x}{3} = 0$

$M(x) = \frac{Qx^3}{27L^2}$, $\frac{dM}{dx} = \frac{Qx^2}{9L^2} = 0 \Rightarrow x = 0$

$M(0) = 0$

$M(2L) = \frac{8QL}{27}$

$2L < x < 3L$

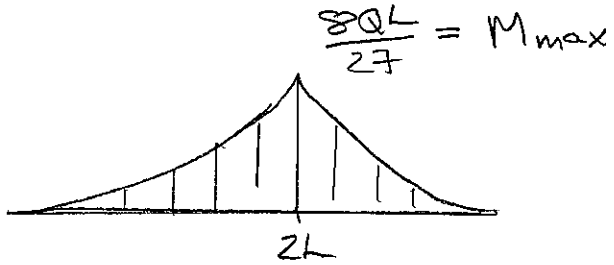


$$\curvearrowright : M(x) = \frac{Qx^3}{27L^2} - Q(x-2L)$$

$$M(2L) = \frac{8QL}{27}, \quad M(3L) = QL - Q(3L-2L) = 0$$

$$\frac{dM}{dx} = \frac{Qx^2}{9L^2} - Q = 0 \Rightarrow x = 3L$$

$M(x)$:

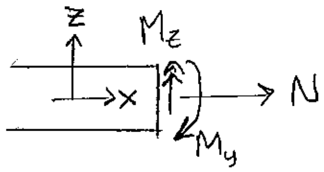


2011-04-12
Tisdag Lv 4

Naviers formel

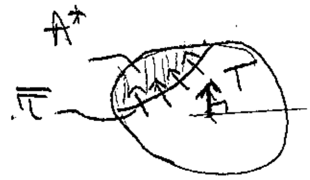
$$\sigma = \frac{N}{A} + \frac{M_y z}{I_y} - \frac{M_z \cdot y}{I_z} \quad (7\text{-}91)$$

$$\left[D_{yz} = \int_A yz\,dA = 0 \right]$$

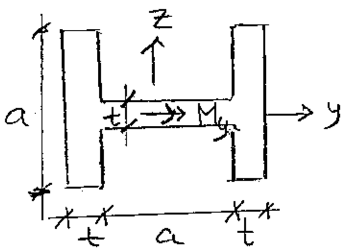


$$\tau = \frac{T S_{A^*}}{I b} \quad (7\text{-}48)$$

Jourawskis formel



5.18



$a = 50$ mm, $t = 10$ mm
$M_y = 1$ kNm

Bestäm σ_{max} & σ_{min}

7-91 med $N=0$ och $M_z = 0$ ger

$$\sigma = \frac{M_y \cdot z}{I_y}, \quad z_{max} = \frac{a}{2}, \quad z_{min} = \frac{-a}{2}$$

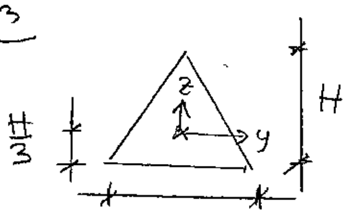
$$I_y = \left[\frac{ta^3}{12} + ta\cdot 0^2\right]\cdot 2 + \frac{at^3}{12} = \left\{ t = \frac{a}{5} \right\} = \frac{17a^4}{500}$$

$$\sigma = \frac{M_y \cdot z \cdot 500}{17a^4}$$

$$\sigma_{max} = \sigma\left(z = \frac{a}{2}\right) = \frac{250\,M_y}{17a^3} = \underline{118 \text{ MPa}}$$

$$\sigma_{min} = -\sigma_{max} = \underline{-118 \text{ MPa}}$$

5.23



$M_y \neq 0,\ N = 0,\ M_z = 0$

σ_{max} & σ_{min} söks

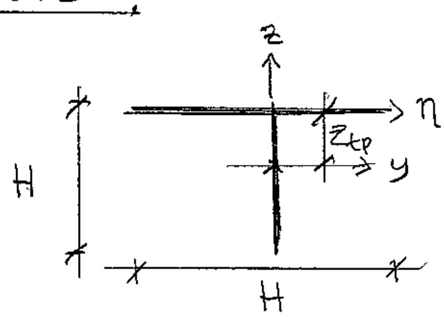
7-91 ger $\sigma = \frac{M_y z}{I_y}$. F.s. sid 5: $I_y = \text{"}\frac{h^3 b}{36}\text{"} = \frac{H^4}{36}$

$\sigma = \frac{36 M_y z}{H^4}$

$\sigma_{max} = \sigma\left(z = \frac{2H}{3}\right) = \underline{\underline{\frac{24 M_y}{H^3}}}$

$\sigma_{min} = \sigma\left(z = \frac{-H}{3}\right) = \underline{\underline{\frac{-12 M_y}{H^3}}}$

5.20



godstjocklek $t \ll H$

$N = \frac{2M}{H},\ M_y = M,\ M_z = \frac{M}{2}$

σ_{max} & σ_{min}?

$S_\eta = A \cdot z_{tp} = H \cdot t \cdot \frac{H}{2} + H \cdot t \cdot 0,\ A = 2Ht \Rightarrow z_{tp} = \frac{H}{4}$

$I_y = \frac{tH^3}{12} + Ht \cdot \left(\frac{H}{2} - z_{tp}\right)^2 + \frac{Ht^3}{12} + Ht \cdot z_{tp}^2 =$

$= \frac{5tH^3}{24} + \underbrace{\cancel{\frac{Ht^3}{12}}}_{\approx 0}$ $t \ll H$

$$I_z = \frac{Ht^3}{12} + \frac{tH^3}{12} \approx \frac{tH^3}{12}$$

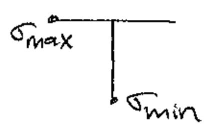
$$(7\text{-}91): \quad \sigma = \frac{N}{A} + \frac{M_y z}{I_y} - \frac{M_y y}{I_z} = \frac{2M}{H \cdot 2Ht} + \frac{M \cdot z \cdot 24}{5tH^3} -$$

$$- \frac{M \cdot y \cdot 12}{2 \cdot tH^3} = \frac{M}{H^2 t}\left(1 + \frac{24z}{5H} - \frac{6y}{H}\right)$$

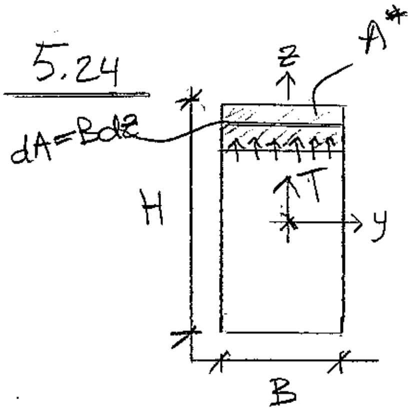
$$\sigma\left(y = \frac{-H}{2}, z = \frac{H}{4}\right) = \frac{M}{H^2 t}\left(1 + \frac{6}{5} + 3\right) = \underline{\underline{\frac{26M}{5H^2 t}}} = \sigma_{max}$$

$$\sigma\left(y = \frac{H}{2}, z = \frac{H}{4}\right) = \frac{M}{H^2 t}\left(1 + \frac{6}{5} - 3\right) = \frac{-4M}{5H^2 t}$$

$$\sigma\left(y = 0, z = \frac{-3H}{4}\right) = \frac{M}{H^2 t}\left(1 - \frac{16}{5} - 0\right) = \underline{\underline{\frac{-13M}{5H^2 t}}} = \sigma_{min}$$



<u>5.24</u>



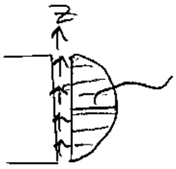
Bestäm $\tau(z)$

$$I = I_y = \frac{BH^3}{12}, \quad b = B$$

$$S_{A^*} = \int_{A^*} z\,dA = B\int_{z}^{H/2} z\,dz = \frac{B}{2}\left[z^2\right]_z^{H/2} = \frac{B}{2}\left(\left(\frac{H}{2}\right)^2 - z^2\right)$$

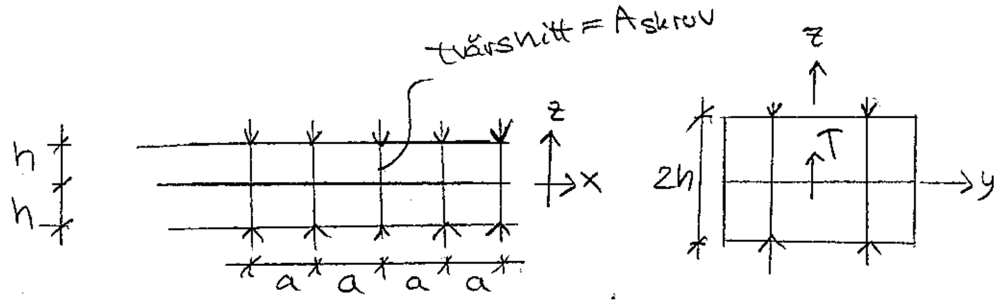
$$= \frac{BH^2}{8}\left(1-4\left(\frac{z}{H}\right)^2\right)$$

$$(7\text{-}48): \quad \tau = \frac{T\cdot BH^2\cdot 12}{8\cdot BH^3\cdot B}\left(1-4\left(\frac{z}{H}\right)^2\right) = \underline{\frac{3T}{2BH}\left(1-4\left(\frac{z}{H}\right)^2\right)}$$

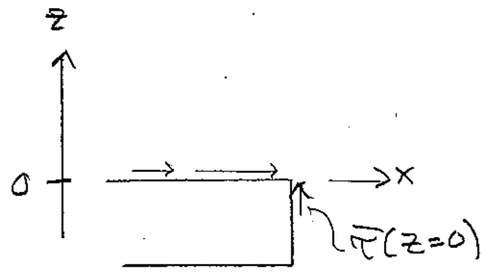


$$\tau_{max} = \tau(0) = \frac{3T}{2BH}$$

5.26

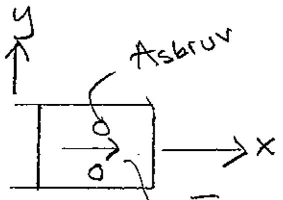


Försumma friktion mellan brädorna och bestäm τ_{skruv}



Från 5.24:

$$\tau(0) = \frac{3T}{2\cdot BH} = \left\{\begin{matrix} B = 3h \\ H = 2h \end{matrix}\right\} = \frac{T}{4h^2}$$

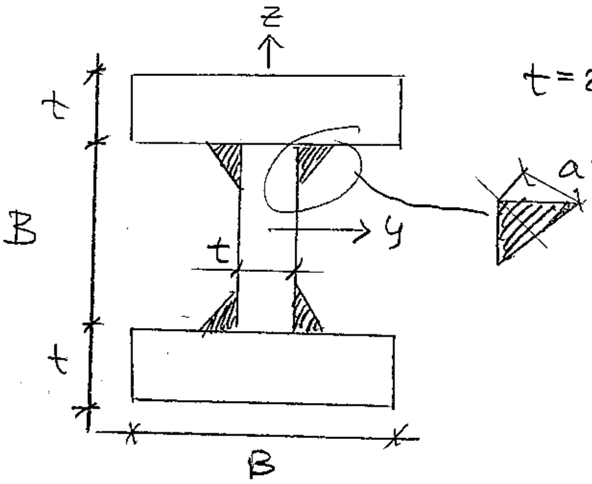


skjuvkraft som ska överföras längs sträckan a i x-led

$$F_{skruv} = \tau(0)\cdot 3h\cdot a = \tau_{skruv}\cdot A_{skruv}\cdot 2$$

$$\Rightarrow \quad \tau_{skruv} = \frac{T\cdot 3h\cdot a}{4h^2\cdot 2A_{skruv}} = \underline{\underline{\frac{3Ta}{8hA_{skruv}}}}$$

5.29



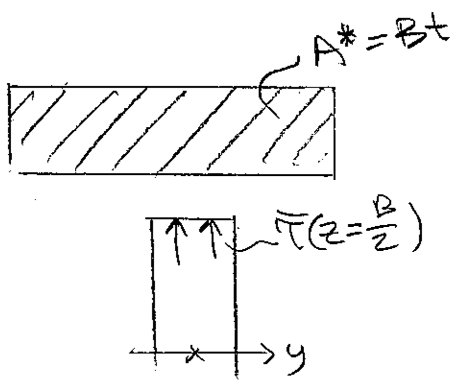
$t = 20\text{mm}, \quad B = 100\text{ mm}$

$\tau_{till} = 10\text{MPa}$

$T_z = 10\text{kN}$

$$I = I_y = \left[\frac{Bt^3}{12} + Bt\cdot\left(\frac{t}{2}+\frac{B}{2}\right)^2\right]\cdot 2 + \frac{tB^3}{12} =$$

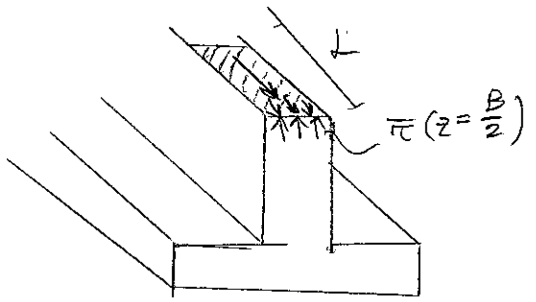
$$= \{B = 5t\} = \frac{405}{4}t^4$$



$$S_{A^*} = A^*\left(\frac{t}{2}+\frac{B}{2}\right) = \{B = 5t\} =$$

$$= 15t^3$$

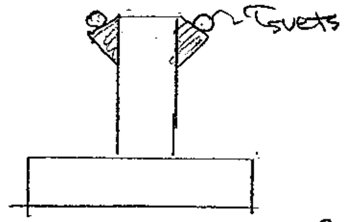
$$\tau\left(z=\frac{B}{2}\right) = \frac{T_z\cdot 15t^3}{\frac{405}{4}\cdot t^4\cdot t} = \frac{4T_z}{27t^2}$$



Skjuvkraft längs en sträcka L i x-led:

$$F_{skjuv} = \tau\left(z=\frac{B}{2}\right)\cdot Lt$$

Denna ska överföras av svetssträngarna.



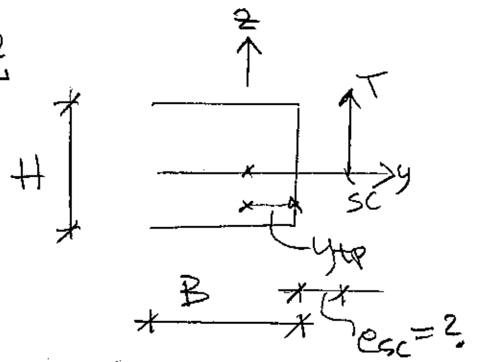
$$F_{skjuv} = \tau_{svets}\cdot 2a\cdot L = \underbrace{\frac{4T_z}{27t^2}}_{\tau(\frac{B}{2})}\cdot Lt$$

$$a = \frac{4T_z Lt}{27t^2\cdot 2L\cdot \tau_{svets}} = \frac{2T_z}{27t\,\tau_{svets}} \geq$$

$$\geq \frac{2T_z}{27t\,\tau_{till}} = \underline{\underline{3{,}7\text{ mm}}}$$

<u>Skjuvcentrum</u>: den punkt genom vilken T måste verka för att få böjning utan rotation.

<u>5.32</u>

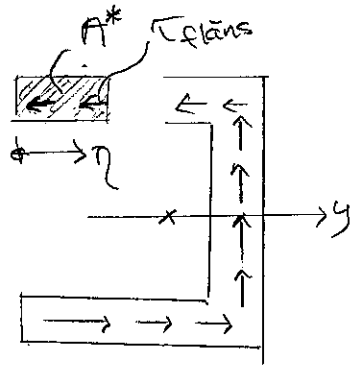


$$A = 2Bt + Ht$$

$$y_{tp}\cdot A = \left(B\cdot t\cdot\frac{B}{2}\right)2 + H\cdot t\cdot 0$$

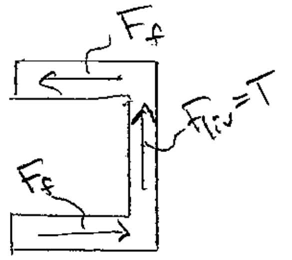
$$y_{tp} = \frac{B^2}{2B+H} \quad \text{(behövs ej)}$$

$$I = I_y = \left[Bt\cdot\left(\frac{H}{2}\right)^2 + \underbrace{\frac{Bt^3}{12}}_{\approx 0}\right]\cdot 2 + \frac{tH^3}{12} = \frac{tH^3}{12} + \frac{tBH^2}{2}$$

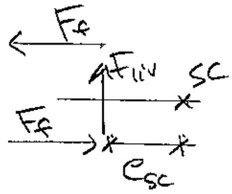


$$S_{A*} = \eta \cdot t \cdot \frac{H}{2}$$

$$\tau_{fläns} = \frac{T S_{A*}}{I_y t} = \frac{TH}{2 I_y} \eta$$



$$F_f = \int_0^B \tau_{fläns} \cdot t \, d\eta = \frac{THtB^2}{4 I_y}$$



$$\overset{\curvearrowright}{sc}: \; F_{liv} \cdot e_{sc} - F_f \cdot \frac{H}{2} \cdot 2 = 0$$

$$e_{sc} = \frac{F_f \cdot H}{F_{liv}} = \frac{H^2 t B^2}{4 I_y} = \underline{\underline{\frac{3B^2}{H + 6B}}}$$

/end tis Lv 4

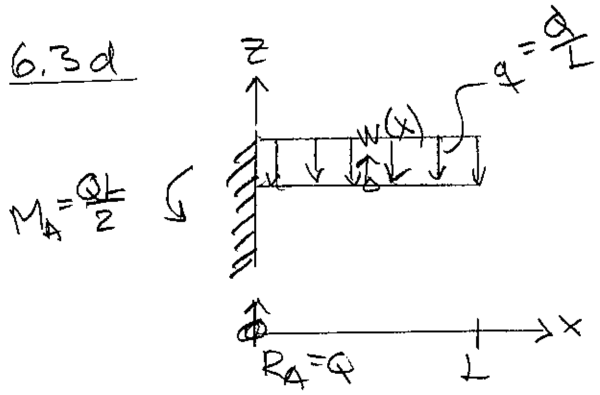
$$-\frac{d^2M}{dx^2} = q(x)$$

2011-04-15
Fredag Lv4

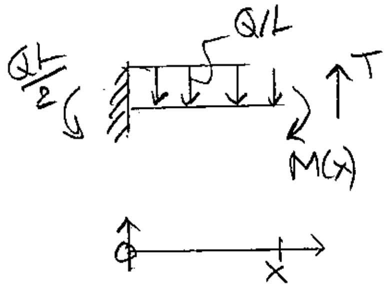
$$\left.\begin{aligned} M(x) &= EI\,K(x) \\ K(x) &= -\frac{d^2w}{dx^2} \end{aligned}\right\} \quad M(x) = -EI\frac{d^2w}{dx^2}$$

$$\frac{d^2}{dx^2}\left[EI\frac{d^2w}{dx^2}\right] = q(x)$$

6.3d



$w(x) = ?$



$$\curvearrowright x: \quad M(x) - \frac{Q}{L} x \frac{x}{L} + Qx - \frac{QL}{2} = 0$$

$$M(x) = \frac{QL}{2}\left(1 - 2\frac{x}{L} + \left(\frac{x}{L}\right)^2\right)$$

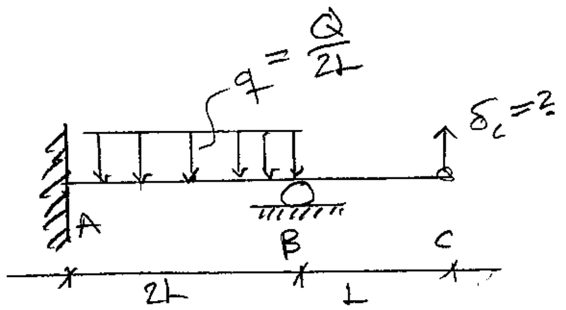
$$\frac{d^2w}{dx^2} = -\frac{M}{EI}, \quad \frac{dw}{dx} = \frac{-QL^2}{2EI}\left(\frac{x}{L} - \left(\frac{x}{L}\right)^2 + \frac{1}{3}\left(\frac{x}{L}\right)^3\right) + C_1$$

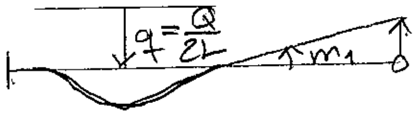
$$\left.\frac{dw}{dx}\right|_{x=0} = 0 \Rightarrow C_1 = 0$$

$$w(x) = \frac{-QL^3}{2EI}\left(\frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{1}{3}\left(\frac{x}{L}\right)^3 + \frac{1}{12}\left(\frac{x}{L}\right)^4\right) + C_2, \quad w(0) = 0 \Rightarrow C_2 = 0$$

$$Q = qL \Rightarrow w(x) = \frac{-qL^4}{24EI}\left(\left(\frac{x}{L}\right)^4 - 4\left(\frac{x}{L}\right)^3 + 6\left(\frac{x}{L}\right)^2\right)$$

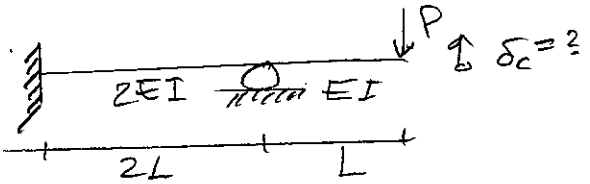
6.10



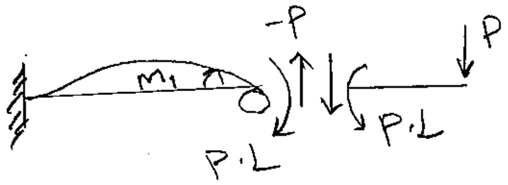


$$\delta_c = m_1 L = \frac{qL^3}{24EI}$$

F.s. sid 11 : $m_1 = \dfrac{\frac{Q}{2L}(2L)^3}{48 \cdot 2EI} = \dfrac{QL^2}{24EI}$



$$m_1 = \frac{PL \cdot 2L}{4 \cdot 2EI} = \frac{PL^2}{4EI}$$

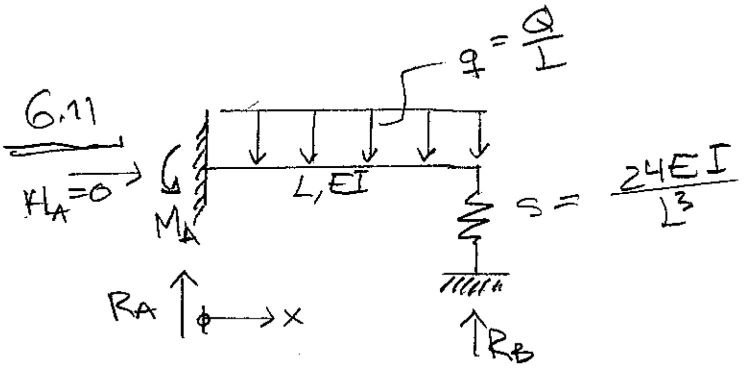


$$m_1 L = \frac{PL^3}{4EI}$$

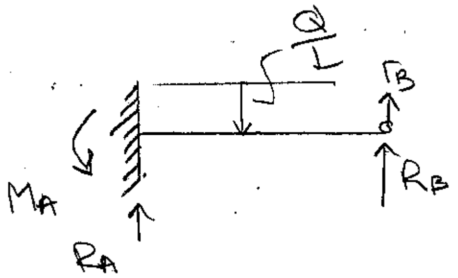
F.s. sid 10 : $p_1 = \dfrac{PL^3}{3EI}$

$$\delta_c = -(p_1 + m_1 L) = \frac{-7PL^3}{12EI}$$

6.11



$M(x) = ?$



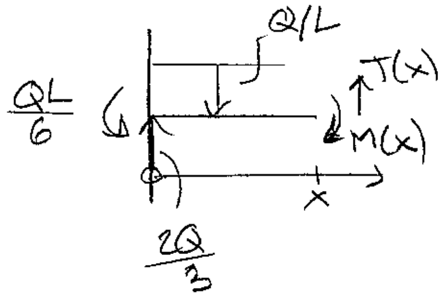
F.S. sid 10: $r_B = \frac{-\frac{Q}{L}L^4}{8EI} + R_B\frac{L^3}{3EI} = \frac{L^3}{EI}\left(\frac{R_B}{3} - \frac{Q}{8}\right)$

$r_B \cdot S = -R_B$

$$\Rightarrow \quad -R_B = \frac{L^3}{EI}\left(\frac{R_B}{3} - \frac{Q}{8}\right)\cdot\frac{24EI}{L^3}, \quad R_B = \frac{Q}{3}$$

$\uparrow: \; R_A + R_B - Q = 0 \qquad R_A = \frac{2Q}{3}$

$\circlearrowleft A): \; M_A + R_B \cdot L - Q\frac{L}{2} = 0 \qquad M_A = \frac{QL}{6}$



$\circlearrowright x: \; M(x) - \frac{QL}{6} + \frac{2Q}{3}\cdot x - \frac{Q}{L}\cdot x \cdot \frac{x}{2} = 0$

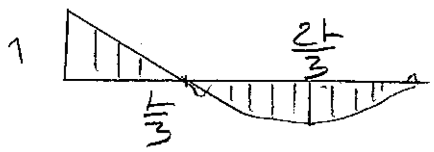
$M(x) = \frac{QL}{6}\left(3\left(\frac{x}{L}\right)^2 - 4\frac{x}{L} + 1\right)$

$M(0) = \frac{QL}{6} \qquad M(L) = 0$

$$T = \frac{dM}{dx} = \frac{Q}{6}\left(6\frac{x}{L} - 4\right) = 0$$

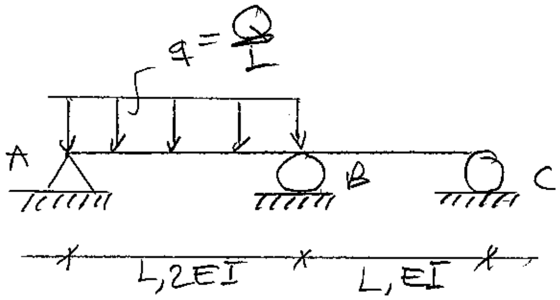
$$\Rightarrow x = \frac{2L}{3}; \quad M\left(\frac{2L}{3}\right) = -\frac{1}{3}\cdot\frac{QL}{6}$$

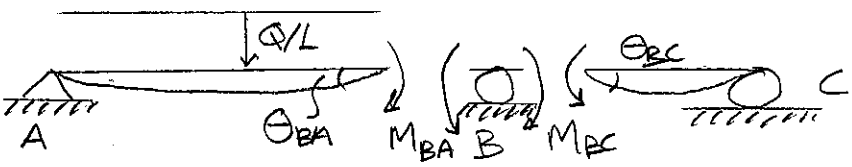
$M(x)$:



$\times \left[\frac{QL}{6}\right]$

<u>6.16</u>



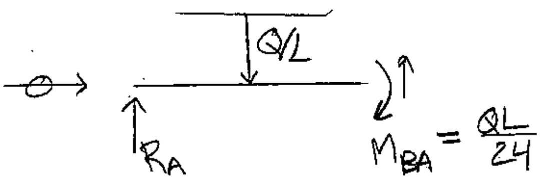


F.S. sid 9: $\theta_{BA} = \dfrac{\frac{Q}{L}L^3}{24\cdot 2EI} - \dfrac{M_{BA}\cdot L}{3\cdot 2EI} = \dfrac{L}{EI}\left(\dfrac{QL}{48} - \dfrac{M_{BA}}{6}\right)$

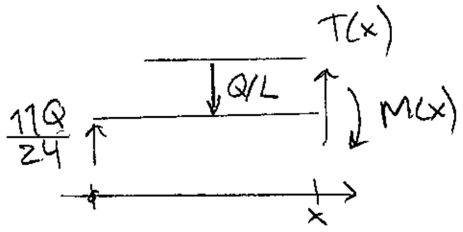
$\theta_{BC} = \dfrac{-M_{BC}L}{3EI}$

Kompatibilitet: $\theta_{BA} + \theta_{BC} = 0$

$\Rightarrow \dfrac{QL}{48} - \dfrac{M_{BA}}{6} - \dfrac{M_{BC}}{3} = 0, \quad M_{BA} = M_{BC} = \dfrac{QL}{24}$



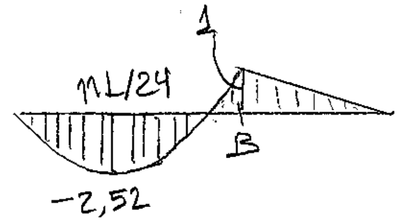
$\overset{\curvearrowright}{B}: \; R_A\cdot L - \dfrac{Q}{L}\cdot L\cdot \dfrac{L}{2} + \dfrac{QL}{24} = 0, \quad R_A = \dfrac{11Q}{24}$



$$\curvearrowright x \; : \quad M(x) + \frac{11Q}{24}\cdot x - \frac{Q}{L}\cdot x \cdot \frac{x}{2} = 0$$

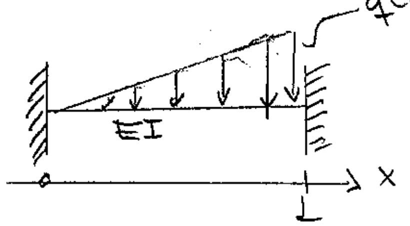
$$M(x) = \frac{QL}{24}\left(12\left(\frac{x}{L}\right)^2 - 11\,\frac{x}{L}\right)$$

$M(x)$:



$\times \left[\frac{QL}{24}\right]$

6.27



$M(x) = ?$

EI konstant $\Rightarrow$ $w^{IV} = \frac{q}{EI}$

$$\begin{cases} w^{IV} = \dfrac{-q_0 x}{EI \cdot L} & 0 < x < L \\ w(0) = 0\,, \quad w'(0) = 0 \\ w(L) = 0\,, \quad w'(L) = 0 \end{cases}$$

$$w(x) = \frac{-q_0 x^5}{120\, EI \cdot L} + C_1 x^3 + C_2 x^2 + C_3 x + C_4$$

$$w'(x) = \frac{-q_0 x^4}{24 EI\, L} + 3C_1 x^2 + 2C_2 x + C_3$$

$$w(0) = 0 \Rightarrow C_4 = 0, \quad w'(0) = 0 \Rightarrow C_3 = 0$$

$$w(L) = 0 \Rightarrow \begin{bmatrix} L^3 & L^2 \\ 3L^2 & 2L \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{q_0 L^4}{120 EI} \\ \frac{q_0 L^3}{24 EI} \end{bmatrix}$$

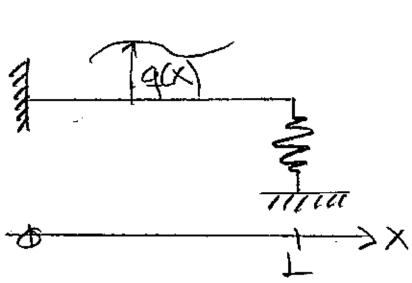
$$\Rightarrow C_1 = \frac{q_0 L}{40 EI} \qquad C_2 = \frac{-q_0 L^2}{60 EI}$$

$$M(x) = -EI \frac{d^2 w}{dx^2} = \frac{q_0 x^3}{6L} - 6C_1 x EI - 2C_2 EI =$$

$$= \underline{\frac{q_0 L^2}{60}\left(10\left(\frac{x}{L}\right)^3 - 9\left(\frac{x}{L}\right) + 2\right)}$$

$$M_A = M(0) = \frac{q_0 L^2}{30} \qquad M_B = M(L) = \frac{q_0 L^2}{20}$$

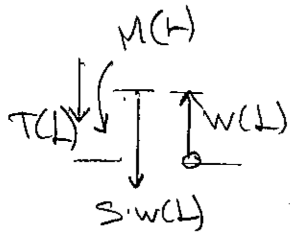
6.31



Formulera R.V.

$w(0) = 0$
$w'(0) = 0$

$x = L$



$\circlearrowleft$: $M(L) = 0, \quad M = -EIw'' \Rightarrow \underline{w''(L) = 0}$

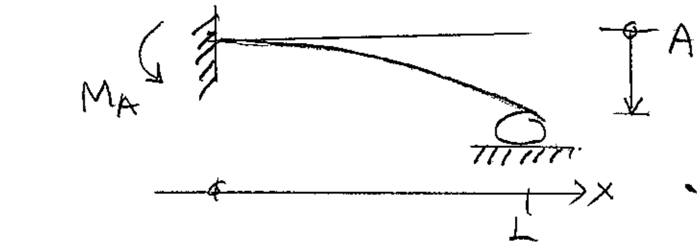
$\downarrow$: $T(L) + Sw(L) = 0$

om EI konst

$$T = \frac{dM}{dx} = -\frac{d}{dx}\left[EIw''\right] = \underline{-EIw'''}$$

$$w'''(L) - \frac{S}{EI}\,w(L) = 0$$

6.35



M_A p.g.a. Δ söks

EI konstant

$\Rightarrow w^{IV} = 0 \quad 0 < x < L$

$\uparrow$

ty $q = 0$

$$w = C_1 x^3 + C_2 x^2 + C_3 x + C_4$$

$$w(0) = 0 \Rightarrow C_4 = 0$$

$$w' = 3C_1 x^2 + 2C_2 x + C_3 \qquad w'(0) = 0 \Rightarrow C_3 = 0$$

: $\quad M(L) = 0 \Rightarrow w''(L) = 0$

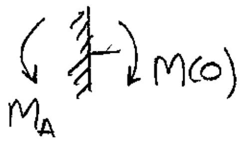
$$w'' = 6C_1 x + 2C_2$$

$$w''(L) = 0 \Rightarrow 6C_1 L + 2C_2 = 0 \qquad C_2 = -3C_1 L$$

$$w(L) = -\Delta \Rightarrow C_1 = \frac{\Delta}{2L^3}, \quad C_2 = \frac{-3\Delta}{2L^2}$$

$$w'' = 6C_1 x + 2C_2 = \frac{3\Delta}{L^2}\left(\frac{x}{L} - 1\right)$$

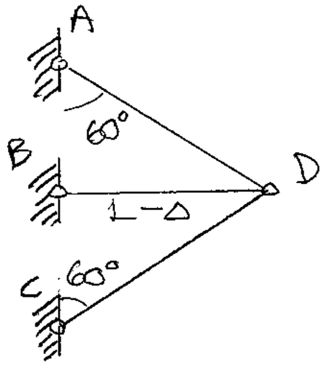
$$M = -EI \cdot w'' = \frac{3\Delta EI}{L^2}\left(1 - \frac{x}{L}\right)$$



$$M_A = M(0) = \underline{\underline{\frac{3\Delta EI}{L^2}}}$$

Lend fre Lv 4

1)

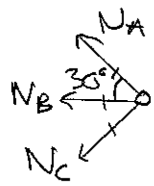


2011-05-03
Tisdag Lv5

Δ passningsfel, $\Delta \ll L$

Beräkna spänningarna i stängerna.

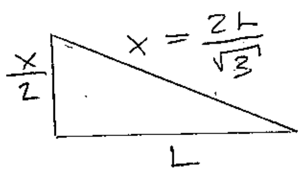
Frilägg:



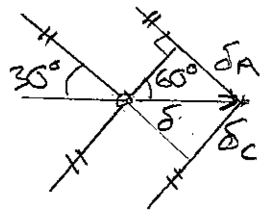
$\uparrow: \; N_A = N_C$

$\leftarrow: \; N_A \cos 30° + N_B + N_C \cos 30° = 0$

$N_B = -\sqrt{3}\, N_A \quad (1)$



Kompatibilitet:



$$\delta_A = \delta \sin 60° = \frac{\sqrt{3}}{2}\delta = \delta_C$$

$$N_A = \frac{EA}{\frac{2L}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{2}\delta = \frac{3EA\delta}{4L} \quad (2)$$

$$\delta_B = \delta + \Delta, \qquad N_B = \frac{EA}{L}(\delta + \Delta) \quad (3)$$

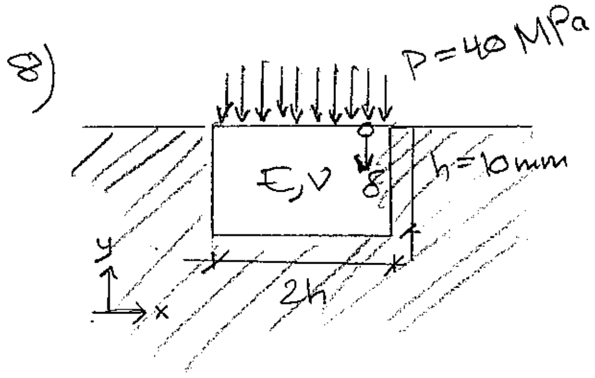
(2) och (3) in i (1) ger $\frac{EA}{L}(\Delta + \delta) = \frac{-3\sqrt{3}\,EA\delta}{4L}$

$$\Rightarrow \delta = \frac{-4\Delta}{3\sqrt{3}+4}$$

(2): $N_A = N_C = \underline{\underline{\dfrac{-3\Delta EA}{(3\sqrt{3}+4)L}}}$

(1): $N_B = \underline{\underline{\dfrac{+3\sqrt{3}\,EA\Delta}{(3\sqrt{3}+4)L}}}$

8)



$E = 375$ MPa, $\nu = 0{,}25$

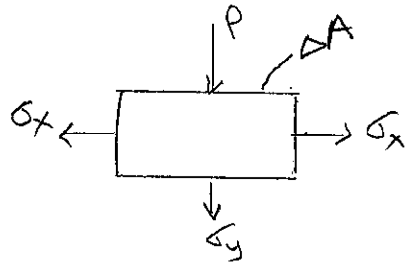
Fri expansion i z-led

$\Rightarrow \sigma_z = 0$

Ingen friktion

$\Rightarrow \tau_{ij} = 0$

a) spänningarna, σ_x och σ_y



$\downarrow: \; p \cdot \Delta A + \sigma_y \cdot \Delta A = 0$

$\underline{\sigma_y} = -p = \underline{-40 \text{ MPa}}$

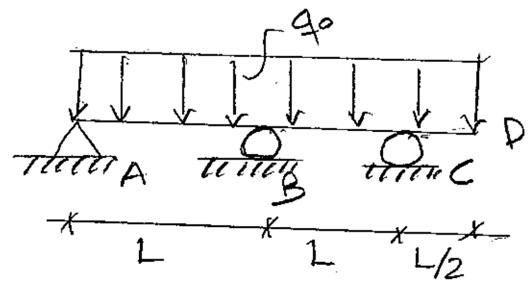
Kompatibilitet: $\varepsilon_x \cdot 2h = 0 \qquad \varepsilon_x = 0$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right) = 0 \Rightarrow \sigma_x = \nu\sigma_y = \underline{\underline{-10 \text{ MPa}}}$$

b) δ?

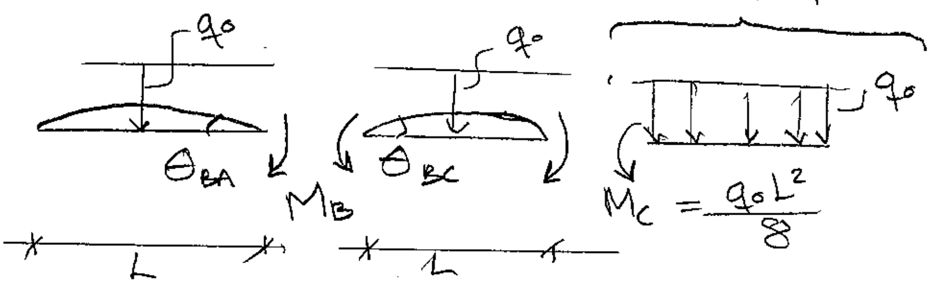
$$\delta = -\varepsilon_y h = -h \cdot \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \underset{0}{\cancel{\sigma_z}})\right) = \underline{\underline{\frac{+ph}{E}(1-\nu^2)}}$$

5)



a) Tvärkraften?

$$\curvearrowleft : M_C - q_0 \cdot \frac{L}{2} \cdot \frac{L}{4} = 0$$



$$M_C = \frac{q_0 L^2}{8}$$

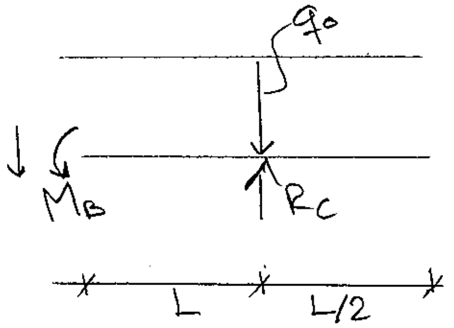
F.s. sid 9: $\theta_{BA} = \frac{M_B \cdot L}{3EI} - \frac{q_0 L^3}{24EI}$

$$= \frac{q_0 L^2}{8}$$

$$\theta_{BC} = \frac{M_B L}{3EI} - \frac{q_0 L^3}{24EI} + \frac{M_C L}{6EI}$$

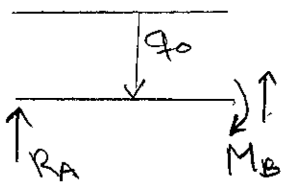
Kompatibilitet: $\theta_{BA} + \theta_{BC} = 0$

$$\Longrightarrow M_B = \frac{3 q_0 L^2}{32}$$



$$\curvearrowleft B : M_B + R_C \cdot L - q_0 \frac{3L}{2} \cdot \frac{3L}{4} = 0$$

$$R_C = \frac{33 q_0 L}{32}$$



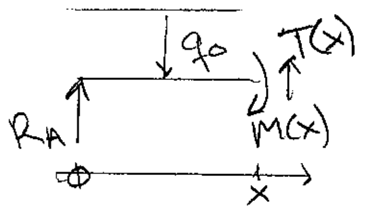
$$\overset{\curvearrowright}{B}: \; M_B + R_A \cdot L - q_0 \cdot L \cdot \frac{L}{2} = 0$$

$$R_A = \frac{13 q_0 L}{32}$$

$\uparrow$: (hela) $\quad R_A + R_B + R_C - q_0 \frac{5L}{2} = 0$

$$R_B = \frac{17 q_0 L}{16}$$

$0 < x < L$



$\uparrow$: $T(x) + R_A - q_0 x = 0$

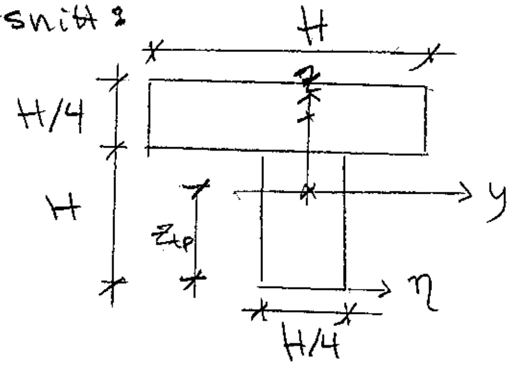
$$\underline{T(x) = q_0 L\left(\frac{x}{L} - \frac{13}{32}\right)}$$

$0 < x < L$

b) $T_{max} = 4\,kN$. Tvärsnitt:

$F_{till} = 1\,kN$ i en skruv

Bestäm avst. c mellan skruvarna i x-led.



$$\tau = \frac{T\, S_{A*}}{I \cdot b}$$

$A = 2 \cdot \frac{H}{4} \cdot H$

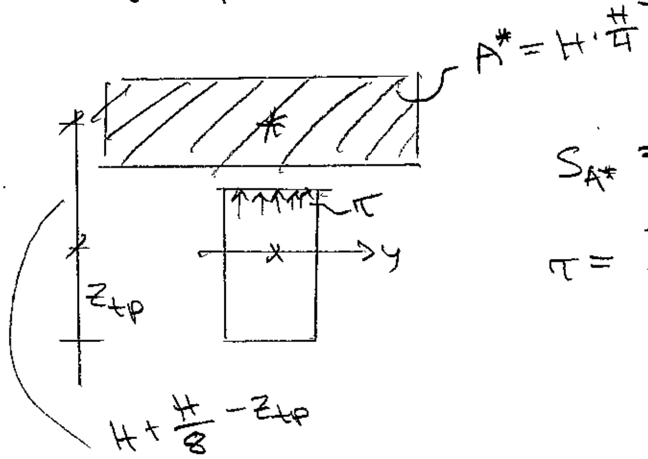
$$\underline{S_\eta = A \cdot z_{tp} = H \cdot \frac{H}{4}\left(H + \frac{H}{8}\right) + H \cdot \frac{H}{4} \cdot \frac{H}{2}}$$

$$\Rightarrow \underline{z_{tp} = \frac{13H}{16}}$$

$$I = I_y = \frac{H\left(\frac{H}{4}\right)^3}{12} + H \cdot \frac{H}{4}\left(H + \frac{H}{8} - z_{tp}\right)^2 + \frac{\frac{H}{4} H^3}{12} +$$

$$+ H \frac{H}{4}\left(z_{tp} - \frac{H}{2}\right)^2 = \frac{109}{1536} H^4$$

Skjuvspänning vid övergången liv | fläns.

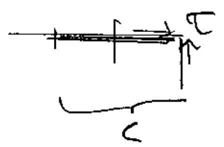


$$S_{A^*} = H\frac{H}{4}\left(H+\frac{H}{8}-z_{tp}\right) = \frac{5}{64}H^3$$

$$\tau = \frac{T_{max}\cdot\frac{5}{64}H^3}{\frac{109}{1539}H^3\cdot\frac{H}{4}} = \frac{480}{109}\cdot\frac{T_{max}}{H^2}$$

Skjuvkraft längs en sträcka c i x-led:

$$F = c\cdot\frac{H}{4}\cdot\tau$$

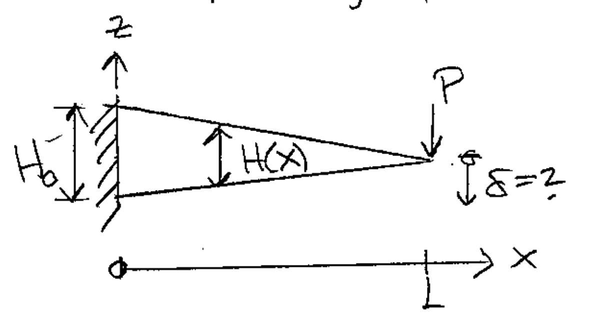


$$c = \frac{4F_{till}}{\tau H} = \frac{109\,F_{till}\cdot H}{120\cdot T_{max}} \approx 45\,mm$$

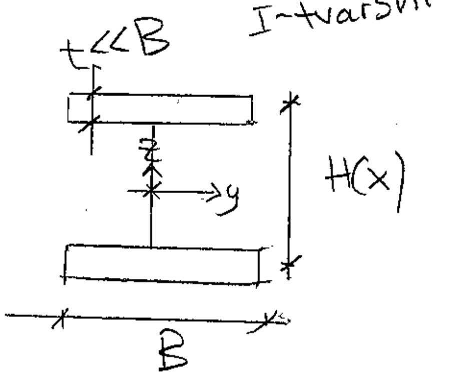
/end tis Lv5

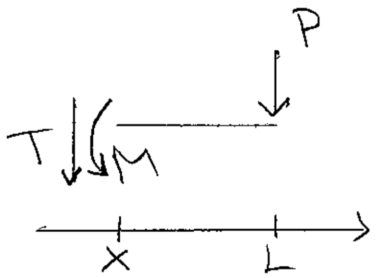
2011-05-06
Fredag Lv5

Rep. uppg. 3



Bestäm balkens nedböjning under lasten P.

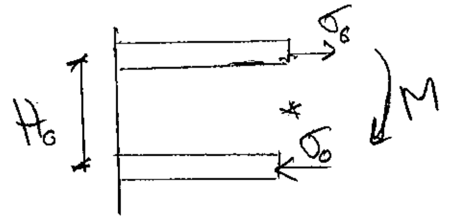




$$\curvearrowleft : M = P(L-x)$$

$$M = \sigma_0 \cdot t \cdot B \,\frac{H(x)}{2} \cdot 2 = \sigma_0 B t H(x)$$

$$M(0) = PL = \sigma_0 B t H_0, \quad \sigma_0 = \frac{PL}{BtH_0}$$



$$H(x) = \frac{M}{Bt\sigma_0} = \frac{H_0(L-x)}{L}$$

$$I = \left[\frac{t^3 B}{12} + Bt\left(\frac{H(x)}{2}\right)^2\right] \cdot 2 = \frac{BtH_0^2}{2L^2}(L-x)^2$$

$$M = EI\kappa = -EIw''$$

$$w'' = \frac{-M}{EI} = \frac{-2PL^2}{EBtH_0^2(L-x)} = \frac{-2PL}{EIBtH_0^2\left(1-\frac{x}{L}\right)}$$

$$w' = C_1 + \frac{2PL^2}{EIBtH_0^2} \ln\left(1-\frac{x}{L}\right)$$

$$w'(0) = 0 \Rightarrow C_1 = 0$$

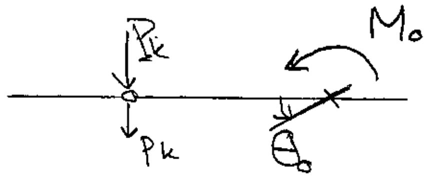
$$w = C_2 - \frac{2PL^2}{EBtH_0^2}\left(x + (L-x)\ln\left(\frac{L-x}{L}\right)\right)$$

$$w(0) = 0 \Rightarrow C_2 = 0$$

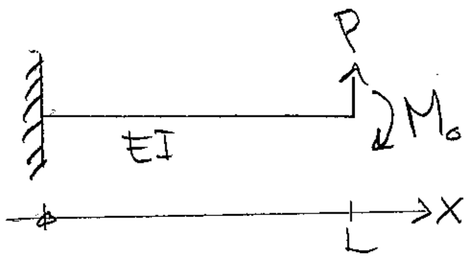
$$\delta = -w(L) = \underline{\underline{\frac{2PL^3}{EBtH_0^2}}}$$

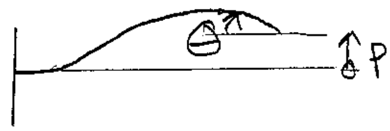
$$W_i = \int_0^L \frac{N^2}{2EA} + \frac{M_y^2}{2EI_y} + \frac{M_z^2}{2EI_z} + \frac{M_v^2}{2GK} + \beta_y \frac{T_y^2}{2GA} + \beta_z \frac{T_z^2}{2GA}\, dx$$

$$\frac{\partial W_i}{\partial P_k} = p_k \quad , \quad \frac{\partial W_i}{\partial M_0} = \theta_0$$



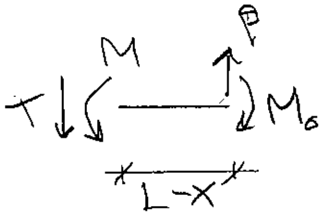
Modellproblemet från f.ö. 10, onsdag 2011-05-04, med EI konstant och $q = 0$





Försumma inverkan av skjuvning och beräkna p & θ.

$$W_i = \int_0^L \frac{M^2}{2EI}\, dx$$



$$\curvearrowleft: \; M + P(L-x) - M_0 = 0$$

$$M(x) = M_0 - P(L-x)$$

$$\frac{\partial M}{\partial P} = -(L-x), \quad \frac{\partial M}{\partial M_0} = 1$$

$$p = \frac{\partial}{\partial P} \int_0^L \frac{M^2}{2EI}\, dx = \int_0^L \frac{\partial}{\partial P}\left[\frac{M^2}{2EI}\right] dx = \int_0^L \frac{\partial M}{\partial P} \underbrace{\frac{\partial}{\partial M}\left[\frac{M^2}{2EI}\right]}_{M/EI} =$$

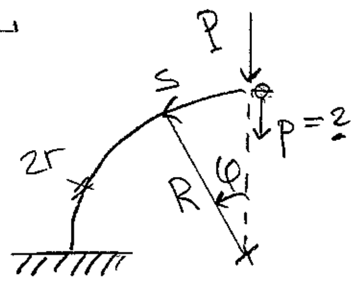
$$= \frac{1}{EI}\int_0^L -(L-x)(M_0 - P(L-x))\, dx = -\frac{M_0 L^2}{2EI} + \frac{PL^3}{3EI}$$

(jmf. F.s. sid 10)

$$\theta = \frac{\partial W_i}{\partial M_0} = \int_0^L \frac{\partial M}{\partial M_0} \overbrace{\frac{\partial}{\partial M}\left[\frac{M^2}{2EI}\right]}^{M/EI} dx = \int_0^L 1 \cdot \frac{M_0 - P(L-x)}{EI}\, dx =$$

$$= \frac{M_0 L}{EI} - \frac{PL^2}{2EI}$$

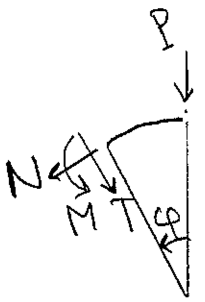
13.1



$r \ll R$

$S = R\varphi$

$$W_i = \int_0^{\pi R/2} \left(\frac{N^2}{2EA}\right) + \left(\beta \frac{T^2}{GA}\right) + \left(\frac{M^2}{2EI}\right) ds$$



$\swarrow$: $N + P\sin\varphi = 0$

$N = -P\sin\varphi \qquad \frac{\partial N}{\partial P} = -\sin\varphi$

$\searrow$: $T + P\cos\varphi = 0, \quad T = -P\cos\varphi, \quad \frac{\partial T}{\partial P} = -\cos\varphi$

$\curvearrowleft$: $M - P \cdot R\sin\varphi = 0, \quad M = PR\sin\varphi, \quad \frac{\partial M}{\partial P} = R\sin\varphi$

$$p = \frac{\partial W_i}{\partial P} = \int_0^{\pi/2} \frac{\partial N}{\partial P} \underbrace{\frac{\partial}{\partial N}\left[\frac{N^2}{2EA}\right]}_{N/EA} + \frac{\partial T}{\partial P} \underbrace{\frac{\partial}{\partial T}\left[\frac{\beta T^2}{2GA}\right]}_{\beta T/GA} + \frac{\partial M}{\partial P} \underbrace{\frac{\partial}{\partial M}\left[\frac{M^2}{2EI}\right]}_{M/EI} \underbrace{R\,d\varphi}_{ds}$$

$$\int_0^{\pi/2} \left(\frac{P\sin^2\varphi}{EA} + \frac{\beta P\cos^2\varphi}{GA} + \frac{PR^2\sin^2\varphi}{EI}\right) R\,d\varphi = \frac{\pi PR}{4EA} + \frac{\beta\pi PR}{4GA} +$$

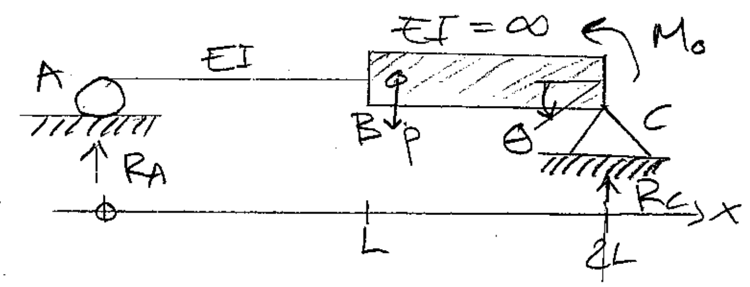
$$+ \frac{\pi PR^3}{4EI} = \left\{\begin{array}{l} A = \pi r^2 \\ I = \frac{\pi}{4} r^4 \text{ (F.s. sid 6)} \\ \beta = 10/9 \end{array}\right\} =$$

$$= \frac{PR}{Er^2}\Big(\underbrace{\frac{1}{4}}_{N} + \underbrace{\overset{\approx 0{,}5}{\frac{5E}{18G}}}_{T} + \underbrace{\left(\frac{R}{r}\right)^2}_{M}\Big) \approx \frac{PR^3}{Er^4}$$

$\gg 1$

$$G = \frac{E}{2(1+\nu)}$$

13.16



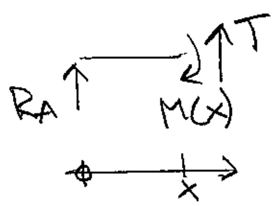
P & θ sökes

$$W_i = \int_0^{2L} \frac{M^2}{2EI}dx = \int_0^{L} \frac{M^2}{2EI}dx + \int_L^{2L} \frac{M^2}{\infty}dx$$

$$\circlearrowright: R_A \cdot 2L - P\cdot L - M_0 = 0, \quad R_A = \frac{P}{2} + \frac{M_0}{2L}$$

$$\uparrow: R_A + R_C - P = 0, \quad R_C = \frac{P}{2} - \frac{M_0}{2L}$$

$0<x<L$



$$\circlearrowright x: \quad M(x) = -R_A \cdot x = \frac{-Px}{2} - \frac{M_0 x}{2L} =$$

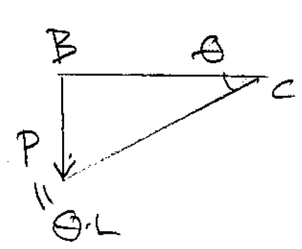
$$= \{P=0\} = \frac{-M_0 x}{2L}$$

$$\frac{\partial M}{\partial P} = -\frac{x}{2}, \quad \frac{\partial M}{\partial M_0} = \frac{-x}{2L}$$

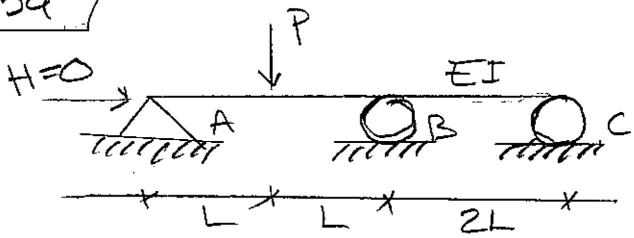
a) $\theta = ?$ $\quad \theta = \frac{\partial W_i}{\partial M_0} = \int_0^L \frac{\partial M}{\partial M_0} \cdot \frac{\partial}{\partial M}\left[\frac{M^2}{2EI}\right]dx =$

$$= \frac{1}{EI}\int_0^L \frac{x}{2L}\,\frac{M_0 x}{2L}\,dx = \underline{\underline{\frac{M_0 L}{12EI}}}$$

b) $p = \frac{\partial W_i}{\partial P} = \int_0^L \frac{\partial M}{\partial P}\frac{M}{EI}dx = \underline{\frac{M_0 L^2}{12EI}}$



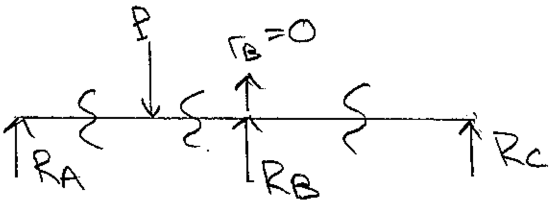
13.15a



Beräkna stödreaktionerna.

Låt R_B vara en yttre last och beräkna

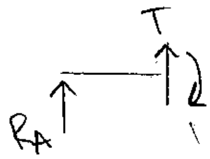
$$r_B = \frac{\partial W_i}{\partial R_B} = 0$$



$$\curvearrowright: \; R_A \cdot 4L - P \cdot 3L + R_B \cdot 2L = 0, \quad R_A = \frac{3P}{4} - \frac{R_B}{2}$$

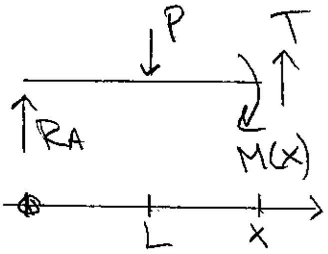
$$\curvearrowleft A: \; R_C = \frac{P}{4} - \frac{R_B}{2}$$

$0 < x < L$



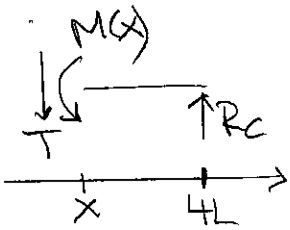
$$M(x) = -R_A x = \frac{R_B x}{2} - \frac{3Px}{4}$$

$L < x < 2L$



$$M(x) = -R_A x + P(x-L)$$
$$= P(x-L) + \frac{R_B x}{2} - \frac{3Px}{4}$$

$2L < x < 4L$



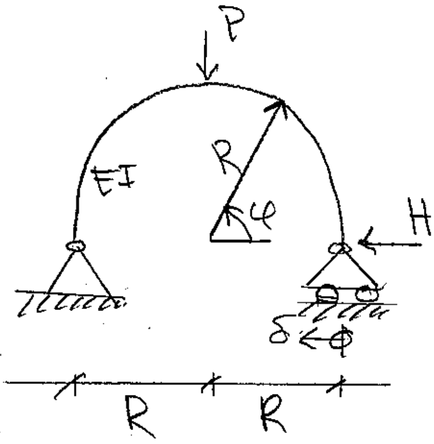
$$M(x) = \frac{R_B}{2}(4L-x) - \frac{P}{4}(4L-x)$$
$$(-R_c(4L-x))$$

$$r_B = \frac{\partial}{\partial R_B}\int_0^{4L} \frac{M^2}{2EI}\,dx = \int_0^{4L} \frac{\partial M}{\partial R_B}\,\frac{M}{EI}\,dx =$$
$$= \int_0^{L} \frac{x}{2}\,\frac{1}{EI}\left(\frac{R_B x}{2} - \frac{3Px}{4}\right)dx + \int_L^{2L} \frac{x}{2}\,\frac{1}{EI}\left(\frac{Px}{4} + \frac{R_B x}{2}\right)dx +$$
$$+ \int_{2L}^{4L} \frac{(4L-x)}{2}\,\frac{1}{EI}\left(\frac{R_B}{2} - \frac{P}{4}\right)(4L-x)\,dx =$$
$$= \frac{L^3}{12EI}(16R_B - 11P) = 0 \Rightarrow R_B = \frac{11P}{16}$$

$$R_c = -\frac{3P}{32} \qquad R_A = \frac{13P}{32}$$

13.2

2011-05-10
Tisdag Lv6

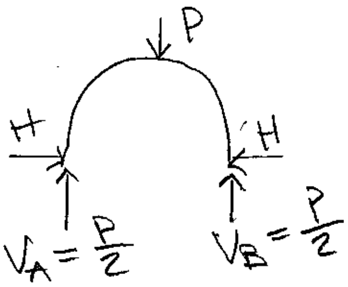


Beräkna δ.

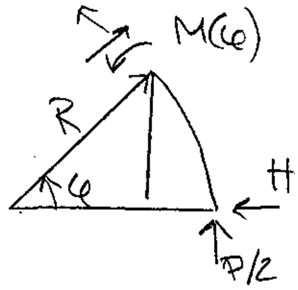
$$\delta = \frac{\partial W_i}{\partial H} \quad \text{(Castigliano)}$$

$$W_i = \int \frac{M^2}{2EI}\,ds = \{ s = R\varphi \} = \int_0^{\pi} \frac{M^2}{2EI} R\,d\varphi =$$

$$= 2\int_0^{\pi/2} \frac{M^2}{2EI} R\,d\varphi$$



$0 < \varphi < \frac{\pi}{2}$:



$$\curvearrowleft : M(\varphi) - HR\sin\varphi + \frac{PR}{2}(1-\cos\varphi) = 0$$

$$M(\varphi) = HR\sin\varphi - \frac{PR}{2}(1-\cos\varphi)$$

$$\frac{\partial M}{\partial H} = R\sin\varphi$$

$$\delta = 2\int_0^{\pi/2} \frac{\partial M}{\partial H}\underbrace{\frac{\partial}{\partial M}\left[\frac{M^2}{2EI}\right]}_{M/EI} R\,d\varphi =$$

$$= \frac{2R^3}{EI}\int_0^{\pi/2} H\sin^2\varphi - \frac{P}{2}\left(\sin\varphi - \cos\varphi\sin\varphi\right) d\varphi =$$

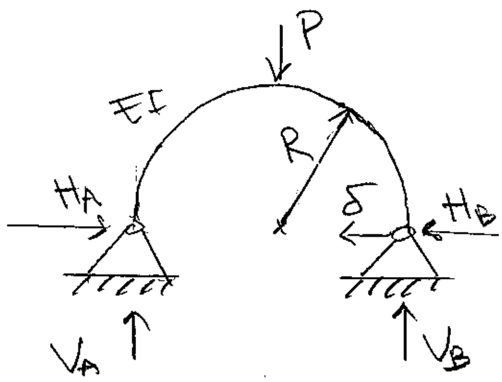
↑ Här kan vi egentligen sätta $H=0$

$$= \frac{2R^3}{EI}\left[H\left(\frac{\varphi}{2} - \frac{1}{4}\sin 2\varphi\right) - \frac{P}{2}\left(-\cos\varphi + \frac{1}{2}\cos^2\varphi\right)\right]_0^{\pi/2} =$$

$$= \frac{R^3}{2EI}(\pi H - P) = \delta$$

$$H = 0 \Rightarrow \underline{\underline{\delta = \frac{-PR^3}{2EI}}}$$

13.3



Beräkna stödreaktionerna.

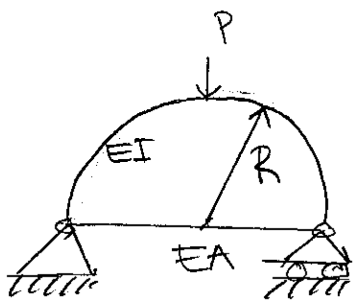
$\uparrow + \curvearrowright$ ger $V_A = V_B = P/2$

$\rightarrow$: $H_A = H_B$

Enligt 13.2 har vi $\delta = \frac{\partial W_i}{\partial H} = \frac{R^3}{2EI}(\pi H_B - P)$

Villkoret $\delta = 0$ ger då $\underline{\underline{H_B = \frac{P}{\pi}}} = H_A$

13.4

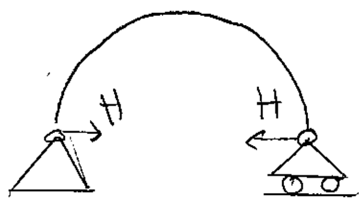


Bestäm stångkraften H.

Snitta ut stången.

$$H \longleftarrow \underset{EA}{\overset{2R}{\text{———}}} \longrightarrow H$$

$$\phi \rightarrow \delta_s = \frac{H \cdot 2R}{EA}$$



$$\leftarrow \phi$$

$$\delta = \frac{R^3}{2EI}(\pi H - P)$$

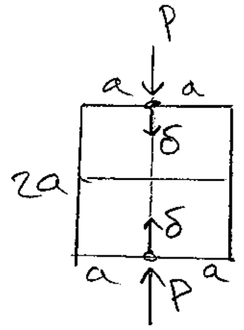
enl. 13.2

Kompatibilitet: $\delta_s + \delta = 0$

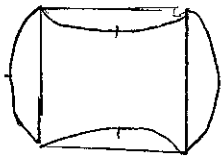
$$H\left(\frac{2R}{EA} + \frac{\pi R^3}{2EI}\right) = \frac{PR^3}{2EI}, \quad \underline{\underline{H = \frac{P}{\pi + \frac{4I}{R^2 A}}}}$$

$$A \to 0 \Rightarrow H = 0, \quad A \to \infty \Rightarrow H = \frac{P}{\pi}$$

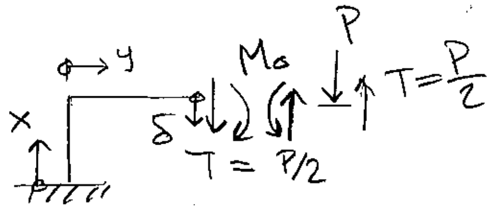
13.16



Alla delar har böjstyvhet EI.
Bestäm hoptryckningen 2δ.



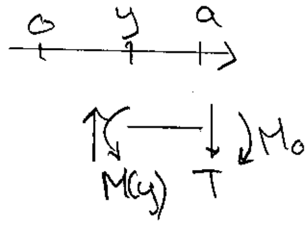
Utnyttja symm.



Villkor $\theta_0 = 0$

$$\theta_0 = \frac{\partial W_i}{\partial M_0}$$

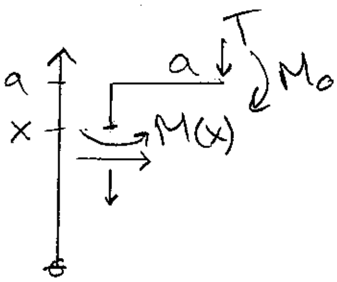
$$\delta = \frac{\partial W_i}{\partial T}$$



$$\curvearrowleft: \; M(y) = M_0 + T(a-y)$$

$$\frac{\partial M}{\partial M_0} = 1$$

$$\frac{\partial M}{\partial T} = a-y$$



$$\curvearrowleft: \; M(x) = M_0 + Ta,$$

$$\frac{\partial M}{\partial M_0} = 1, \quad \frac{\partial M}{\partial T} = a$$

$$\Theta_0 = \int \frac{\partial}{\partial M}\left[\frac{M^2}{2EI}\right]\frac{\partial M}{\partial M_0}\,ds = \frac{1}{EI}\int_0^a (M_0 + Ta)\,dx +$$

$$+\frac{1}{EI}\int_0^a M_0 + T(a-y)\,dy = 2M_0 a + \frac{3Ta^2}{2} = 0$$

$$M_0 = \frac{-3Ta}{4} = \left\{T = \frac{P}{2}\right\} = -\frac{3Pa}{8}$$

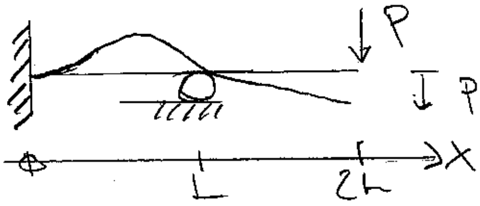
$$\delta = \int \frac{\partial M}{\partial T}\frac{\partial}{\partial M}\left[\frac{M^2}{2EI}\right]ds = \frac{1}{EI}\int_0^a (M_0 a + Ta^2)\,dx +$$

$$+\frac{1}{EI}\int_0^a M_0(a-y) + T(a-y)^2\,dy =$$

$$= \frac{1}{EI}\left(\frac{4Ta^3}{3} + \frac{3M_0 a^2}{2}\right) = \left\{\begin{array}{l} T = \frac{P}{2} \\ M_0 = -\frac{3Pa}{8}\end{array}\right\} = \frac{5Pa^3}{48EI}$$

$$2\delta = \frac{5Pa^3}{24EI}$$

13.6



$$\downarrow p = -w(2L) = \frac{7PL^3}{12EI}$$

(se uppg. 13.5)

Approximera w med ett 3e-gradspolynom och minimera potentiella energin.

$$U(\bar{w}) = \underbrace{\frac{1}{2}\int_0^{2L} EI(\bar{w}'')^2 dx}_{W_i} - P\left(-\bar{w}(2L)\right) \qquad \text{[Ekv (15-74)]}$$

$U(w) < U(\bar{w})$ $\forall \bar{w}$ som uppfyller villkor på w & w'

$w \approx w_a = C_0 + C_1 x + C_2 x^2 + C_3 x^3$

Vi vet att $w(0) = 0 \Rightarrow w_a(0) = 0 \Rightarrow C_0 = 0$

$w'(0) = 0 \Rightarrow w_a'(0) = 0 \Rightarrow C_1 = 0$

$w(L) = 0 \Rightarrow w_a(L) = 0 \Rightarrow C_2 L^2 + C_3 L^3 = 0$, $C_2 = -C_3 L$

$w_a = C_3(x^3 - Lx^2)$

$w_a(2L) = 4C_3 L^3$, $w_a'' = 6C_3 x - 2LC_3$

$$U(w_a) = \frac{EI}{2}\int_0^{2L} (6C_3 x - 2C_3 L)^2 dx + 4C_3 P L^3 =$$

$$= C_3^2 EI \cdot 28L^3 + 4C_3 P L^3$$

$$\frac{\partial U}{\partial C_3} = 56 EI L^3 C_3 + 4PL^3 = 0, \quad C_3 = \frac{-P}{14EI}$$

$$\left(\frac{\partial^2 U}{\partial C_3^2} = 56EIL^3 > 0\right)$$

$$w_a = \frac{PL^3}{14EI}\left(\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)^3\right)$$

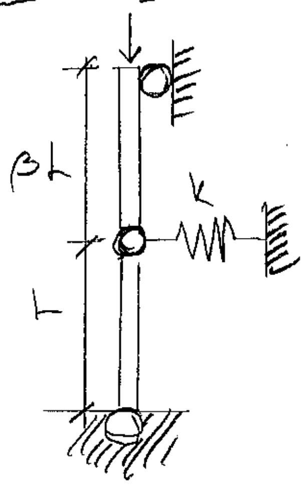
$$\delta_a = -w_a(2L) = \underline{\underline{\frac{2PL^3}{7EI}}}$$

$$\frac{\delta_a}{\delta} = \frac{2/7}{7/12} = \frac{24}{49} \approx \underline{0{,}49}$$

/end tis Lv6

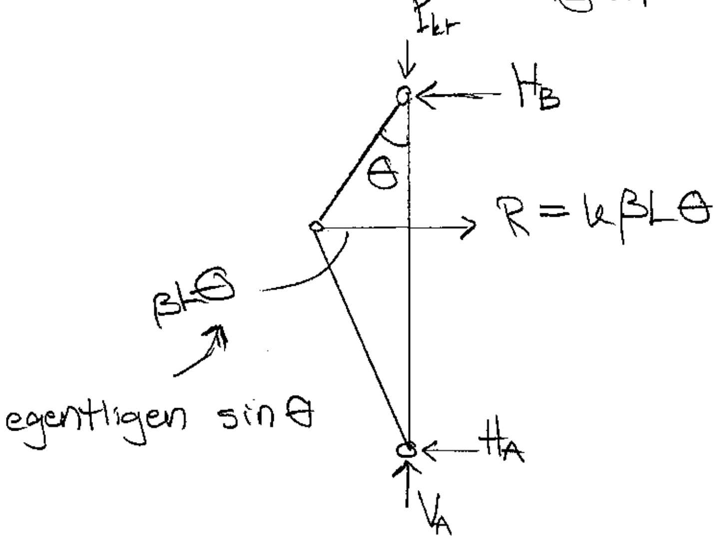
2011-05-13
Fredag Lv6

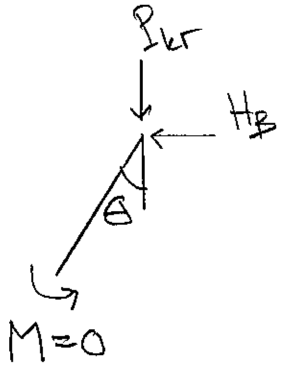
7,1d



Bestäm P_{kr}.

$\theta \ll 1$





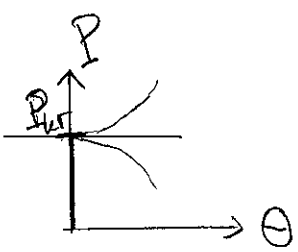
$\curvearrowright$: $P_{kr}\beta L\theta - H_B\beta L = 0$

$H_B = \theta P_{kr}$

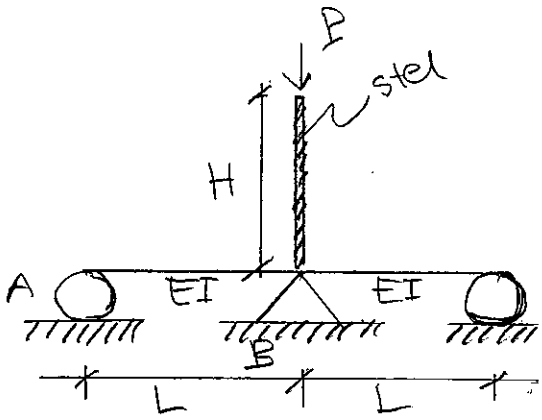
$\curvearrowleft A$: $R \cdot L - H_B(L + \beta L) = 0$

$\theta L\left(k\beta L - P_{kr}(1+\beta)\right) = 0$

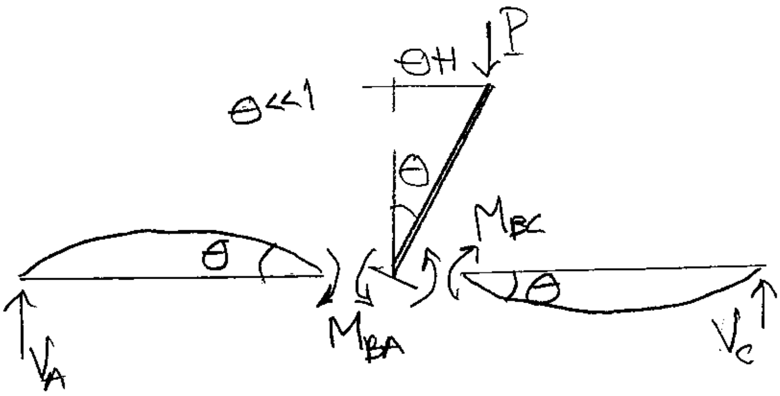
$\theta \neq 0$ möjligt om $P = \underline{P_{kr} = \dfrac{k\beta L}{1+\beta}}$



7.3



Bestäm P_{kr}.



F.s. sid 9: $\theta = \dfrac{M_{BA} \cdot L}{3EI}$, $\theta = \dfrac{M_{BC} L}{3EI}$ $\Rightarrow M_{BA} = M_{BC} = \dfrac{3EI}{L} \cdot \theta$

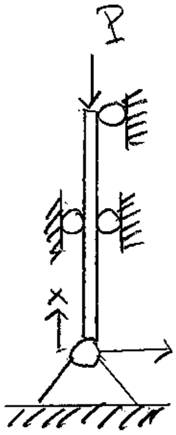
Momentjämvikt för pelaren:

$\curvearrowright B: \; P\theta H - M_{BA} - M_{BC} = 0$

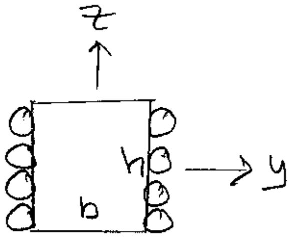
$\theta \left(PH - \dfrac{6EI}{L} \right) = 0$

$\theta \neq 0$ möjligt om $\underline{\underline{P = P_{kr} = \dfrac{6EI}{HL}}}$

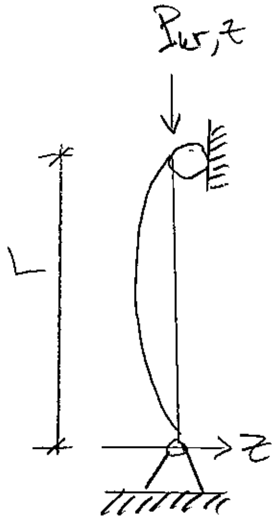
7.7



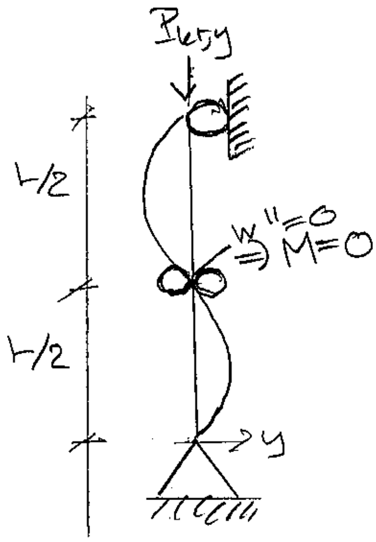
Tvärsnitt:



Bestäm h/b så P_{kr} blir samma för knäckning i (x,y)- och (x,z)-planet.



Euler 2: $$P_{kr,z} = \frac{\pi^2 E I_y}{L^2} = \left\{ I_y = \frac{bh^3}{12} \right\} = \frac{\pi^2 E b h^3}{12 L^2}$$



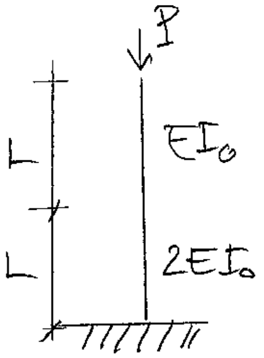
Euler 2 med längd $\frac{L}{2}$:

$$P_{kr,y} = \frac{\pi^2 E I_z}{\left(\frac{L}{2}\right)^2} = \left\{ I_z = \frac{hb^3}{12} \right\} = \frac{\pi^2 E h b^3}{3L^2}$$

$$P_{kr,y} = P_{kr,z} \Rightarrow \frac{hb^3}{3} = \frac{bh^3}{12}$$

$$\frac{h^2}{b^2} = 4 \qquad \underline{\underline{\frac{h}{b} = 2}}$$

7.20



a) Bestäm övre och undre gräns för P_{kr}.

Euler 1 med böjstyvhet EI_0: $P_{kr,undre} = \dfrac{\pi^2 EI_0}{4(2L)^2}$

—— 〃 —— $2EI_0$: $P_{kr,övre} = \dfrac{\pi^2 \cdot 2EI_0}{4\cdot(2L)^2}$

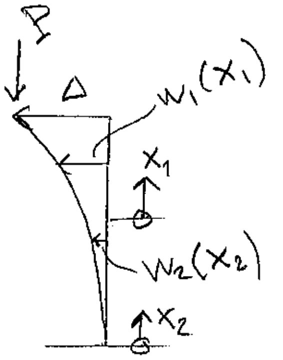
$$\underline{\frac{\pi^2 EI_0}{16L^2} < P_{kr} < \frac{\pi^2 EI_0}{8L^2}},$$

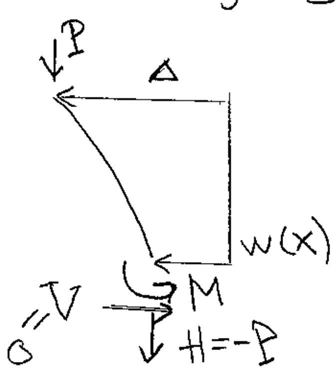
$$\frac{\pi^2}{16} < (nL)^2 < \frac{\pi^2}{8}, \quad \frac{\pi}{4} < nL < \frac{\pi}{2\sqrt{2}}$$

$\dfrac{PL^2}{EI_0}$

b) Bestäm P_{kr}; utnyttja att $M = -EIw''$

Snitta vid godtycklig höjd:





$\curvearrowleft: M + P(\Delta - w) = 0, \quad M = -EIw''$

$$w'' + n^2 w = \frac{P\Delta}{EI}, \quad n^2 = \frac{P}{EI}$$

$$\therefore \quad w_1'' + n_1^2 w_1 = \frac{P\Delta}{EI_0} = n_1^2 \Delta, \quad n_1^2 = \frac{P}{EI_0}$$

$$w_2'' + n_2^2 w_2 = \frac{P\Delta}{2EI_0} = n_2^2 \Delta, \quad n_2^2 = \frac{P}{2EI_0}$$

Part. lösning: ansätt $w_{1,p} = C \Rightarrow 0 + n_1^2 C = n_1^2 \Delta, \; C = \Delta$

Kar. ekv. : $r^2 + n_1^2 = 0 \Rightarrow r_{1,2} = \pm i n_1$

$$\Rightarrow w_{1,h} = A\sin(n_1 x_1) + B\cos(n_1 x_1)$$

$$w_1 = A\sin(n_1 x_1) + B\cos(n_1 x_1) + \Delta$$

$$w_2 = C\sin(n_2 x_2) + D\cos(n_2 x_2) + \Delta$$

$$w_1' = An_1\cos(n_1 x_1) - Bn_1\sin(n_1 x_1)$$

$$w_2' = Cn_2\cos(n_2 x_2) - Dn_2\sin(n_2 x_2)$$

Randvillkor: $w_2(0) = 0 \Rightarrow D + \Delta = 0, \; D = -\Delta$

$w_2'(0) = 0 \Rightarrow C = 0$

$$w_2 = \Delta(1 - \cos(n_2 x_2)), \quad w_2' = \Delta n_2 \sin(n_2 x_2)$$

Kontinuitetsvillkor (kompatibilitet)

$$w_1(0) = w_2(L) \Rightarrow B + \Delta = \Delta - \Delta\cos(n_2 L)$$

$$B = -\Delta\cos(n_2 L)$$

$$w_1'(0) = w_2'(L) \Rightarrow An_1 = \Delta n_2 \sin(n_2 L)$$

$$A = \frac{n_2}{n_1}\Delta\sin(n_2 L) =$$

$$= \frac{\Delta}{\sqrt{2}}\sin(n_2 L)$$

$$\Rightarrow \quad w_1(x_1) = \Delta\left(\frac{\sin(n_2L)}{\sqrt{2}}\sin(n_1x_1) - \cos(n_2L)\cos(n_1x_1) + 1\right)$$

$$= \left\{n_2 = \frac{n_1}{\sqrt{2}}\right\} = \Delta\left(\frac{\sin\left(\frac{n_1L}{\sqrt{2}}\right)\sin(n_1x)}{\sqrt{2}} - \cos\left(\frac{n_1L}{\sqrt{2}}\right)\cos(n_1x_1) + 1\right)$$

$$\Delta = w_1(L) \Rightarrow \underset{\neq 0}{\Delta}\underbrace{\left(\frac{\sin\left(\frac{n_1L}{\sqrt{2}}\right)\sin(n_1L)}{\sqrt{2}} - \cos\left(\frac{n_1L}{\sqrt{2}}\right)\cos(n_1L)\right)}_{=0} = 0$$

Lägsta positiva rot n_1L ger knäcklasten.
Enligt a) ligger roten: $\frac{\pi}{4} \doteq 0{,}785 < n_1L < 1{,}111 \doteq \frac{\pi}{2\sqrt{2}}$

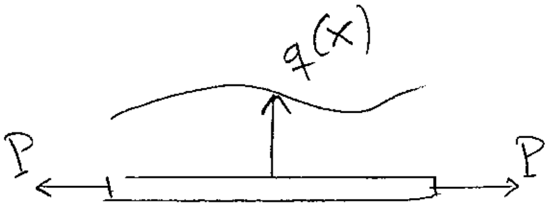
Numeriskt fås $n_1L \approx 1{,}0167$

$$(n_1L)^2 = \frac{P_{kr}}{EI_0}L^2 \qquad P_{kr} = \frac{(n_1L)^2 EI_0}{L^2} \approx \frac{\pi^2 EI_0}{9{,}55\,L^2}$$

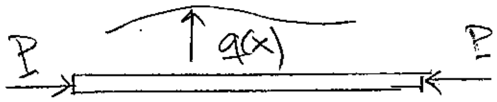
/end fre Lv6

2011-05-16
Måndag Lv 7

$$n^2 = \frac{P}{EI}$$

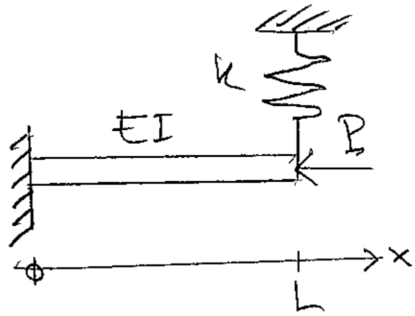


$$w^{IV} - n^2 w'' = q \Rightarrow w_h = A + Bx + C\cosh(nx) + D\sinh(nx)$$



$$w^{IV} + n^2 w'' = q \Rightarrow w_h = A + Bx + C\cos(nx) + D\sin(nx)$$

7.8



a) Bestäm en övre och undre gräns för den kritiska lasten P_{kr}.

a) $k = 0 \Rightarrow$ Euler 1 med $P_{kr} = \dfrac{\pi^2 EI}{4L^2}$

$k \to \infty \Rightarrow$ Euler 3 med $P_{kr} \approx \dfrac{2{,}05\pi EI}{L^2}$

$$0 < k < \infty \Rightarrow \frac{\pi^2 EI}{4L^2} < P_{kr} < \frac{2{,}05\pi^2 EI}{L^2}$$

b) Härled knäckekvationen. Lös den för $k=0$ och $k \to \infty$.

$$w(x) = A + Bx + C\cos(nx) + D\sin(nx), \quad n = \sqrt{\frac{P}{EI}} \quad (8\text{-}66)$$

$$w'(x) = B - Cn\sin(nx) + Dn\cos(nx)$$

Randvillkor:

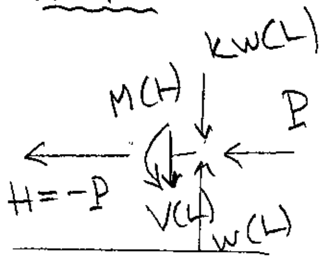
$w(0) = 0$ (1)

$w'(0) = 0$ (2)

$w''(L) = 0$ (3)

$w'''(L) + n^2 w'(L) - \frac{k}{EI} w(L) = 0$ (4)

$\underline{x = L}$:



$\curvearrowleft$: $M(L) = 0, \quad M = -EIw''$

$\downarrow$: $V(L) + kw(L) = 0$

8-59: $V = T + Hw' = -EIw''' - Pw'$

$$w''(x) = -Cn^2\cos(nx) - Dn^2\sin(nx)$$

$$w'''(x) = Cn^3\sin(nx) - Dn^3\cos(nx)$$

(1): $A + C = 0$, (2): $B + Dn = 0$

(3): $C\cos(nL) + D\sin(nL) = 0 \Rightarrow C = -D\tan(nL)$ om $\cos(nL) \neq 0$

$A = -C = D\tan(nL), \quad B = -Dn$

(4) $D\Big(\underbrace{-\tan(nL)\sin(nL)\cdot n^3 - n^3\cos(nL)}_{w'''(L)} -$

$\underbrace{-n^3 + n^3\tan(nL)\sin(nL) + n^3\cos(nL)}_{n^2 w'(L)} -$

$- \frac{k}{EI}\big(\tan(nL) - nL - \tan(nL)\cos(nL) + \sin(nL)\big)\Big) = 0$

$$D\underbrace{\left(n^3+\frac{k}{EI}\left(\tan(nL)-nL\right)\right)}_{=0}=0\ ,\quad D=0 \Rightarrow w\equiv 0$$

$$\Rightarrow \underbrace{n^3 EI\cos(nL)+k\left(\sin(nL)-nL\cos(nL)\right)=0}_{\text{knäckekvationen}}$$

$k=0 \Rightarrow \quad \cos(nL)=0 \quad \Rightarrow nL=\frac{\pi}{2},$

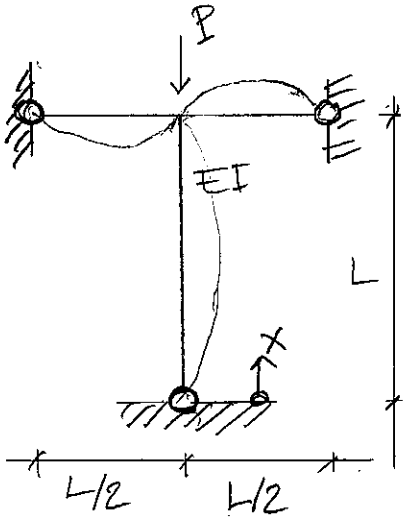
$(nL)^2=\frac{P}{EI}L^2=\frac{\pi^2}{4} \quad \Rightarrow P_{kr}=\frac{\pi^2 EI}{4L^2}$

$k\to\infty \qquad \frac{n^3 EI\cos(nL)}{k}+\left(\sin(nL)-nL\cos(nL)\right)=0$

$\tan(nL)=nL\ ,\ nL\approx 4{,}4934$

$P_{kr}=(nL)^2\frac{EI}{L^2}\approx\frac{2{,}05\pi^2 EI}{L^2}$

7.11

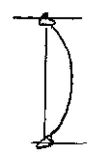


Försumma axial-deformationer

$\Rightarrow$ tryckkraft P i pelaren

Bestäm P_{kr}.

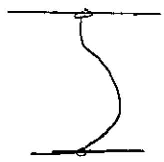
Led mellan delarna $\Rightarrow P_{kr} = \dfrac{\pi^2 EI}{L^2}$



Euler 2

$EI \to \infty$ för horisontella delen

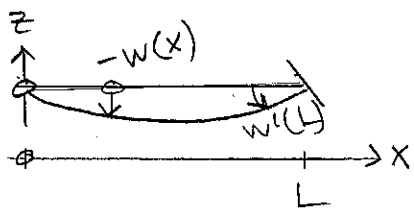
$\Rightarrow P_{kr} = \dfrac{2{,}05\,\pi^2 EI}{L^2}$



Euler 3

$$\frac{\pi^2 EI}{L^2} < P_{kr} < \frac{2{,}05\,\pi^2 EI}{L^2}$$

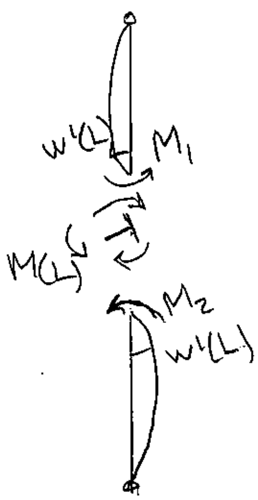
$$\pi^2 < \underbrace{\frac{P_{kr} L^2}{EI}}_{(nL)^2} < 2{,}05\,\pi^2 , \qquad \pi < nL < \sqrt{2{,}05}\,\pi \approx 4{,}49$$



F.s. sid 9:

$$w'(L) = \frac{M_1 \frac{L}{2}}{3EI}$$

$$w'(L) = \frac{M_2 \frac{L}{2}}{3EI}$$



$$\Rightarrow M_1 = M_2 = \frac{6EI}{L} w'(L)$$

$$\curvearrowleft : \; M(L) - M_1 - M_2 = 0$$

$$\underline{\underline{M(L) = \frac{12EI}{L} w'(L) = -EI\,w''(L)}}$$

Randvillkor:

$$w(0) = 0 \qquad (1)$$
$$w''(0) = 0 \qquad (2)$$
$$w(L) = 0 \qquad (3)$$
$$w''(L) + \frac{12}{L} w'(L) = 0 \qquad (4)$$

För den tryckta balken har vi

$$w = A + Bx + C\cos(nx) + D\sin(nx)$$

$$w' = B - Cn\sin(nx) + Dn\cos(nx)$$

$$w'' = -Cn^2\cos(nx) - Dn^2\sin(nx)$$

$$(1): A + C = 0, \quad (2): -Cn^2 = 0 \Rightarrow A = C = 0$$

$$(3): BL + D\sin(nL) = 0, \quad B = -D\frac{\sin(nL)}{L}$$

$$(4): D\Big(\underbrace{-n^2\sin(nL)}_{w''(L)} \underbrace{- \frac{12\sin(nL)}{L^2} + \frac{12n}{L}\cos(nL)}_{\frac{12}{L}\cdot w'(L)}\Big) = 0$$

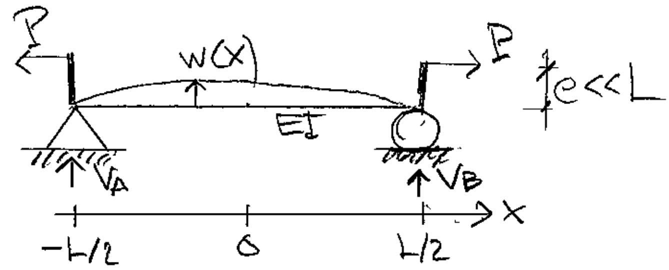
$$-Dn^2\cos(nL)\underbrace{\left(\tan(nL) + \frac{12}{(nL)^2}\tan(nL) - \frac{12}{nL}\right)}_{=0} = 0, \quad D = 0 \Rightarrow w \equiv 0$$

$$\left(12 + (nL)^2\right)\tan(nL) = 12nL, \quad \tan(nL) = \frac{12(nL)}{12 + (nL)^2}$$

Lägsta positiva rot ligger i intervallet $(\pi, \sqrt{2{,}05}\,\pi)$. Numeriskt hittar man

$$nL \approx 4{,}1811; \quad P_{kr} = (nL)^2\frac{EI}{L^2} \approx \frac{1{,}77\pi^2 EI}{L^2}$$

7.26

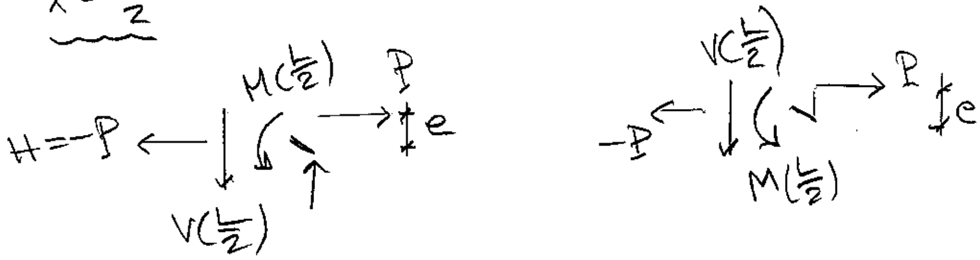


a) $w(x)$ och $M(x)$ söks

$$w^{IV} - n^2 w'' = 0, \quad -\frac{L}{2} < x < \frac{L}{2}, \quad n^2 = \frac{P}{EI}$$

$$w\left(-\frac{L}{2}\right) = w\left(\frac{L}{2}\right) = 0$$

$x = \frac{L}{2}$



$$\curvearrowleft : M\left(\frac{L}{2}\right) - Pe = 0, \quad -EIw''\left(\frac{L}{2}\right) = Pe$$

$$w''\left(-\frac{L}{2}\right) = w''\left(\frac{L}{2}\right) = -\frac{P}{EI}e = -n^2 e$$

$$w = A + Bx + C\cosh(nx) + D\sinh(nx)$$

Symmetri: $w(x) = w(-x) \Rightarrow B = D = 0$

$$w\left(\pm\frac{L}{2}\right) = A + C\cosh\left(\frac{nL}{2}\right) = 0, \quad A = -C\cosh\left(\frac{nL}{2}\right)$$

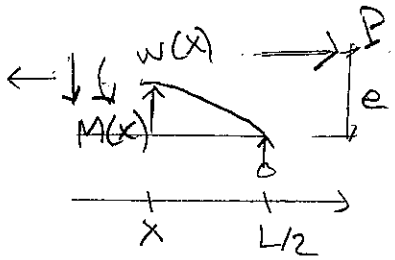
$$w'' = Cn^2\cosh(nx)$$

$$w''\left(\pm\frac{L}{2}\right) = Cn^2\cosh\left(\frac{nL}{2}\right) = -n^2 e$$

$$\Rightarrow \quad C = \frac{-e}{\cosh\left(\frac{nL}{2}\right)} \qquad \underline{\underline{w(x) = e\left(1 - \frac{\cosh(nx)}{\cosh\left(\frac{nL}{2}\right)}\right)}}$$

$$M(x) = -EIw'' = -EIC\overset{\frac{P}{EI}}{n^2}\cosh(nx) = Pe\,\frac{\cosh(nx)}{\cosh\left(\frac{nL}{2}\right)}$$

Alt:



$$\curvearrowleft x:\quad M(x) - P(e - w(x)) = 0$$

$$M(x) = Pe\left(1 - 1 + \frac{\cosh(nx)}{\cosh\left(\frac{nL}{2}\right)}\right)$$

b) $w(0)$ a $M(0)$ då $P \to \infty$

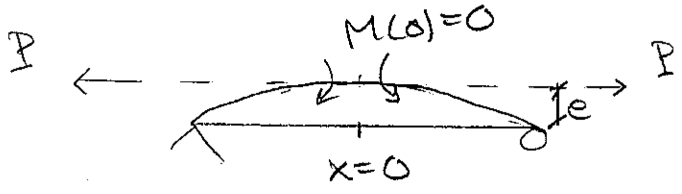
$$\frac{w(0)}{e} = 1 - \frac{1}{\cosh\left(\frac{nL}{2}\right)} \quad ; \quad n = \sqrt{\frac{P}{EI}} \to \infty \text{ då } P \to \infty$$

$$\cosh\left(\frac{nL}{2}\right) \to e^{\frac{nL}{2}} \text{ då } n \to \infty$$

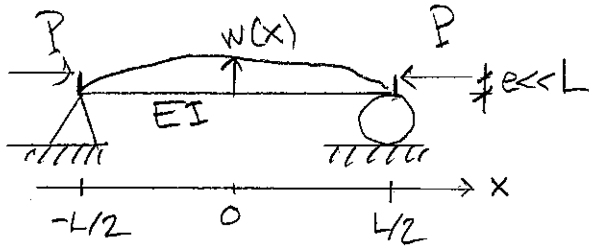
$$\lim_{n \to \infty} \frac{w(0)}{e} = 1, \quad w(0) = e \text{ då } P \to \infty$$

$$\frac{M(0)}{Pe} = \frac{1}{\cosh\left(\frac{nL}{2}\right)} \to 0 \text{ då } P \to \infty \qquad \left(M(0) = \frac{Pe}{\cosh\left(\frac{nL}{2}\right)} \to 0\right)$$

$$\Longrightarrow M(0) \equiv 0 \text{ då } P \to \infty$$



7.27

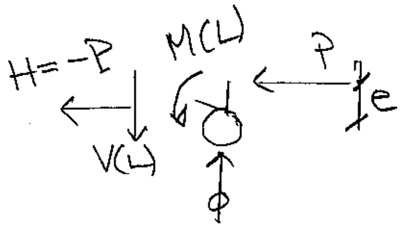


Bestäm $w(x)$ och $M(x)$.

Bestäm P_{kr}

$$\begin{cases} w^{IV} + n^2 w'' = 0 & -\frac{L}{2} < x < \frac{L}{2}, \quad n^2 = \frac{P}{EI} \\ w\left(\pm\frac{L}{2}\right) = 0 \end{cases}$$

$x = \frac{L}{2}$



$$\curvearrowleft : M\left(\tfrac{L}{2}\right) = -Pe = -EIw''\left(\tfrac{L}{2}\right)$$

$$\left\{ w''\left(\pm\frac{L}{2}\right) = n^2 e \right.$$

8-66: $w = A + Bx + C\cos(nx) + D\sin(nx) =$

$= \left\{ \begin{array}{l} \text{sym: } w(x) = w(-x) \\ \Rightarrow B = D = 0 \end{array} \right\} = A + C\cos(nx)$

$w\left(\pm\frac{L}{2}\right) = 0: \quad A + C\cos\left(\frac{nL}{2}\right) = 0, \quad A = -C\cos\left(\frac{nL}{2}\right)$

$w''\left(\pm\frac{L}{2}\right) = n^2 e: \quad -Cn^2\cos\left(\frac{nL}{2}\right) = n^2 e \Rightarrow C = \dfrac{-e}{\cos\left(\frac{nL}{2}\right)}$

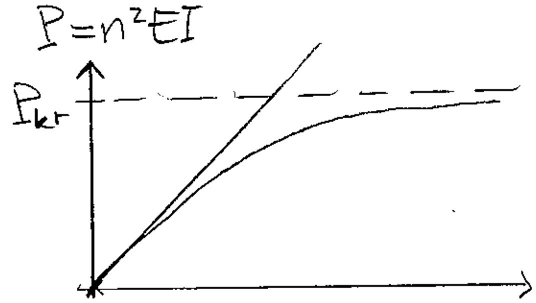
$\Rightarrow A = e$

$$\underline{w(x) = e\left(1 - \frac{\cos(nx)}{\cos\left(\frac{nL}{2}\right)}\right)}$$

$$M(x) = -EIw''(x) = -EIe\,\frac{\cos(nx)}{\cos\left(\frac{nL}{2}\right)} \cdot n^2 = \underline{-Pe\,\frac{\cos(nx)}{\cos\left(\frac{nL}{2}\right)}}$$

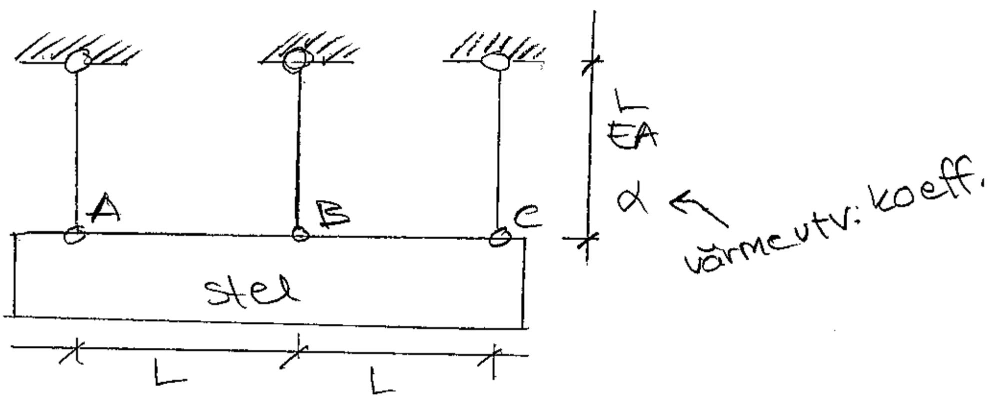
$|w| \to \infty$ då $\cos\left(\frac{nL}{2}\right) \to 0$, $\frac{nL}{2} = \frac{\pi}{2}$

$(nL)^2 = \pi^2 = \frac{P_{kr}}{EI} L^2$ $\qquad$ $\underline{P_{kr} = \frac{\pi^2 EI}{L^2}}$ (Euler 2)



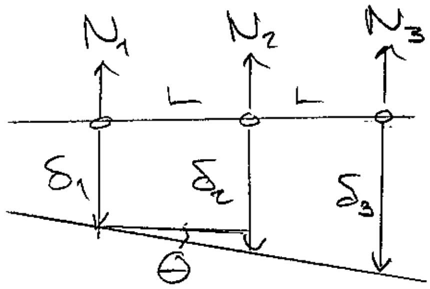
$$-\frac{w(0)}{e} = \frac{1}{\cos\left(\frac{nL}{2}\right)} - 1$$

2004-05-28, #1 (tentauppg.)



Högra stången värms ΔT, bestäm spänningarna.

Frilägg:



$$\tan\theta = \underbrace{\frac{\delta_2-\delta_1}{L} = \frac{\delta_3-\delta_1}{2L}}_{\delta_2 = \frac{\delta_1+\delta_3}{2}}$$

$$\delta_1 = \frac{N_1 L}{EA} \qquad \delta_3 = \frac{N_3 L}{EA} + \alpha L \Delta T$$

$$N_1 = \frac{EA}{L}\delta_1 \qquad N_2 = \frac{EA}{L}\delta_2 = \frac{EA}{L}\left(\frac{\delta_1+\delta_3}{2}\right)$$

$$N_3 = \frac{EA}{L}(\delta_3 - \alpha L \Delta T)$$

$$\uparrow:\ N_1+N_2+N_3=0 \qquad \delta_1 + \frac{\delta_1+\delta_3}{2} + \delta_3 = \alpha L \Delta T$$

$$\delta_1+\delta_3 = \frac{2}{3}\alpha L \Delta T$$

$$\overset{\curvearrowleft}{B}:\ N_1 L - N_3 L = 0 \Rightarrow \delta_1 - \delta_3 + \alpha L \Delta T = 0$$

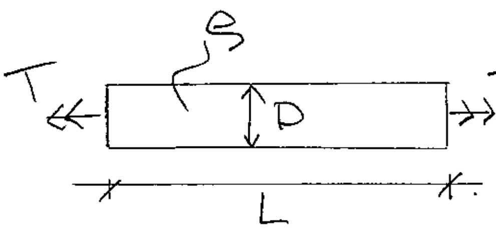
ger $\delta_1 = \frac{-1}{6}\alpha L \Delta T$, $\delta_3 = \frac{5}{6}\alpha L \Delta T$

$$\delta_2 = \frac{\delta_1+\delta_3}{2} = \frac{1}{3}\alpha L \Delta T$$

$$\sigma_1 = \frac{N_1}{A} = \frac{E}{L}\delta_1 = \underline{\underline{\frac{-\alpha E \Delta T}{6}}} \qquad \sigma_2 = \frac{N_2}{A} = \underline{\underline{\frac{\alpha E \Delta T}{3}}}$$

$$\sigma_3 = \frac{N_3}{A} = \frac{E}{L}(\delta_3 - \alpha L \Delta T) = \underline{\underline{\frac{-\alpha E \Delta T}{6}}}$$

2008-01-12, #5

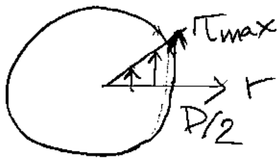


$T = M_{vs}$ ger $\tau_{max} = \tau_s$

Bestäm M_{max}

$\downarrow$

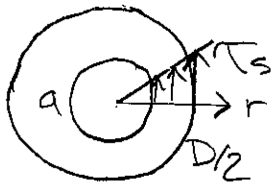
(1)



$A_1 = \frac{\pi D^2}{4}$

$M_1 = \rho L A_1$

(2)



$A_2 = \pi\left(\frac{D^2}{4} - a^2\right)$

$M_2 = \rho L A_2 = \alpha M_1 = \alpha \rho L A_1$

$A_2 = \alpha A_1 \Rightarrow a = \frac{D}{2}\sqrt{1-\alpha}$

Fall 1:

$$\tau_{max,1} \overset{6-14}{=} \frac{2 M_{vs} b}{\pi(b^4 - a^4)} = \left\{ \begin{matrix} b = \frac{D}{2} \\ a = 0 \end{matrix} \right\} = \frac{16 M_{vs}}{\pi D^3} = \tau_s$$

Fall 2:

$$\tau_{max,2} = \frac{2 M_{max}\left(\frac{D}{2}\right)}{\pi\left(\left(\frac{D}{2}\right)^4 - \left(\frac{D}{2}\right)^4 (1-\alpha)^2\right)} = \frac{16 M_{max}}{\pi D^3 (2\alpha - \alpha^2)} = \tau_s$$

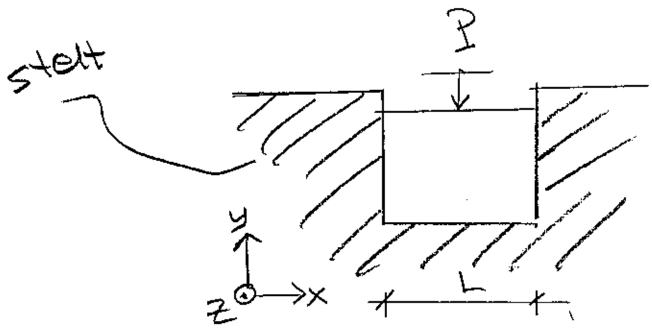
$$\Rightarrow \frac{16 \cdot M_{max}}{\pi D^3 (2\alpha - \alpha^2)} = \frac{16 M_{vs}}{\pi D^3} \Rightarrow \underline{\underline{M_{max} = (2\alpha - \alpha^2) M_{vs}}}$$

$$\left(\begin{matrix} \text{T.ex. } \alpha = \frac{1}{2} \\ M_{max} = \frac{3}{4} M_{vs} \end{matrix} \right)$$

/end tis Lv7

6/3 2006, #5 (tentauppgift)

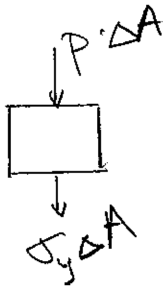
2011-05-18
Onsdag LV7



Bestäm τ_{max}.

Ingen friktion: $\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$

Fri expansion i z-led
$\Rightarrow \sigma_z = 0$



$\downarrow: \sigma_y = -p$

$$\varepsilon_x \cdot L = 0 \Rightarrow \varepsilon_x = \frac{1}{E}\underbrace{\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)}_{=0} = 0$$

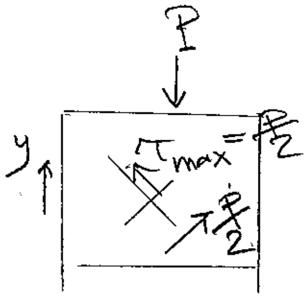
$$\sigma_x = \nu\sigma_y = -\nu p$$

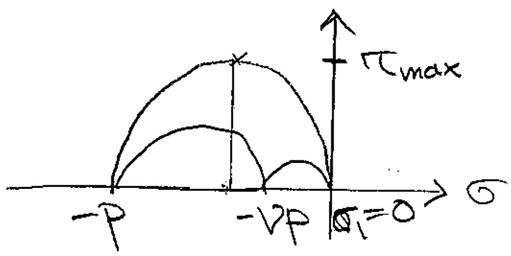
$\tau_{ij} = 0 \Rightarrow \sigma_x, \sigma_y, \sigma_z$ är huvudsp.

$\sigma_1 = 0$, $\sigma_2 = -\nu p$, $\sigma_3 = -p$ $\quad (-1 < \nu < \frac{1}{2})$

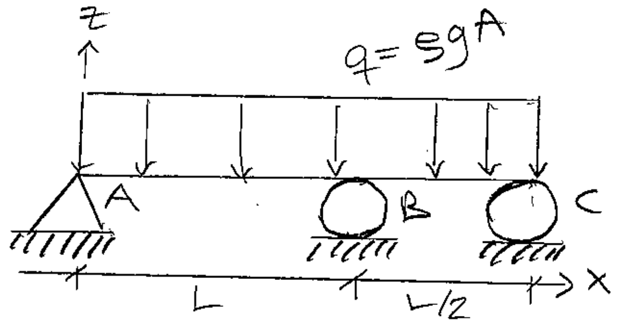
antagit $\nu > 0$

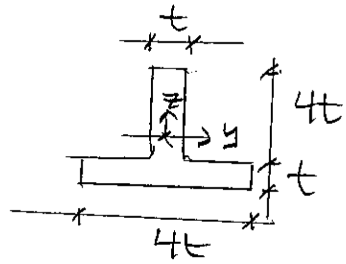
$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{p}{2}$$





#1



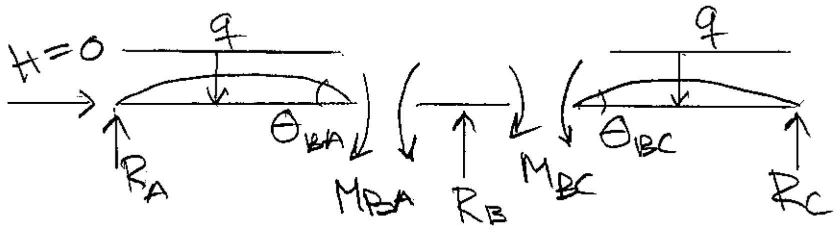


$\boxed{\sigma_{max} = ?}$

$g = 9.81\ m/s^2$, $L = 2m$, $t = 3cm$,
$\rho = 7850\ kg/m^3$, $E = 210\ GPa$

$$\text{Egentyngd} = Mg = L_{tot} A \rho g \ , \quad q = \frac{Mg}{L_{tot}} = \rho g A$$

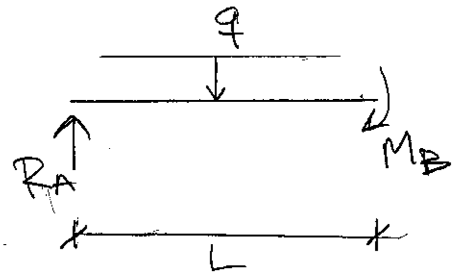
$$\sigma = \cancel{\frac{N}{A}} + \frac{M_y z}{I_y} - \cancel{\frac{M_z y}{I_z}}$$



F.S. sid. 9:
$$\theta_{BA} = M_B \frac{L}{3EI} - q\frac{L^3}{24EI}$$
$$\theta_{BC} = M_B \frac{L/2}{3EI} - q\frac{(L/2)^3}{24EI}$$

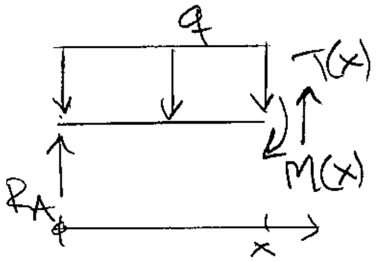
Kompatibilitet: $\theta_{BA} + \theta_{BC} = 0$

$$\Rightarrow M_B = \frac{3qL^2}{32} \approx 0{,}09375 qL^2$$



$$\overset{\curvearrowright}{B}: R_A \cdot L + M_B - qL \cdot \frac{L}{2} = 0$$

$$R_A = \frac{13qL}{32}$$



$$\overset{\curvearrowright}{x}: R_A \cdot x - M(x) - q\frac{x^2}{2} = 0$$

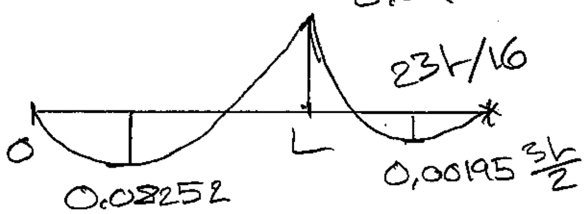
$$M(x) = \frac{qx^2}{2} - \frac{13qLx}{32}$$

$$\frac{dM}{dx} = T = qx - \frac{13qL}{32} = 0 \Rightarrow x = \frac{13L}{32}$$

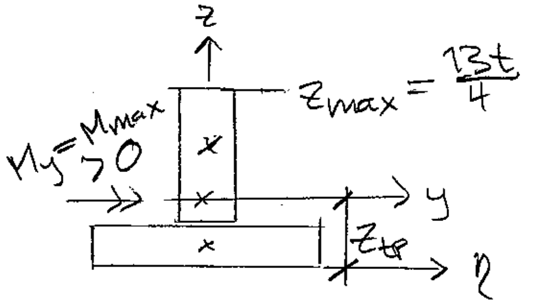
$$M\left(\frac{13L}{32}\right) = -0{,}08252\,qL^2$$

$$0.09375 = M_{max} = \frac{3qL^2}{32}$$

M(x):



$\times\, qL^2$



$$S_\eta = A z_{tp} = 4t^2 \cdot 3t + 4t^2 \cdot \frac{t}{2}$$

$$= \underline{\underline{14t^3}}$$

$$z_{tp} = \frac{7t}{4}$$

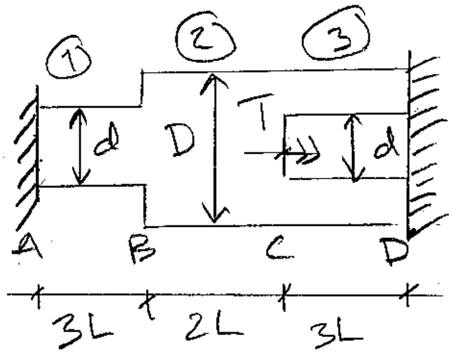
$$I_y = \frac{t(4t)^3}{12} + 4t^2(3t - z_{tp})^2 + \frac{4t \cdot t^3}{12} + 4t^2\left(z_{tp} - \frac{t}{2}\right)^2$$

$$= \frac{109}{6}t^4$$

$$|\sigma_{max}| = \sigma_{max} = \frac{\frac{3qL^2}{32}\cdot\frac{13t}{4}}{\frac{109}{6}t^4} = \left\{ q = \rho g A,\ A = 8t^2 \right\} =$$

$$= \frac{117}{872}\,\frac{\rho g L^2}{t} \approx \underline{\underline{1{,}38\ \text{MPa}}}$$

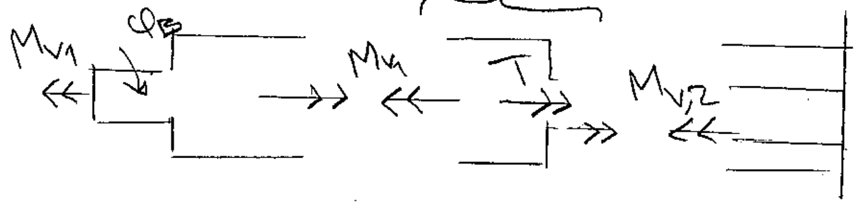
8/1 1992, # 2



$D = 1{,}5d$

Bestäm vridvinkeln i B.

$$\rightrightarrows :\ T + M_{v,2} - M_{v,1} = 0 \qquad (1)$$



$\underline{6\text{-}11}$: $\varphi = \frac{M_v L}{GK}$

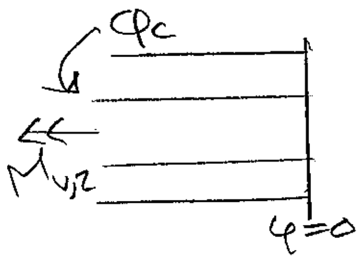
$\underline{\text{Delen (1)}}$: $$\varphi_B = \frac{M_{v,1}\cdot 3L}{GK_1} = \left\{ K_1 = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32} \right\}$$

$$= \frac{96\, M_{v,1} L}{\pi G d^4} \qquad (2)$$

$\underline{\text{Delen (2)}}$: $$\varphi_C = \varphi_B + \frac{M_{v,1}\cdot 2L}{GK_2} =$$

$$= \left\{ K_{②} = \frac{\pi D^4}{32},\ d = \frac{2}{3}D \right\} = 550 \frac{M_{v,1} L}{\pi G D^4} \qquad (3)$$

Delen ③:



$$\varphi_c = \frac{-M_{v,2} \cdot 3L}{G K_{③}} = \left\{ K_{③} = \frac{\pi}{2}\left(\left(\frac{D}{2}\right)^4 - \left(\frac{d}{2}\right)^4 \right) \right\} =$$

$$= \frac{-7776}{65} \cdot \frac{M_{v,2} L}{\pi G D^4}$$

$$(3) = (4) \Rightarrow M_{v,2} = \frac{-35750}{7776} M_{v,1}$$

In i (1) $\rightarrow$ $M_{v,1} = \frac{7776}{43526} T \approx 0{,}179\, T$

(2) ger då $\varphi_B \approx 5{,}46 \frac{TL}{Gd^4}$

/end ons LV7
end Hållf. räkneövn.