EXEMPELTENTA FUF040, Kvantfysik

Kurs: FUF040, Kvantfysik, HT18
Tid:
Plats:
Tillåtna hjälpmedel: Physics Handbook/Beta
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1. A particle with mass m and energy $E = V_0$ is moving in one dimension under the influence of the potential energy:
$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 \leq x \leq a \\ 0 & x > a \end{cases}$$
where $V_0 > 0$ is a constant.
   (a) Solve the stationary Schrödinger equation in the three regions, as given above.
   (b) Use the boundary conditions to obtain the relations between the constants appearing in part a of the problem.
   (c) What is the probability of transmission for a particle coming from the $x < 0$ region and moving to the right?

2. Determine the expectation values of $\langle x \rangle$, $\langle \hat{p} \rangle$, $\langle x^2 \rangle$, and $\langle \hat{p}^2 \rangle$, for the nth state of the one-dimensional harmonic oscillator, by expressing x and $\hat{p}$ in terms of the creation and annihilation operators. Also, show that Heisenberg's uncertainty principle is satisfied.

3. A hydrogen atom is the mixed state
$$\Psi(r, \theta, \phi, t) = N \left(3e^{-iE_1 t/\hbar} \psi_{100} + e^{-iE_2 t/\hbar} \psi_{200} \right) \quad (1)$$
where $\psi_{n\ell m} = R_{n\ell}(r) Y_\ell^m(\theta, \phi)$, and E_1 and E_2 are the two consecutively lowest energy levels of the hydrogen atom.
   (a) Determine the normalization constant N.
   (b) What is the expectation value of the energy, in terms of the lowest energy E_1?
   (c) What is the expectation value of the radial position r?

4. Prove the following statements:
   (a) The eigenvalues of hermitian operators are real.
   (b) The Hamiltonian operator
$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) \quad (2)$$
is hermitian.
   (c) $[x^n, \hat{p}] = i\hbar n x^{n-1}$.

5. An electron is in the spin state

$$\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}, \tag{3}$$

in the basis of the eigenstates of $\hat{S}_z$.

(a) Normalize the state.

(b) Determine the eigenstates of

$$\hat{S}_y \rightarrow \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{4}$$

(c) Express the electron spin state χ in terms of the eigenstates from (b).

(d) What is the probability to find the electron in the spin-down state, if you measure $\hat{S}_y$?

6. A particle is moving in a potential $V(\mathbf{r})$.

(a) Show

(b) Derive the relation between the time change of the *expectation value* of the angular momentum $\mathbf{L}$ and the *expectation value* of the torque $\mathbf{N} = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (-\nabla V)$.

(c) What happens with the evolution of the expectation value of the angular momentum when the potential V is spherically symmetric?

Lycka till!